C. R. DASGUPTA

A HANDBOOK OF PRACTICAL PHYSICS

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A HANDBOOK OF PRACTICAL PHYSICS

For B. Sc. (PASS)

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Preface to the second edition

I have the pleasure in presenting the second edition of the book entitled 'A Handbook of Practical Physics' to the readers. Copies of the first edition having been exhausted in a reasonably short time, the book may be supposed to have gained popularity among the students. In this edition, the syllabus of the newly established Vidyasagar University has been fully covered.

In this edition, some mistakes that crept in the first edition, have been rectified. I hope that this edition will be able to meet the laboratory requirements of the students better than before.

Department of Physics
City College, Calcutta
15th May 1987

C. R. Dasgupta

Preface to the first edition

The book is written according to the B.Sc. (Pass) Physics practical syllabus of Indian Universities—particularly the Universities of Calcutta, Burdwan and North Bengal. In writing the manuscript of the book, I have always kept in mind the deficiencies of the common students which I experienced during my long association with the students in the laboratory. Detailed instructions about each experiment, the possible sources of error, the precautions to be taken against those sources etc. have been thoroughly discussed in the text. In following the practical syllabus of degree course, the students will have to handle some instruments like microscope, barometer etc. with which they are not previously acquainted. In order to acquaint them with such instruments, I have given a few preliminary experiments—like the determination of the bore of a capillary by microscope etc—at the beginning of the book. It will be good for the students if they perform these experiments before going in for the actual syllabus.

The first edition, as it has been hurriedly passed through the press, may contain mistakes both printing and otherwise. I shall feel very much obliged if such mistakes are brought to my notice. I shall feel happy if the book is found helpful by the students for whom it is intended.

Department of Physics
City College, Calcutta
10th January, 1985

C. R. Dasgupta

Syllabus of Practical (Pass) Physics

Calcutta University

1. Determination of gamma by Searle's method.
2. Determination of Y by flexure of beam (single length only).
3. Determination of modulus of rigidity by (a) Statical method (b) Dynamical method.
4. To determine the M.I. of a body about an axis passing through its centre of gravity.
5. Measurement of surface tension of water with the help of capillary tubes (capillary tubes to be supplied).
6. Viscosity of water by Poiseuille's method (diameter of the tube to be measured by microscope).
7. Coefficient of linear expansion of a metal by (a) Travelling microscope (b) Pullinger's method by using optical lever.
8. Pressure coefficient of air.
9. Refractive index of liquid and the material of convex lens by using a convex lens and a plane mirror.
10. Focal length of a concave lens by method of combination.
11. Adjustment of spectrometer for parallel rays by Schuster's method and to determine R.I of the material of a prism by minimum deviation method.
12. To draw i—δ curve of a prism by a spectrometer and hence to find out the angle of minimum deviation.
13. To determine the radius of curvature of a lens by Newton's ring method.
14. To calibrate a polarimeter and hence determine the concentration of sugar solution (Mother solution to be supplied).
15. To determine the moment of a magnet and horizontal component of Earth's magnetic field by deflection magnetometer.
16. Dip by Earth inductor.
17. Determination of resistance per unit length of bridgewire by Carey Foster's method and hence determination of unknown resistance.
18. Determination of resistance of a galvanometer by the method of half deflection.
19. Determination of figure of merit of a galvanometer.
20. Measurement of current by potentiometer (using P.O. Box for measurement of resistance of potentiometer).
21. Determination of temperature coefficient of the material of a coil of wire using a metre bridge.
22. To determine the reduction factor of a tangent galvanometer using copper voltameter.
23. I—V characteristics of a diode (Vacuum tube and *p—n* junction).
24. Static characteristics of a triode and hence determine amplification factor.
25. Characteristics of a common-emitter type of transistor.

Burdwan University

[It is expected that the students should show proficiency in operating Physical balance and should know use of diagonal scale, vernier scale, vernier slide callipers, screw gauge and spherometer. They should also know to read the barometer and make corrections as well as to know to weigh a body by the method of oscillation. Besides they will be required to perform the following experiments.]

Group A

1. To determine the Young's modulus of the material of a wire by Searle's method.
2. To determine the rigidity modulus of the material of a wire by (a) statical method and (b) by dynamical method.
3. To determine the moment of inertia of a body about an axis passing through its centre of gravity.
4. To determine the frequency of a tuning fork by sonometer.
5. To draw *n–l* curve and hence to find the frequency of an unknown tuning fork.
6. To determine the density of the material of wire by sonometer method.
7. To determine the frequency of a tuning fork by Melde's experiments (Transverse arrangement).
8. To measure the coefficient of linear expansion of a metal (a) by the travelling microscope (b) by Pullinger's apparatus using an optical lever.
9. To determine the coefficient of apparent expansion of a liquid by weight thermometer method.
10. To determine the coefficient of absolute expansion of mercury by Dulong and Petit's method.
11. To determine the pressure coefficient of air.
12. To determine the volume coefficient of air.
13. To determine the thermal conductivity of a metal by Searle's method.

Group B

14. To determine the R.I. of a liquid by a travelling microscope.
15. To determine the R.I. of a liquid by lens and mirror method.
16. To determine the focal length of a concave lens (i) by combination method (ii) by auxiliary lens method (iii) of a convex lens by displacement method.
17. To adjust a spectrometer by Schuster's method and to determine the refractive index of a prism by the method of minimum deviation.
18. To determine the R.I. of the material of a thin prism by the method of normal incidence.
19. To determine the earth's horizontal component of magnetic field and the moment of the bar-magnet by the magnetometer method.
20. To determine the value of the resistance of a wire by a metre bridge making end-corrections and hence to calculate the specific resistance of the material.
21. To determine the resistance per unit length of the wire of a metre-bridge by Carey Foster's method.
22. To find the mechanical equivalent of heat by Joule's calorimeter.
23. To determine the temperature coefficient of the material of a coil using a metre-bridge.
24. To determine the reduction factor of a tangent galvanometer.
25. To determine the resistance of a galvanometer by half-deflection method.
26. To determine the figure of merit of a moving coil galvanometer.
27. To determine the high resistance by substitution method using a shunt box.
28. To find the value of a low resistance by drop of potential.
29. To measure the e.m.f. of a cell by potentiometer using a milliammeter.
30. To measure a current by potentiometer using a known low resistance (Resistance of the potentiometer to be measured by P.O. Box).
31. To draw the characteristic curves of a triode valve.

[*N.B.* Two experiments are to be performed in the final examination. The duration of the examination is six hours.]

North Bengal University

Each candidate will perform two experiments, one from Group A and another from Group B (Time = 6 hours).

Group A

1. To determine the rigidity modulus of a wire by (a) statical method (b) dynamical method.
2. To determine the moment of inertia of a body about an axis passing through its centre of gravity.
3. To determine the surface tension by capillary tube method. (Capillary tube to be supplied).
4. To determine the viscosity of a liquid by flow through a capillary tube.
5. To draw the *n*—*l* curve and hence to find the frequency of an unknown fork.
6. To determine the frequency of a tuning fork by Melde's experiment.

7. & 8. To determine *H* and *M* by magnetometers (two experiments).

9.& 10. To measure the coefficient of linear expansion of a metal by (i) Travelling microscope and (ii) by Pullinger's apparatus using an optical lever.

11. To determine the absolute expansion of mercury by Dulong and Petit's method.
12. To determine the pressure coefficient of air.
13. To determine the volume coefficient of air.
14. To determine the thermal conductivity of a metal by Searle's method.
15. To determine the refractive index of a liquid by the travelling microscope.
16. To find the power of a convex lens by displacement method.
17. To determine the focal length of a concave lens by auxiliary lens method.

Group B

18. To adjust a spectrometer by Schuster's method and to determine the r. i. of a prism by the method of minimum deviation.
19. To draw the i—δ curve of a prism by spectrometer and hence to find the angle of minimum deviation.
20. To determine the r.i. of the material of a thin prism by the method of normal incidence.
21. To determine the width of a narrow slit by diffraction method.
22. To determine the wave length of a monochromatic light by Newton's ring method.
23. To determine the value of resistance of a wire by Wheatstone bridge method making end-corrections and hence to calculate the specific resistance of the wire.
24. To find the figure of merit of a moving coil galvanometer and determine the resistance of the galvanometer by half-deflection method.
25. To determine the value of low resistance by the fall of potential method.
26. To determine the resistance per unit length of the wire of a metre bridge by Carey Fosters' method.
27. To measure the current by potentiometer using a low resistance.
28. To determine the temperature coefficient of the material of a coil using a metre bridge.
29. To draw the characteristic curves of a triode valve.
30. To measure the thermo-e.m.f. with a potentiometer and draw *E*—*T* curve. (Thermo-couple to be supplied).
31. To study the characteristics of a junction diode and transistors.

Vidyasagar University

1. To determine modulus of elasticity : (a) Y by flexure of a beam (b) Modulus of rigidity by dynamical method.
2. To determine the surface tension of a liquid by capillary tube method.
3. To determine the coefficient of viscosity of water by Poiseullie's flow method.
4. To determine the coefficient of linear expansion of a metal by a travelling microscope and (by optical lever).
5. To determine the pressure coefficient of air.
6. To determine the refractive index of (i) a liquid and (ii) the material of a convex lens.
7. To determine the focal length of a concave lens by method of combination.
8. To adjust a spectrometer for parallel rays by Schuster's method and to determine the refractive index of the material of a prism.
9. To draw the $(i-\delta)$ curve of a prism and hence to determine the minimum deviation angle.
10. To determine the radius of curvature of a lens by Newton's ring method.
11. To use a polarimeter to determine the strength of the solution of an optically active substance (e.g. sugar etc).
12. To determine by a deflection magnetometer (i) the moment of a magnet and (ii) the horizontal component of Earth's magnetic field.
13. To determine by Carey Foster's method (i) resistance/cm of the bridge wire and (ii) unknown resistance.
14. To determine for a moving coil galvanometer (i) the resistance and (ii) figure of merit.
15. To use a potentiometer to measure (i) current (ii) potential difference and (iii) resistance.
16. To determine the reduction factor of a tangent galvanometer using a copper voltameter.
17. To study the (I—V) characteristics of a (i) vacuum tube diode (ii) p—n junction diode.
18. To study the static characteristics of a triode and hence to determine the amplification factor.
19. To determine the characteristics of a common emitter transistor.

Miscellaneous :

1. Use of common measuring instruments e.g. C.R.O., multimeters, L-C-R & Q bridges, P.O. Box, Potentiometer (built in).
2. Use of different temperature measuring devices.
3. Maintenance (and charging) of storage cells & batteries.
4. Use of the devices for measuring intensity of light and sound.
5. Use of a digital voltmeter.
6. Setting up of a public address system.

One experiment is to be performed by the candidate. The mark distribution is as follows : Experiment—50 ; Proportional error or probable error calculation/discussion where necessary—10 ; class note book records—10 ; Viva—20 ; Fault finding of any laboratory instrument which is not working ; Measurement of current, resistance, voltage etc. using multimeter, voltmeter, ammeter, C.R.O. etc—10 ; Full marks—100.

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A
HANDBOOK
OF
PRACTICAL
PHYSICS

GENERAL DISCUSSION ON METHODS OF MEASUREMENT

All natural phenomena that happen around us daily are connected by definite laws. To unravel these natural laws is the aim of science. When a stone falls from a certain height, it falls according to certain laws. The planets in the solar system move round the sun in accordance with some definite laws. Nature does not tolerate lawlessness in its realm. But to establish these natural laws, we require experiment and observation. The basis of whole science—specially the science of Physics—is experiment, observation and measurement. Observations, experiment and determination of physical quantities are the province of Practical Physics. With a view to conduct an experiment successfully and to arrive at the proper conclusion by accurate measurement, physicists have not only been trying to invent more and more advanced and sensitive instruments but have also been developing the theory of errors which enable us to perform our investigation free from possible mistakes.

The theory of errors is based on the following two assumptions :—

(i) Although everybody wishes that he should be careful enough to get a correct measurement, yet he is, to some extent, careless by nature and commits some error in reading an instrument. This natural carelessness, of course, varies from person to person. Every reading taken with a given apparatus, is liable to be more or less erroneous. Divergence in different readings will be greater, if the instrument is very sensitive and fine. Such errors are known as *chance errors*. It is to be noted that chance error, although branded as an error, does not carry any sense of blame ; it is not a 'mistake'. It is inherent in everybody. The magnitude of chance error, however, depends on the quality of the instrument as well as the efficiency of the observer. If the instrument is free from defects and the observer is sufficiently careful, the error may be minimised to a great extent.

(ii) Every instrument is, to some extent, defective. Faultless instrument is an ideal thing. So, the reading given by an instrument is liable to contain some error. Such error arising out of some inherent defect in the instrument, is known as —*instrument* or *constant error*.

ELIMINATION OF THE ERRORS :

(i) **Chance errors :** As chance error is purely accidental, according to the law of probability, the errors will be distributed equally on the positive (*i.e.* greater) and the negative (*i.e.* less) side of the correct value Measurement of a quantity by an instrument may be likened with the hitting of a fixed target by bullets. The deviations of the shots from the exact target point are comparable to the errors of observations, both obeying the law of probability. It may be proved in the following way that *the arithmetic mean of a large number of observations of the same quantity will give us the most accurate value free from chance errors.*

Suppose the accurate value of a certain quantity is x and let the readings of various observations of the quantity be $x_1, x_2, x_3 \ldots . x_n$ and the errors of the corresponding observations be respectively, $e_1, e_2, e_3 . . e_n$.

Then, we have,

$$\begin{aligned} x_1 &= x+e_1 \\ x_2 &= x+e_2 \\ x_3 &= x+e_3 \\ &\cdots\cdots \\ &\cdots\cdots \\ x_n &= x+e_n \end{aligned}$$

$$\therefore \quad (x_1+x_2+x_3+\ldots.+x_n)=nx+(e_1+e_2+e_3+\ldots.+e_n)$$

As e_1, e_2, e_3 etc are chance errors, they will be so distributed that some will be positive while others will be negative; for there is no reason why mere chance should favour positive errors in preference to negative ones. Hence,

$$e_1+e_2+e_3+\ldots.+e_n=0$$

$$\therefore \quad x_1+x_2+x_3+\ldots.+x_n=nx$$

$$\text{or, } x=\frac{x_1+x_2+x_3+\ldots.+x_n}{n}$$

=arithmetic mean of all the observations

Let us take a concrete case. Suppose the true value of the focal length of a convex lens is 15 cm. A large number of experimental observations gave the gollowing readings :

15·1 cm ; 15·2 cm ; 15·3 cm ; 14·7 cm ; 14·8 cm ; 14·9 cm and 15 cm.

Note that some errors are positive (*i.e.* the reading is more than the true value) and some are negative (*i.e.* the reading is less than the true value). Here the arithmetic mean or the average value

$$=\frac{15.1+15.2+15.3+14.7+14.8+14.9+15}{7}=15 \text{ cm}$$

(ii) **Instrument or constant error :**

Elimination of instrument error is not so easy as that of the chance error. To eliminate the error due to some defect in the design or manufacture of the instrument, the experimenter has to study carefully, the factors which govern the two chief qualities of the instrument, *viz*, *accuracy* and *sensitivity*. These two qualities are, however, antagonistic to each other. One increases at the cost of other. It is very difficult—almost impossible to increase both of them simultaneously.

Probable errors : Probable error denotes the range on either side of the most probable value of a quantity, so that there is equal chance of the true result lying between this range. If x be the arithmetic mean of a set of measurments *carrying chance errors only* and β the probable error, it denotes that the true value is as likely to lie within the range $x \pm \beta$ as outside it.

To calculate the probable error, suppose $x_1, x_2, x_3 \ldots x_n$ be the readings of n observations of a quantity and x the arithmetic mean. Then x may be taken as the nearest approach to the correct value of the quantity. Now, find the difference between each observational reading and the arithmetic mean. Let the differences (they are known as deviations) be denoted by d *i.e.* $d_1 = (x_1 - x)$; $d_2 = (x_2 - x) \ldots$ $d_n = (x_n - x)$. Now find the average value of the deviations without having any regard to their signs. Let it be δ. It represents the mean error and the correct value of the quantity, for all practical purposes, may be taken to be equal to $x \pm \delta$.

Sometimes, instead of calculating the average value of the deviations the *standard deviation* is found out. Standard deviation is the root mean square of the deviations of individual observations. Thus, if C be the standard deviation, then,

$$C = \sqrt{\frac{d_1^2 + d_2^2 + \ldots + d_n^2}{n}}$$

The probable error can be shown to be approximately equal to

$\beta = \frac{2C}{3\sqrt{n}}$. Hence the correct value $= x \pm \beta$.

Let us again take the case of a convex lens. The readings of six observations in cm are

(i) 15·1, (ii) 15·2 (iii) 15·3 (iv) 14·7 (v) 14·8 and (vi) 14·9.

The arithmetic mean of the observations, as has already been mentioned is 15 cm. The deviations are as follows :

$d_1=15-15{\cdot}1=-0{\cdot}1$; $d_2=15-15{\cdot}2=-0{\cdot}2$; $d_3=15-15{\cdot}3=-0{\cdot}3$; $d_4=15-14{\cdot}7=+0{\cdot}3$; $d_5=15-14{\cdot}8=+0{\cdot}2$ and $d_6=15-14{\cdot}9=+0{\cdot}1$.

The standard deviation

$$C=\sqrt{\frac{(-0{\cdot}1)^2+(-0{\cdot}2)^2+(-0{\cdot}3)^2+(0{\cdot}3)^2+(0{\cdot}2)^2+(0{\cdot}1)^2}{6}}=0{\cdot}216$$

$$\text{Hence, probable error}\quad \beta=\frac{2\times0{\cdot}216}{3\sqrt{6}}=0{\cdot}06 \text{ cm (nearly)}$$

So, the correct value of the focal length may be written as $15\pm0{\cdot}060$ cm. This means that the correct value is as likely to lie between 15·06 cm and 14·94 cm as outside it.

Calculation of proportional error and percentage error :

It is very often seen that the evaluation of the final result of an experiment involves measurement of several quantities by different instruments, the quantities being connected by a formula. It may be shown that the measured quantities do not influence the final result equally,—some may have more influence than the other. Suppose X is a quantity which depends on three other quantities A, B and C in a manner given by the relation, $X=A^p.B^q.C^r$ .. (i)

Then taking logarithm and then differentiating we get,

$$\left(\frac{\delta X}{X}\right)_{max}=p.\ \frac{\delta A}{A}+q.\ \frac{\delta B}{B}+r.\ \frac{\delta C}{C} \quad \text{.. (ii)}$$

Here, $\frac{\delta X}{X}$, $\frac{\delta A}{A}$..etc are known as *proportional errors* in the measurement of the respective quantities. Each multiplied by 100, is the corresponding *percentage error*. The quantities p, q and r in the eqn (ii) should be regarded as positive, even though in the expansion for X in eqn (i), one or more of the indices may be negative.

So, from eqn (ii) we may say that the proportional error in $X=p\times$ the proportion error in $A+q\times$ the proportional error in $B+r\times$ the proportional error in C. Therefore, for given possible errors in A, B and C, the influence on the final result is greater for the factor having higher power and that (the factor) having smaller value. For example, if B is the smallest quantity and q is the highest power, the measurement of B should be most carefully made so that δB becomes the least.

A few examples will illustrate the application of eqn (ii) for the purpose of calculating the maximum error.

1. Determination of *Y* by flexure of a beam :

When a beam of length *l*, breadth *b* and width *d* is loaded in the middle by a weight *mg*, causing a depression *x* of the centre, the Young's modulus *Y* of the material of the beam is given by :

$$Y=\frac{m.g.l^3}{4b.d^3.x}$$

Taking logarithm and then differentiating, we get,

$$\left(\frac{\delta Y}{Y}\right)_{max}=\frac{\delta m}{m}+\frac{\delta g}{g}+3.\ \frac{\delta l}{l}+\frac{\delta b}{b}+3.\ \frac{\delta d}{d}+\frac{\delta x}{x}\quad \cdot\cdot\quad \text{(i)}$$

[Maximum error is obtained by *adding* the proportional errors]. Let the data of a set of observations and their probable errors be as follows :

$m=2$ kg$=2000$ gm ;	$\delta m=4$ gm assuming an error of 1 gm per ½ kg.
$g=980$ cm/s^3	$\delta g=1$ cm/s^3 (assumed)
$b=2{\cdot}54$ cm	$\delta b=0{\cdot}01$ cm (L.C. of slide calipers)
$d=0{\cdot}982$ cm	$\delta d=0{\cdot}01$ cm (,, ,,)
$l=90$ cm	$\delta l=0{\cdot}1$ cm (L.C. of a metre scale)
$x=0{\cdot}173$ cm	$\delta x={\cdot}001$ cm (L.C. of travelling microscope)

Substituting these values in eqn (i), we get,

$$\begin{aligned}\left(\frac{\delta Y}{Y}\right)_{max}&=\frac{4}{2000}+\frac{1}{980}+\frac{3\times0{\cdot}1}{90}+\frac{0{\cdot}01}{2{\cdot}54}+\frac{3\times0{\cdot}01}{0{\cdot}982}+\frac{0{\cdot}001}{0{\cdot}173}\\&=0{\cdot}0020+0{\cdot}0010+0{\cdot}0033+0{\cdot}0038+0{\cdot}0305+0{\cdot}0052\\&=0{\cdot}0458\end{aligned}$$

Thus, the maximum proportional error in $Y=0{\cdot}0458$ and percentage error$=0{\cdot}0458\times100=4{\cdot}58\,\%$. Of all the errors, the proportional error in the width($=0{\cdot}0305$) being the maximum, has the greatest contribution to the maximum error of *Y*. Hence width should be measured very carefully. It is to be noted that the percentage error of *d* (3%) is much higher than that of *x* (0·5%). The percentage error of other quantities are smaller than that of *x*. Hence, the relative probable error in *d* has to be brought down, at least, to $\frac{1}{6}$th of the present value if the contribution of *d* to the total error is not to exceed that of *x*. There is another difficulty. The width of the beam may not be uniform throughtout. To eliminate the error due to non-uniformity of width, *d* should be measured at various places of the beam.

2. Determination of surface tension by capillary tubes :

If the height of a liquid column in a capillary tube of radius r be h, the surface tension T of the liquid is given by,

$$T=\frac{r.h.\rho.g}{2} \quad [\rho=\text{density of the liquid}]$$

Taking logarithm and differentiating, we get,

$$\left(\frac{\delta T}{T}\right)_{max}=\frac{\delta r}{r}+\frac{\delta h}{h}+\frac{\delta \rho}{\rho} \text{ [Assuming that the value of } g \text{ is accurate]}$$

Let the data of a set of observations and their probable errors be as follows :

$r=0{\cdot}022$ cm $\delta r=0{\cdot}001$ cm (L.C. of a microscope)
$h=5{\cdot}624$ cm $\delta h=0{\cdot}002$,, (twice the L.C. because two readings are taken)
$\rho=0{\cdot}8$ gm/c.c. $\delta\rho=0{\cdot}01$ gm/c.c. (approximate error in sp. gravity bottle)

$$\therefore \left(\frac{\delta T}{T}\right)_{max}=\frac{0{\cdot}001}{0{\cdot}022}+\frac{0{\cdot}002}{5{\cdot}624}+\frac{0{\cdot}01}{0{\cdot}08}=0{\cdot}0454+0{\cdot}0003+0{\cdot}0125=0{\cdot}0582$$

So, the maximum proportional error in the measurement of $T=0{\cdot}0582$ and percentage error$=5{\cdot}82\%$. It is clear that the proportional error in r $(=0{\cdot}0454)$ being the highest, its contribution towards the total error is maximum. The percentage error in the measurement of $r=4{\cdot}5\%$ and hence all possible care should be taken in the measurement of the radius of the capillary tubes.

Percentage error : In measuring a quantity, what is important is the percentage error and not the absolute value of the error. Now,

$$\text{Percentage error}=\frac{\text{Observed value}-\text{real value}}{\text{observed value}}\times 100$$

For example, observed value of the focal length of a convex lens is 15·7 cm whereas the real value is 15·5 cm. Hence, the percentage error, in this case, is

$$\frac{15{\cdot}7-15{\cdot}5}{15{\cdot}7}\times 100=1{\cdot}3\% \text{ (nearly)}$$

We also say that the *accuracy* of the measurement is 1·3 in 100 or 1 in 77.

Significant figures :

It has been mentioned earlier that no instrument is faultless nor any observation exact. The accuracy of the result depends on how fine is the instrument and how skilful is the experimenter. The last digit of the reading is always approximate and hence doubtful regarding its accuracy. Still, the last digit is significant because it gives us some information about the quantity under measurement. *A significant figure is defined to be one which may be regarded as fairly trustworthy regarding its accuracy.*

Metre scale is usually graduated in millimetres. Suppose, measurement of length of a rod with the help of a metre scale yields a value 25·7 cm. It is clear that the measurement has been done with the accuracy of 1 mm. Although the scale does not show fraction of a millimetre, yet by eye-estimation we can read upto the fraction of a millimetre and can say that the length of the rod may be half millimetre more or less than the above reading (*i.e.* 25·7 cm). Neglecting this part, if the length of the rod is taken as 25·7 cm, the last digit 7—not to speak of the first two digits—may be generally taken as trustworthy. Under the circumstances, the reading is said to contain three significant digits. The maximum probable error in this reading is about 1 mm and hence the percentage error = 0·35%.

Now, if the above reading be taken as 25 cm, the significant figures are two in number and the percentage error involved is also greater. Here, the reading has been taken neglecting the fraction of a centimetre and hence upto the accuracy of 1 cm. Further, if the reading be taken as 25·0 cm, the significant figures are again three in number. Hence, 25 *cm reading and* 25·0 *cm readings do not indicate the same accuracy.*

So, the significant figures of a reading give us an idea of the accuracy of the observation. Again in calculating the final result from several readings, such figures are to be retained in the final result as are consistent with the accuracy with which individual readings have been taken. In the final result, the insignificant figures are to be dropped. For example, readings of length, breadth and height of a cube are respectively 4·15 cm, 2·15 cm, and 1·75 cm. Here, the accuracy of measurement is upto three significant figures. Multiplying the length, breadth and height, the volume of the cube may be computed as 15·614375 c.c. But it is not proper to put the volume like that because none of the length, breadth or height has been measured with so much accuracy. Since the accuracy of measurement is upto three significant figures, the volume should also be expressed in terms of only three significant figures and it should be written as 15·6 c.c. The figures after the digit 6 are all insignificant and should be dropped. However, while dropping the insignificant figures and writing the final result the last figure of the final result is to be retained unchanged if the first figure dropped is less than 5. If the first figure dropped is, however, greater than 5, the last figure should be increased by 1. If the dropped figure is 5, the preceding digit should be kept

unchanged if it is even but it should be increased by 1 if it is odd. According to this rule, 20·75 will be written as 20·8
20·73 ,, ,, ,, ,, 20·7
20·79 ,, ,, ,, ,, 20·8
20·65 ,, ,, ,, ,, 20·6
Some people, however, prefer to retain the first insignificant figure in the final result and since it is insignificant they write it after the last significant digit in small letter and somewhat below the main line. Thus, the volume of the cube mentioned above, may be expressed according to this rule, as $15{\cdot}6_1$.

METHOD OF DRAWING GRAPHS

In many experiments of Physics, graphs have to be drawn. When two quantities are so related to each other that the value of one changes when the value of other is changed, a graph drawn between the quantities will very clearly bring out the nature of such change. For example, we know Boyle's law which says that the temperature remaining constant, the volume of a certain quantity of gas changes with the change of pressure. If a graph is drawn between the volumes occupied by the mass of gas at different pressures and the corresponding pressures, this change will be clearly understood. Furthermore, if an experiment is repeated and several observations taken, then from the graph we can easily check whether all the observations are correct or any one of them is incorrect. So, graphs have got enough practical importance.

Squared paper : A paper divided into a number of small squares by drawing horizontal and vertical lines is called a squared paper. Graphs are to be drawn in such squared papers. Generally, each small square of the squared paper measures 1 sq mm *i.e.* its length and height are each 1 mm and every fifth and tenth lines are thicker than others so that they may be easily distinguished. This makes graduation easier. The squared papers, commonly known as graph papers, described above are available in the market.

Abscissa or X-axis and Ordinate or Y-axis : In going to draw a graph, at first a line is to be drawn along a horizontal line at the bottom of the graph paper. This line will be called the abscissa or the X-axis. On the left-end of the graph paper, another line is to be drawn, perpendicular to the X-axis, along a vertical line of the graph paper and this will be referred to as the ordinate or the Y-axis. So, the two axes are mutually perpendicular to each other. The point where the axes intersect is called the origin and is, usually, denoted by the letter 'O'

Of the two variable quantities between which a graph is to be drawn, one will be independent variable and the other dependent variable. For example, y and x are two quantities related by the equation $y=mx+c$. Now, if we give to x different values, y will assume different values according to the above equation. y cannot take up values according to its own choice and it will have to depend on x for its different values. For this reason, x may be called an independent variable and y a dependent one. Before going to draw a graph, dependent and independent variables are to be marked out. It is customary to plot the values of independent variable along the X-axis and those of dependent variable along the Y-axis. *Name and unit of each quantity must be mentioned along the axis against which they are plotted.*

Fixation of scale : To plot the dependent and independent variables along their respective axes, appropriate scales should be fixed for them. The fixation of scale should be done very carefully, because if the scale is a reduced one, errors are likely to come in the process of plotting points, while if the scale is a magnified one, then any mistake done in taking a reading will also be magnified in the graph and the points on the graph will be so scattered that it will be difficult to draw any consistent graph from them. As a general rule, however, a reduced scale for quantities of large magnitude and an enlarged scale for quantities of small magnitude may be used. There is, of course, no hard and fast rule about it. All that can be said is that according to the situation scales are to be fixed judiciously. A point is to be remembered in this connection. Scales along the two axes need not be identical. Scales may be different, if found convenient. But whatever may be the scales—identical or different—distinct mention of it must have to be made.

Another point : It is not binding that the origin of the axes must always indicate 0-0 (zero-zero) values of the two variables. If the minimum values of the quantities are much higher than zero, then this minimum value or any other value *slightly less* than the minimum may be taken as the origin and the scale along the axis is to be so fixed that the point furthest from the origin along that axis may indicate the maximum value or any other value *slightly greater* than the maximum value of that quantity. While drawing a graph, it should always be borne in mind that *the graph should be spread over the entire paper.*

Plotting of points : According to the values of the two variable quantities, when the position of a point has been ascertained on the graph paper, it may be marked by a cross sign (×) or a dot with a circle round it. Then the points are to be joined by a line. *The joining must not be done by short straight lines with the help of a scale.* It should be a free-hand joining, without scale, graph passing through most of the points in such a way that there is no abrupt bend anywhere. If a particular point happens to be away from the graph, it is to be understood that some serious mistake had occurred during that particular observation.

To find the value of an unknown quantity from the graph :

If a particular value of the X-axis (or Y-axis) is given, then to get the value of the corresponding Y-axis (or X-axis) a perpendicular in broken line is to be drawn from the given point on X-axis (or Y-axis) to cut the curve at a point, *P* say. The value of Y-axis (or X-axis) corresponding to the point *P* will give the value of the unknown quantity.

If the pressure-volume graph shown in page 11, be taken as an illustration and the volume of the gas to be found at a pressure of 80 cm, a perpendicular is to be drawn on pressure-axis through the 80 cm-pressure point to cut the curve. The intersection point has the value 10 for the Y-axis which means that the volume of the gas is 10 c.c. at a pressure of 80 cm of mercury.

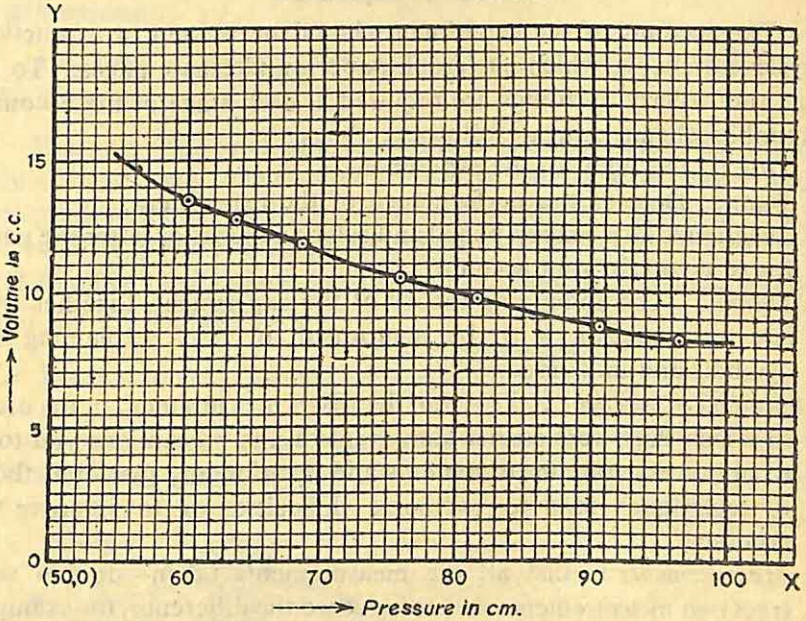
Example : Suppose in Boyle's law experiment, the following are the volumes obtained for a certain mass of gas at the following different pressures :

Pressure (P) in cm	60·0	63·2	68·4	76·2	81·3	90·9	96·2
Volume (V) in c.c.	13·5	12·8	11·8	10·7	10·0	8·9	8·5

A graph is to be drawn between pressure and volume. In this case, pressure is the independent variable. So, pressure is to be plotted along the X-axis and volume along the Y-axis. Fig A shows how the graph is to be drawn.

OX and *OY* are the two axes, *O* being the origin. Pressure (P) is plotted along the X-axis and volume (V) along the Y-axis. Since the minimum value of pressure (60 cm) is much higher than zero, 50 cm pressure is taken as the origin. But the volume at the origin is taken zero. Now, the scale along the X-axis is so chosen that each small division indicates 1 cm pressure but since difference of volume is very small, the scale along the Y-axis is so selected that each small division is equivalent to 0·5 c.c. or two small divisions=1 c.c.

According to these scales, the values from the previous table are plotted. For instance, at 60 cm pressure, the volume is 13·5 c.c. Along the X-axis, if you move from the origin ten divisions (50 cm pressure being represented at the origin) you will get 60 cm pressure point and then moving vertically parallel to the Y-axis, if you go 27 divisions, you will get 13·5 c.c. volume point. Put a dot mark there and draw a small circle round it. This is the first point. Similarly, plot other points. Next, the points are to be joined by a pointed pencil by free-hand drawing. From the graph it is clear that the volume diminishes with the increase of pressure —in other words,the volume and the pressure are inversely proportional to each other.



X – *Axis → small division* = *1 cm*
Y – *Axis →* " = *0·5 cc.*
Origin → (50 cm; 0 c.c.)

Fig. A

It is worthwhile to note that by simply examining the equation connecting the variables, one can have an idea of the nature of the graph between those variables. The following are the equations of some of the well-known curves :

(i) $ax+by+c=0$ represents a straight line

(ii) $x^2+y^2=a^2$ represents a circle.

(iii) $y^2=4ax$ represents a parabola.

(iv) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ represents an ellipse.

Here, a, b and c are constants.

Account of experiment

The account of the experiment should be written in a practical physics notebook, which has graph paper on alternate pages. To be neat and orderly, headings are required at each stage of the account, a suitable scheme being as follows :

1. *Title* : The name of the experiment.
2. *Date* : Date of the performance of the experiment.
3. *Diagram* : Use a sharp pencil and ruler and label or letter the parts for reference in your account.
4. *Theory* : The underlying theory of the experiment, together with relevant equation is to be mentioned, in brief, explaining the symbols and units used.
5. *Method* or *Account* : Give full details of measurement in the order in which they were carried out, mention the precautions you took to obtain accurate measurements and mention any special methods or techniques used to overcome difficulties or to improve the accuracy.
6. *Measurements* : List all the measurements taken—do not subtract two measurements and only record the difference, for example. Enter the measurements in a ruled table if possible or list them below each other in a column, giving units in which each is measured.
7. *Calculation* : Convert measured values to c.g.s. units and substitute them in the formula. Final result with proper unit should be shown on the right hand side page. Detailed calculations, with the help of log-table are to be shown on the left-hand side page.
8. *Graph* : If a graph is to be plotted, choose scales to use as much of the graph paper as possible, label the two axes, and mention the units. Mark the points plotted with crosses or with a dot and a small circle ; a sharp and hard pencil is essential.
9. *Result* : Your final result must be given to a sensible order of accuracy and it must state the units concerned.

For completeness, percentage error or uncertainties in the experiment should also be discussed and the order of accuracy of the final result then estimated.

1. GENERAL PROPERTIES OF MATTER

1·1. Determination of the radius of curvature of a spherical surface by a spherometer :

Apparatus : A spherometer, a convex (or a concave lens), a plane glass plate etc.

Theory : The pitch of the instrument is the perpendicular distance between any two consecutive threads of the central screw and is measured by the vertical distance travelled through by the screw when the screw is given one complete rotation. The pitch divided by the total number of divisions on the circular scale gives the least count (L. C.) of the instrument.

$$\text{Least count} = \frac{\text{Screw-pitch}}{\text{Total no. of divisions on the circular scale } (N)}$$

If d be the mean length of the three arms of the equilateral triangle ABC [Fig 1(a)] formed by the tripod stand of the spherometer and h

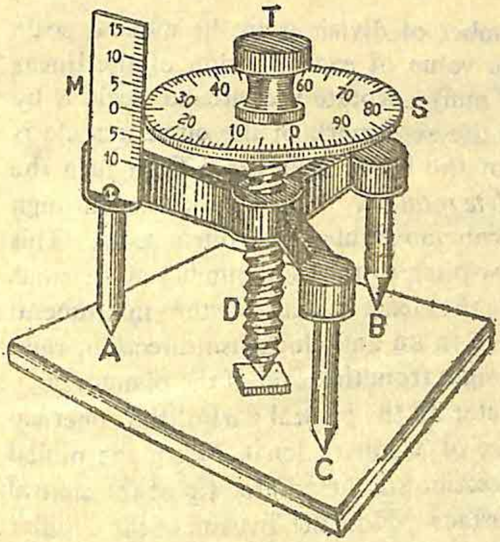


Fig. 1 (a)

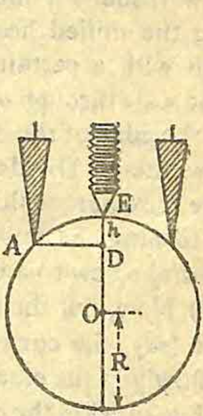


Fig. 1 (b)

be the displacement of the central screw-tip when it touches consecutively the given spherical surface and a plane surface [Fig 1(b)], then the radius of curvature R of the spherical surface is given by the following equation :

$$R=\frac{d^2}{6h}+\frac{h}{2}$$

Description of the instrument :

Fig 1(*a*) shows the appearance of a spherometer. It consists essentially of a fine, accurate and pointed micrometer screw D, known as the central screw which can turn in a nut fixed to a metal tripod stand. The three pointed legs A, B and C of the tripod are of equal length and arranged at the corners of an equilateral triangle. The axis of the screw D is perpendicular to the plane ABC and passes through the centre of the circumscribed circle of the triangle ABC. The screw carries at its top a milled head T, by turning which the screw D can be moved up and down. Below the milled head T, there is a circular disc S, the circumference of which is divided into equal parts which form the *circular scale*. To the side of the disc and nearly in contact with its rim, a short, vertical and linear scale M is fixed to the frame, with its edge parallel to the axis of the screw. The linear scale is graduated in divisions equal to the pitch of the screw.

Experimental procedure :

(1) Ascertain the total number of divisions in the circular scale (usually 100 divisions) and the value of each division of the linear scale M (usually 1 mm or 0·5 mm). Rotate the circular scale S by turning the milled head T till the zero-mark of the circular scale is in flush with a certain mark of the linear scale M. Then turn the circular scale through *one complete rotation*. Note the distance through which the edge of the circular scale moves along the linear scale. This is screw pitch. Divide the screw-pitch by the total number of divisions on the circular scale to get the least count of the instrument. Next, turning the milled head T in an anti-clockwise direction, raise the central screw to a certain height from the level of the plane ABC.

(2) Now, put the spherometer on the central part of the spherical surface (say, the convex surface of a convex lens). Turn the milled head slowly in the clockwise direction till the pointed tip of the central screw just touches the convex surface. Note the division of the circular scale standing against the linear scale [24 division in fig, 1(*a*)]. This is circular scale reading. Suppose it is p.

(3) Without altering the position of the circular scale, remove the spherometer carefully from the convex surface and place it on a plane glass plate. Turn the milled head T slowly in the *same direction*

.*e*. clockwise till the central screw just touches the glass plate. In doing so, if the total number of complete rotations given to the circular scale be n and if n' be the *extra** reading of the circular scale, then the total circular scale reading $m = n \times N + n'$ [N=total no. of divisions in the circular scale].

According to the circular scale reading, $h = m - p$.

(4) Multiply the above value of h by the least count (expressed in cm). This gives the value of h in centimeter.

(5) Repeat the observations to get several values of h and find the mean of them.

(6) Raise the central screw to a certain height and place the spherometer on a sheet of white paper. Press the spherometer lightly. This produces impressions of the points of the three legs of the spherometer on the paper. From these impressions, draw the equilateral triangle ABC. With the help of a metre scale find the lengths of the three arms of the triangle and therefrom, the mean length (d).

(7) Substituting the values of h and d in the equation mentioned in the theory, find the value of R, the radius of curvature of the spherical surface.

Measurements : (*a*) *Least count of the instrument* :

The value of each division of the linear scale= . . mm.

Total number of circular divisions= . . (N).

Screw-pitch= . . mm.

$$\text{Least count (L.C.)} = \frac{\text{Screw pitch}}{N} = \text{.. mm} = \text{.. cm.}$$

(*b*) *The mean length of the three legs of the tripod* :

(i) . . cm (ii) . . cm (iii) . . cm.

∴ Mean length (d)= . . cm.

---

* n' is obtained from the initial circular scale reading (p) and final circular scale reading (p' say) in the following way :

(i) If the direction of the movement of the screw *increases* the circular scale reading, then, $n' = (100 - p) + p'$ when $p > p'$ and $n' = p' - p$ when $p' > p$.

(ii) If the direction of the movement of the screw *decreases* the circular scale reading then $n' = (100 - p') + p$ when $p' > p$ and $n' = p - p'$ when $p > p'$.

(*c*) *The value of* h :

(Numerical figures given in the table are simply for illustration).

No. of Obs.	Initial cir. scale reading when the screw touched the convex surface (p)	Cir. scale reading when the screw touched the glass plate			Total no. of cir. scale div rotated ($m=n\times N+n'$)	Value of h in cir. scale divisions ($m-p$)	Value of h in cm. $(m-p)\times L.C.$	mean value of h
		No. of complete rotations (n)	final cir. scale reading (p')	Additional no. of cir. scale divisions rotated (n')				
1.	25	3	87	$(100-87)+25=38$	$3\times100+38=338$	$338-25=313$	$313\times$ L.C.	..
2.	..	..	..	..	..	..	..	
3.	..	..	..	..	..	..	..	

[In the above table N is taken as 100]

(*d*) *Calculations* : $R=\frac{d^2}{6h}+\frac{h}{2}=$.. cm.

Remarks : (i) While taking readings, the central screw should be turned always in the same direction to avoid back-lash error.

(ii) Several observations should be taken at different points of the spherical surface.

Oral questions

1. For what purpose is the spherometer used ?

Ans. For measuring small lengths like the thickness of a sheet or the radius of curvature of a spherical surface.

2. What is screw-pitch ?

Ans. The perpendicular distance between any two consecutive threads of a screw is called the pitch of the screw.

3. Why is the screw rotated always in one direction ?

Ans. To avoid back-lash error.

4. What is the radius of curvature of a spherical surface ?

Ans. The radius of the sphere of which the surface is a part is called the radius of curvature of the spherical surface.

5. What is the most reliable test for finding that all the legs of a spherometer **and the tip** of the central screw lie in the same plane ?

Ans. When the instrument is placed with its three legs on an accurately plane surface and the central screw is turned downwards until its pointed end just touches the surface, both the linear scale and the circular scale should read zero, if the instrument is perfect.

6. Which of the two quantities *d* and *h* should you measure more accurately **and why ?**

Ans. *d* ; because its value is small and its power is 2 in the formula.

1·2. To read Fortin's barometer :

Apparatus : A Fortin's barometer.

Theory : From Toricelli's experiment it is known that if a glass tube of about 1 metre length, open at one end and closed at the other be filled with dry and clean mercury and then inverted over a trough containing mercury with the open end dipping in the mercury, then a column of mercury stands in the tube. The weight of the mercury column standing in the tube per unit area is equal to the atmospheric pressure. Generally the atmospheric pressure is expressed in terms of the vertical height of the mercury column.

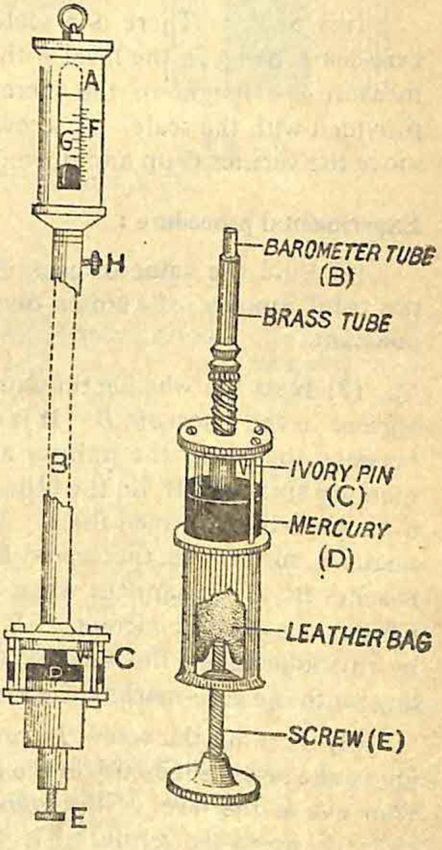


Fig. 2

Description of the apparatus : Fig 2 shows a section of Fortin's barometer. It consists of the following parts :

(i) *Barometer tube :* *AB* is a glass tube of uniform bore, having length about 1 metre with one end closed and the other open. It is filled completely with dry and clean mercury and inverted over another vessel *D* with the open and dipping into the mercury of the vessel *D*. The barometer tube is enclosed in a brass tube (partly shown in the figure). The tube is fitted against a wooden frame which is clamped to the wall of a room in a vertical position. There are two diametrically opposite slits, about 20 cm long and $1\frac{1}{2}$ cm broad, near the top-end of the brass tube. Through this slit, the glass tube and the mercury meniscus become visible.

(ii) *Mercury reservoir* : *D* is a reservoir containing dry and clean mercury. The level of mercury in the reservoir can be raised or lowered by a screw *E* fixed at the bottom of the reservoir. When the screw *E* is turned, a leather bag increases or decreases in volume, thereby lowering or raising the level of mercury. Air can pass through the leather bag but not the mercury. As a result, the pressure on the mercury surface in the reservoir *D* is equal to the atmospheric pressure. An ivory pointer *C* is provided to adjust the level of mercury in the reservoir. [This portion of the barometer has been shown separately in the figure.]

(iii) *Scale* : There is a scale *F* engraved on the brass tube, the zero-mark being in the level with the tip of the ivory pointer *C*. To measure the height of the mercury column, a vernier *G* has been provided with the scale. A screw *H* attached with the brass tube can move the vernier *G* up and down.

Experimental procedure :

(1) Find the value of each small division of the main scale and the total number of vernier divisions. From this find the vernier constant.

(2) Next see whether the ivory pointer *C* just touches the mercury surface in the reservoir *D*. If it does not, there will be some gap left between the tip of the pointer and its image formed by the shining mercury surface. If, on the other hand, the tip dips into the mercury, a dimple will be formed there. Whatever may be the case, adjust the mercury surface by the screw *E* so that the ivory pointer *C* just touches it. This happens when the tip of the pointer just coincides with its image in the mercury. It is known as zero-adjustment because by this adjustment the surface of mercury in the reservoir comes in level with the zero-mark of the scale *F*.

(3) Turning the screw *H*, raise the vernier *G* to a certain height above the level of mercury in the tube and then slowly lower it. Keep your eye at the level of the convex surface of the mercury meniscus and slide down the vernier till its lower edge is tangential to the convex mercury meniscus [Fig 2(*a*)]. To do this adjustment correctly, a milk-white plate is fixed on the wooden board behind the vernier. As

soon as the vernier is adjusted correctly, the milk-white plate being completely obstructed by the vernier, will disappear from view. It is known as vernier adjustment.

(4) Read the main scale and the vernier scale avoiding parallax. Repeat the observations at least, thrice and find the mean height of the mercury column. Note the temperature from the thermometer attached with the barometer.

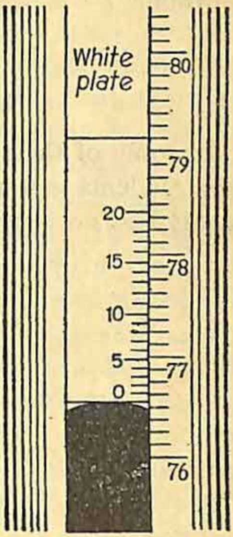


Fig. 2 (*a*)

Measurements :

.. vernier divisions = main scale divisions

= .. mm.

∴ 1 ,, ,, = .. mm.

So, Vernier constant (v.c.) = .. mm.

Temperature at the time of observation = .. °C

Value of h :

(Data given are for illustration only)

No. Obs.	Main scale reading mm. (*a*)	Vernier reading	Vernier reading $\times V.C.$ mm. (*b*)	Total reading $(a+b)$ mm.	Mean height (h) mm.
1.	754	12	$12\times0{\cdot}1=1{\cdot}2$ mm	755·2	..
2.	..	..	..	..	
3.	..	..	..	..	

Atmospheric pressure = ... mm of Hg.

Remarks : (1) The barometer tube must be fixed vertical.

(2) While making zero-adjustment or vernier adjustment, parallax must be avoided.

(3) Temperature correction of the barometric height should be done. If h be the height of mercury column observed at t°C, then, the correct height h_0 is given by the following relation, $h_0=h_t\{1-(\gamma-\alpha)t\}$ where γ = coefficient of cubical expansion of mercury and α = the coefficient of linear expansion of the material of the scale (brass).

(4) Atmospheric pressure in dynes/cm^2, may be obtained from the formula, $P = h_0 \rho_0 . g$ where ρ_0 = density of mercury at 0°C.

1·3. Travelling Microscope

In some of the experiments on general properties of matter and optics, students will have to handle a travelling microscope. As the students had no opportunity of getting acquainted with this instrument earlier, a description of the instrument is given first and then a simple experiment to familiarise the students with the use of it.

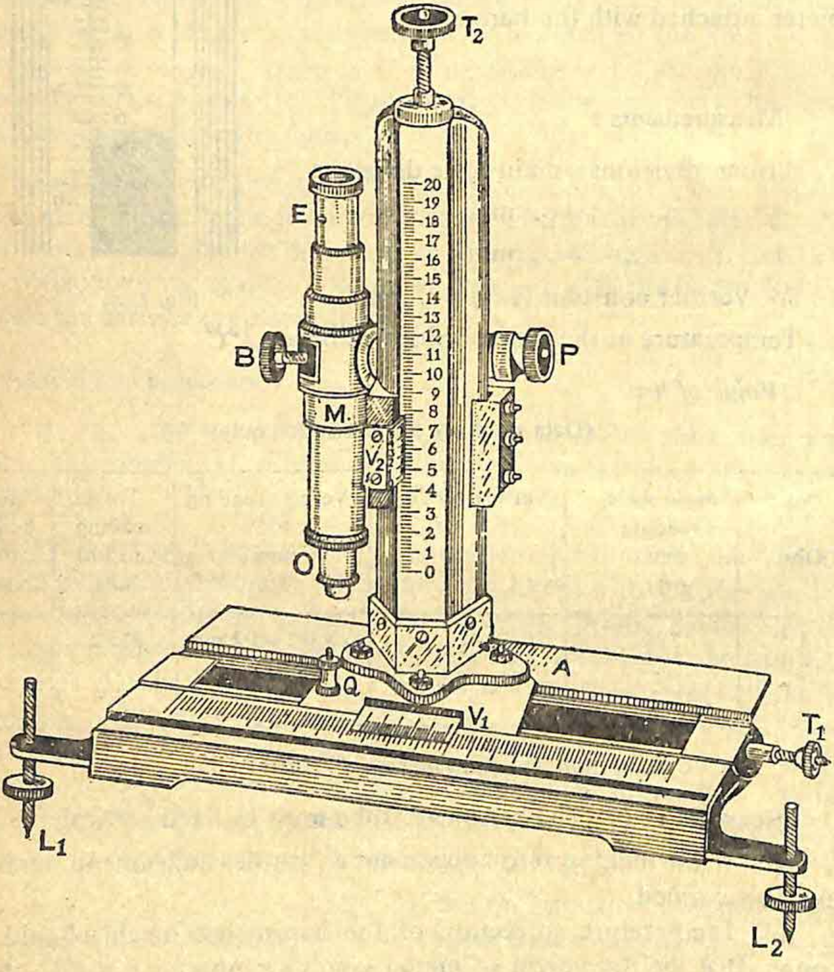


Fig. 3

Fig 3 shows the appearance of a travelling microscope. It consists of a compound microscope M movable in the horizontal as well as in the vertical directions. In both the directions there is a main scale along with a vernier. The vernier V_1 along with its main scale measures the horizontal displacement and the vernier V_2 along with its main scale measures the vertical displacement of the microscope. With the help of the fixing screw P, the microscope M can be set vertical, horizontal or inclined at any angle. The tangent screw T_1 gives the microscope a slow horizontal motion while the tangent screw T_2 a slow vertical motion. The tangent screws will, however, work only when the fixing screw Q and another fixing screw given at the back of the pillar are tight. If the screw Q and that at the back of the pillar are not tight, the microscope can be moved easily and freely by hand. E represents the eye-piece of the compound microscope. Looking through the eye-piece, the image of the object under observation will be visible. O is the objective of the microscope. For horizontal measurement, the object is placed on the platform A. The platform can be made horizontal and the pillar vertical by turning the levelling screws (L_1, L_2, L_3) at the bottom of the instrument.

1·4. Determination of the diameter of the bore of a capillary tube by a travelling microscope :

Apparatus : A travelling microscope, a capillary tube, suitable stands, a spirit level etc.

Theory : If y_1 be the reading when the intersection of the cross-wires of the microscope coincides with the left edge of the bore of the tube and y_2 that when the intersection coincides with the right edge, then the diameter of the bore $D=y_1\sim y_2$.

Experimental procedure :

(1) Find the value of the smallest division of the main scale and the total number of divisions in the vernier. Find how many divisions of the main scale are equal to the total length of the vernier divisions. From this calculate the vernier constants of the two scales—horizontal and vertical.*

(2) Keep the spirit level on the platform of the microscope so that axis of the spirit level is parallel to the line joining the base screws L_1 and L_2. Bring the bubble at the centre by turning the levelling screws L_1 and L_2. Now rotate the spirit level such that its axis is perpendicular to the line joining the screws L_1 and L_2. Turn the third

---

*Usually the two verniers have the same vernier constant.

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levelling screw L_3 to bring the bubble at the centre. When this is done, the platform becomes horizontal and the pillar vertical.

(3) By raising or lowering the eye-piece *E*, focus it on the cross-wires till you see them sharply defined. This is to be done without straining the eyes. If necessary turn the cross-wires so that one is horizontal and the other is vertical. Loosening the screw *P*, turn the microscope till its axis is horizontal. Examine with the spirit level whether the axis is exactly horizontal.

(4) Now fix the capillary tube in a suitable stand and turn its bore towards the objective of the microscope. By turning the focussing screw *B*, focus the bore till a sharp and magnified image is seen. Fix the microscope with the screw *Q*. Turning the tangent screw T_1, slowly move the microscope towards left till the vertical cross-wire becomes tangent on the left side of the bore [Fig 12(*a*)]. Read the horizontal main scale and the vernier V_1. Let the reading be y_1.

(5) Unfixing the screw *Q*, move the microscope slowly with hand towards the right side till the vertical cross-wire is *almost* tangential to the right side of the bore. Now fix the screw *Q* and turn the tangent screw T_1 till the vertical cross-wire is *exactly* tangent to the bore. Again note the main scale and the vernier (V_1) readings. Let the reading be y_2. Repeat the observations, at least, thrice. In this case, the horizontal diameter of the bore $D_1=(y_1\sim y_2)$.

(6) Now raise the microscope along the vertical scale till the horizontal cross-wire is *almost* tangential to the upper end of the bore. Tightening the fixing screw, turn the tangent screw T_2, till the horizontal wire is *exactly* tangential to the bore. Note the readings of the vertical scale and the vernier V_2. Let the reading be y_3. Similarly make the horizontal wire exactly tangential to the lower end of the bore. Let the reading, in this case, be y_4. Repeat the observations thrice. In this case, the vertical diameter of the bore $D_2=(y_3\sim y_4)$.

(7) The values of D_1 and D_2 will be very near to each other, if the bore is circular. Find the mean value of D_1 and D_2.

Measurements :

Value of the smallest division of the main scale= . . mm= . . cm.
. Vernier divisions= . . smallest divisions of the main scale
= . . mm
∴ 1 vernier division= . . mm
Hence, vernier constant= . . mm= . . cm*

---

*In case the vernier constant is different for the two verniers, the vernier constants are to be mentioned separately.

No. of Obs.	Horizontal scale reading (cm)						Horizontal diameter $D_1=(y_1\sim y_2)$ cm.	Mean (D_1) cm	Vertical scale reading (cm)						Vertical diameter $D_2=(y_3\sim y_4)$	Mean (D_2)
	Left end			Right end					Top end			Bottom end				
	Main scale	Vernier scale	Total (y_1)	Main scale	Vernier scale	Total (y_2)			Main scale	Vernier scale	Total (y_3)	Main scale	Vernier scale	Total (y_4)		
1.	..	..	..	..	..	..	..		..	..	..	..	..	..	..	
2.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3.	..	..	..	..	..	..	..		..	..	..	..	..	..	..	

$\therefore$ Diameter of the bore$=\frac{1}{2}(D_1+D_2)=$ ..cm.

Remarks : (1) Parallax between the image and the cross-wires must be avoided.

(2) The bore is not circular if the horizontal diameter is widely different from the vertical one.

1.5. Determination of Young's modulus of the material of a wire by Searle's method :

Apparatus : Searle's apparatus, screw-gauge, slotted weights, metre scale etc.

Theory : Within elastic limit,s tress $\propto$ strain. Now, Young's modulus $Y=\frac{\text{stress}}{\text{strain}}=\frac{Mg/\pi r^2}{l/L}=\frac{Mg.L}{\pi r^2 l}=\frac{4.L.g.}{\pi d^2}\left(\frac{M}{l}\right)$ dynes/cm^2, where Y=Young's modulus, L cm=length of the wire ; Mgm=load applied; l cm=elongation of the wire ; d cm=diameter of the wire.

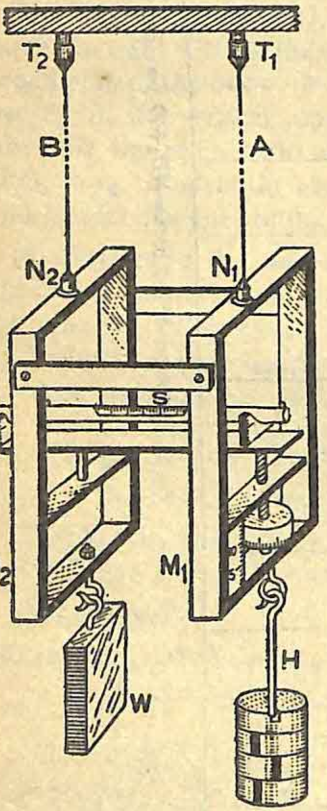

Fig. 4

Description of the apparatus : Fig 4 shows the section of Searle's apparatus. M_1 and M_2 are two metallic rectangular frame-works provided with two torsion heads N_1 and N_2. The frame-work is suspended from two other torsion heads T_1 and T_2 by means of two wires A and B of same length, material and cross-section, the torsion heads being fixed to the roof of a room. The wire A is called the *experimental wire* while the wire B is known as *reference wire.* A weight W is suspended from the hook of the frame-work M_2 in order to keep the wire taut. Suitable weights may be placed on a scale-pan H fixed to the hook of the frame-work M_1. A spirit level S is kept in a horizontal position on a support, one end of which is fixed to a point on the frame-work M_2, the other end resting on the pointed end of a micrometer screw attached to the other frame-work M_1. If the spirit level is exactly horizontal, the bubble will stand symmetrically between two short marks given on the glass case of the spirit level.

When the experimental wire A elongates, the spirit level tilts and the bubble is displaced. Turning the micrometer screw, the spirit level can be restored to its horizontal position. The micrometer screw can move along a straigh scale graduated in millimeters.

Experimental Procedure :

(1) With the help of a screw gauge, measure the diameter (d) of the wire A. This measurement is to be taken, at least, at five different places of the wire and in mutually right angle directions at each place. From these observations, mean diameter is to be calculated, and then compute the value of the cross-sectional area ($\pi d^2/4$) of the wire A.

(2) Multiplying the cross-sectional area by the breaking stress of the material of the wire (to be supplied), breaking weight may be obtained. It is to be noted that *the total load placed on the scale-pan attached to the frame-work M_1 should, under no circumstances, exceed half the breaking weight.* For example, if the breaking weight be 14 Kg, the maximum load permissible is 7 Kg.

(3) Put the maximum permissible load (say, 7 Kg) on the hook of the frame-work M_1. Allow the wire to remain stretched for some time. Then, keeping a small weight (say, 1 Kg) on the hook, remove the others. This small weight left on the hook will keep the wire A free from kinks. It is called *dead-load.*

(4) With the help of a long wooden rod and a metre scale, ascertain the length of the wire A from T_1 to N_1. Repeat the measurement, at least, thrice and find the mean length (L).

(5) Now turning the micrometer screw, bring the bubble at the centre of the spirit level. Take care that *the micrometer screw is rotated always in the same direction* ; otherwise back-lash error will come in. Note the readings of the linear scale and the circular scale. This is the initial reading.

(6) Divide the additional permissible load (over and above the dead-load) that may be put on the hook of the frame-work M_1 (for example, 7 kg $-$ 1 kg $=$ 6 kg) into 10 or 12 equal instalments, each of $\frac{1}{2}$ kg or 1 kg (1 kg instalment is preferable if the load is large while $\frac{1}{2}$ kg instalment in the case where the load is comparatively smaller). Now, go on putting weights by steps of $\frac{1}{2}$ kg over the dead-load till the maximum permissible load is reached. At every step, the bubble of the spirit level will be displaced due to the elongation of the wire A but at every step, the bubble is to be brought back at the centre by turning the micrometer screw *always in the same direction at every step.* The readings of the linear scale and the circular scale are also to be noted.

(7) Now remove the $\frac{1}{2}$ kg weights from the hook, one by one and read the linear and circular scales at every step after bringing the bubble at the centre. This will give two readings for each load—one while the loads were increasing and the other while they were decreasing. the mean value of these two readings in each case. From these, different loads and corresponding elongations can be found out.

(8) Draw a graph plotting the additional load (except the dead-load) expressed in kilogram along the X-axis and the elongation of the wire expressed in centimetre along the Y-axis. Origin should be the (0, 0) point. The graph will be a straight line passing through the origin [Fig 4(*a*)]. Take any point *P* on the straight line and draw *PM* perpendicular on the X-axis. Find from the graph the value of the load *OM* and the corresponding elongation *PM*.

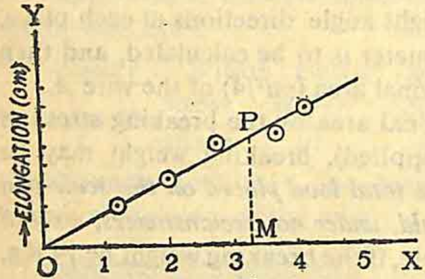

Fig. 4(*a*)

(9) Substitute the values in the equation mentioned in the theory and calculate the value of Young's modulus.

Measurements : (*a*) *Measurement of the diameter of the wire by screw-gauge* :

Value of the smallest division of the linear scale=..mm.

Screw-pitch=...mm ; Total number of divisions in the circular scale=...

Least count (L.C.)=...mm.

No. of Obs.	Linear scale reading (*M*)	Cir. scale reading (*N*)	Cir. scale reading ×*L.C.* (*N*×*L.C.*)	Total reading (*M*+*N*×*L.C.*)	Mean dia-meter	Instru-mental error	Correc-ted dia-meter (*d*)
1. (i)	..mm	..	..mm	..mm	..mm	±..mm	..mm
(ii)	..mm	..	..mm	..mm			
2. (i)							
(ii)							
....				..			
....							
5. (i)	..	..					
(ii)	..	..	..	..			

[Note : Figures (i), (ii) denote mutually perpendicular readings at each place]

The cross-sectional area of the wire$=\frac{\pi d^2}{4}=$..sq. cm.

(*b*) *Breaking weight for the wire* : Breaking wt.=Breaking stress$\times\frac{\pi d^2}{4}=$...kg

$\therefore$ Maximum permissible load=½×breaking wt=..kg.

(*c*) *Length of the wire* : (i) ...cm (ii) ...cm (iii) ...cm.
Mean length of the wire (L)=..cm.

(*d*) *Load-elongation table* :

The value of the smallest division of the linear scale of the micrometer screw=...mm ; Screw-pitch=...mm ; total number of circular scale divisions=....

$\therefore$ Least count (L.C.)=...mm
Dead-load=...Kg

[See table on Page 28]

(*e*) *Table for drawing graph* : [From the table (*d*)]

Additional load in kg→	0	½	1	1½	2	..	..	6
Elongation in cm——→	0	.005	..	..	..	..	..	..

(*f*) *Calculations* : From the graph, elongation (PM)=....cm (l) and corresponding load (OM)=... Kg=... gm(M)

$$Y=\frac{4l.g}{\pi d^2}\left(\frac{M}{l}\right)=\ldots\text{dynes/cm}^2.$$

[**NOTE** : Instead of reading main scale and circular scale everytime during load increasing and load decreasing, only circular scale reading will suffice. In that case, the total number of complete rotation of the circular scale and extra circular scale reading are to be noted at each step of ½ kg load. From this, the elongation can be found out. See expt 1·1 on spherometer.]

Remarks : (1) When a set of readings are taken, the micrometer screw must be rotated in the same direction to avoid back-lash error. (2) The wire may not be uniform or cross-section may not be exactly circular throughout the length of the wire. To avoid consequent error in the measurement of diameter, the screw-gauge reading is to be taken at different places and at mutually perpendicular directions at each place of the wire. (3) After adding a load or

[Data given are for illustration]

No. of Obs.	Additional load put on dead-load	Readings when load increasing (mm)			Readings when load decreasing (mm)			Mean reading (mm) $[\frac{1}{2}(x+y)]$	Mean reading (cm)	Elongation (cm)
		Main scale (S)	Cir. scale (C)	Total (x) $[S+C\times l.c.]$	Main scale (S)	Cir. scale (C)	Total (y) $[S+C\times l.c.]$			
1	0 (dead-load)	4	37	$4+37\times{\cdot}01=4{\cdot}37$	4	36	$4+36\times{\cdot}01=4{\cdot}36$	4·365	0·4365 (i)	(i)-(i) =0
2	½ kg	4	41	$4+41\times{\cdot}01=4{\cdot}41$	4	42	$4+42\times{\cdot}01=4{\cdot}42$	4·415	0·4415 (ii)	·005 (ii)-(i)
3	1 kg	..	..	..	..	..	..	..	..(iii)	..(iii)-(i)
4	1½ „	..	..	..	..	..	..	..	..(iv)	..(iv)-(i)
5	2 „	..	..	..	..	..	..	..	..(v)	..(v)-(i)
etc	..	..	..	..	..	..	..	..	..(vi)	..(vi)-(i)
	etc			etc						....
13	6 „	..	..	..	..	..	..	..	..(xiii)	..(xiii)-(i)

[In the above table, the L.C. of the micrometer has been taken equal to 0.01 mm.]

removing a load, some time should be allowed before the next reading is taken ; this will hep the wire to elongate or contract fully. (4) The dead-load should be of such value as would keep the wire taut. (5) While taking a reading, care should be taken so that the wire does not oscillate or get twisted.

Oral questions

1. Define the following terms :—(i) Stress (ii) Strain (iii) Youngs modulus.

Ans. Consult any text book.

2. What is the necessity of taking a dead-load ?

Ans. Dead-load removes the kinks of the wire and keeps the wire taut.

3. What is the harm if the two wires of the instrument are hung from two seperate supports ?

Ans. Errors will come in if the support buckles due to load. If the wires are hung from the same support, buckling will affect both the wires equally and no error will come in the measurement of elongation.

4. On what factors does the Young's modulus depend ? Will its value be different if a thicker wire is used ?

Ans. Young's modulus depends on the material. Material remaining the same, Young's modulus will be same whether the wire is thicker or thinner. For the thicker wire, the elongation will be less and for a thinner wire, it will be longer.

5. In measuring which quantity in the above experiment would you take utmost care ?

Ans. In measuring the diameter of the wire ; because its power is 2

6. What is the nature of the load-elongation graph ? Why should it pass through the origin ?

Ans. It is a straight line. As zero load produces zero elongation the straight line should pass through the origin.

7. What is the utility of determining the breaking weight ?

Ans. Every wire has a limit of carrying weight. If the limit is exceeded, the wire snaps. To have an idea of the limit, it is necessary to determine the breaking weight.

8. Why is it that the maximum permissible load is not more than half the breaking weight ?

Ans. Within this limit, the stress is proportional to the strain. If the limit is exceeded, the wire cannot recover the strain fully and a permanent set is left in the wire.

9. What will happen to the diameter of the wire if the length increases ? What is the relevant elastic constant called ?

Ans. The diameter decreases. The relevant elastic constant is called Poisson's ratio. Poisson's ratio$=\frac{\text{lateral strain}}{\text{longitudinal strain.}}$

10. What are the units of Young's modulus and Poisson's ratio ?

Ans. Young's modulus→dynes/cm^2 ; Poisson's ratio→no unit ; it is a pure number.

1·6. Determination of Young's modulus of the material of a wire by torsional oscillation according to Searle's method

Apparatus : Two equal bars made of brass of square cross-section having length about 30 cm and breadth about 2 cm provided with suitable screws and clamps, a wire of about 50 cm length and about 1 mm diameter of the material under test (say iron), a stop-watch, a screw-gauge, a pair of slide callipers, metre scale, a weight box, a balance etc.

Description of the apparatus : Referring to fig 5, two equal brass bars *AB* and *CD* of square section are joined at their centres by a fairly short and moderately thin wire *W* of the material whose Young's modulus is to be determined. The system is suspended by two parallel *torsionless* threads so that in the equilibrium position, the bars *AB* and *CD* may be parallel to each other with the plane *ABDC* horizontal and the wire *W* straight. If the two bars be turned through *equal* angles (θ) in opposite directions (faces *A* and *C* coming closer to each other, say) and be, then, set free, the bars will execute flexural vibrations in a horizontal plane with the same period about their suspension threads.

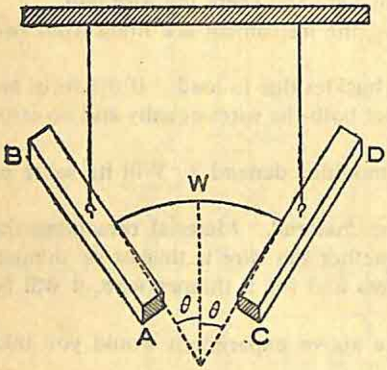

Fig. 5

Theory : (i) When the bars *AB* and *CD* are set into oscillation as mentioned above, the motion of the bars is simple harmonic and hence, the time-period of vibration is given by,

$$T=2\pi\sqrt{\frac{2I.L}{Y.\pi r^4}} \quad \text{or,} \quad Y=\frac{8\pi I.L}{T^2.r^4}$$

where Y=Young's modulus of the material of the wire (W) under test
L=length of the wire
r=radius of the wire
I=moment of inertia of the bar *AB* or *CD* about their suspension thread.
T=time-period of oscillation of *AB* or *CD*.

(ii) If l and b be the length and breadth respectively of the bar *AB* or *CD* and M its mass, then since the bar has rectangular cross-section, its moment of inertia about the suspension thread is given by, $I=\frac{M}{12}(l^2+b^2)$.

Experimental procedure : (1) Take two cotton threads (about 60 cm in length) and suspend the two bars by means of the threads from a rigid support so that when the system is at rest the axes of the bars and the wire (*W*) lie in the same horizontal plane and the wire is straight.

(2) To count the number of oscillations of the bar *AB* conveniently, set up a pointer close to the end *A* and put a chalk-mark on the face *A* in line with the pointer when the bar is at rest.

(3) Make a loop of thin cotton thread and slowly slip the loop over the ends *A* and *C*. The ends will be drawn slightly together and the wire *W* will be bent in the form of an arc (Fig 5). Allow the system to remain constrained for a while.

(4) Burn the thread. The bars will be suddenly released and they commence to oscillate. The ends *A* and *C* (in the same way the ends *B* and *D*) will alternately come closer to each other and recede away from each other.

(5) To count the number of oscillations and the time taken thereon, start the stop-watch when the chalk-mark on the face *A*, moving from right to left crosses the pointer. When the chalk-mark will again cross the pointer, moving from right to left, one complete oscillation is over. Stop the watch as soon as 20 such complete oscillations are executed. Note the time taken for the purpose and find the time-period therefrom.

(6) Repeat the observations, at least, thrice and find the mean time-period (*T*).

(7) Find the length (*L*) of the wire *W* by a metre-scale and radius by a screw-gauge. Find also the length (*l*) and the breadth (*b*) of each bar *AB* and *CD* by slide-callipers. Weigh the bars in a balance and find their mass (*M*).

Measurements : (*a*) *Table for measurement of time-period*

No. of Obs	No. of complete oscillations	Total time (t)	Time-period $=\frac{t}{20}$	Mean time-period (T)
1	20	. . Sec	. . Sec	
2	„	. . Sec	. .	. . Sec
3	„	. . Sec	. .	

(*b*) *Measurement of radius of the wire* (r) :

Make a table for screw-gauge readings like that of the expt. 1·5

(*c*) *Measurement of the length of the wire* (L) :

(i) ... cm (ii) ... cm (iii) ... cm. Mean length = ... cm.

(*d*) *Measurement of length and breadth of the bars* :

Vernier constant of the slide-callipers = .. cm.

Bar	Quantity	No. of Obs	Main scale	Vernier reading	Total (cm)	Mean (cm)	Instrumental error	Corrected value (cm)	Remark
AB	Length (l)	1.	..	..	..				Lengths and breadths are equal
		2.	..	..	..	..	..	..	
		3.	..	..	..				
	Breadth (b)	1.	..	..	..				
		2.	..	..	..	..		..	
		3.	..	..	..				
CD	Length (l)	1.							
		2.						..	
		3.						..	
	Breadth (b)	1.							
		2.							
		3.						..	

(*e*) *Mass of the bars* (M) :

Bar	Weights	Total mass	Remarks
AB	..gm + ..gm + ..mg	...gm	Mass is equal
CD	..gm + ..gm + ..mg	...gm	

(*f*) *Calculations* :

Moment of inertia of the bar, $I = \frac{M}{12}(l^2 + b^2)$ gm–cm^2

$= ..$ gm–cm^2

$\therefore \quad Y = \frac{8\pi L I}{T^2 . r^4} = ...$ dynes/cm^2.

Remarks : (1) While taking observations for the time-period, the amplitude of oscillations should be small ($\theta < 4°$). In that case, threads will remain vertical and there will be no horizontal component of the tension on the wire. Moreover, the strain in the wire will remain within elastic limit. (2) Since the radius of the wire occurs in the fourth power in the formula, and is a small quantity, it should be measured very carefully. Readings should be taken at a number of points equally placed along the wire, two perpendicular diameters being measured at each place. (3) Since time-period occurs in the second power, it should also be measured accurately by timing as large number of oscillations as possible with an accurate stop watch. (4) All undesirable motions should be completely avoided.

Oral questions

1. For the suspension of the bars, why torsionless threads are preferable to meatllic wires ?

Ans. If a metallic wire is used, it will undergo twisting causing a couple to act on the bars. In that case, the formula mentioned in the theory will not be applicable.

2. Is there any condition regarding the amplitude of oscillation while determining the time-period ? If so, why ?

Ans. The amplitude should be small ($\theta < 4°$) ; See remark no. 1.

3. Is there any harm if the cross-section of the bar is circular instead of rectangular ?

Ans. No harm ; only difference is that moment of inertia of the bar

$I = M\left(\frac{l^2}{12} + \frac{D^2}{16}\right)$ where D=diameter of the bar.

4. Why do the bars oscillate? What factor controls the period of oscillation?

Ans. When the ends A and C of the bars approach each other, the wire W bends in the form of an arc. Due to elasticity, the wire develops a shearing couple which sets the bars in torsional oscillation. The bending moment of the wire W controls the period.

5. What are the advantages of Searle's method ?

Ans. By this method, Young's modulus can be very easily determined. In stretching method a long wire is needed but in this method, a short wire will do. Further by Searle's method we can determine the modulus of rigidity and Poisson's ratio of the material of the wire.

6. Why should you take extra care in measuring the radius of the wire ?

Ans. See remark no. 2.

[Questions on expt no. 1.5 are also applicable in this case].

1·7. Cathetometer

Vertical heights are usually measured accurately by a travelling microscope. But the range of a travelling microscope is short, because of its short size. For the measurement of large vertical heights, a cathetometer is a very convenient apparatus.

It consists of a small telescope T [Fig 6] with horizontal axis which can be moved up and down along and clamped to a vertical column AB. The column AB is graduated in centimetre, the position of the telescope on the scale being read by a vernier V on the telescope holder. The telescope T along with AB can be rotated about a vertical axis. The telescope holder is attached to a slider *i.e.* the carriage which slides along AB by a screw P with a large head. By means of the micrometer-screw a fine movement of telescope can be done after the slider has been clamped. The telescope carries a spirit level L. The spirit level along with the three levelling screws (S_1, S_2, S_3) at the three legs of the stand help to make the axis of the telescope horizontal and the column AB vertical. There is a screw Q attached to the telescope which allows the telescope to tilt slightly and thus permits its axis to be adjusted.

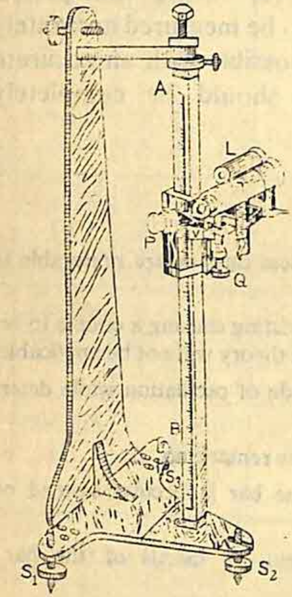

Fig. 6

To use the instrument, it is first adjusted with the aid of the spirit level so that the scale (*i.e.* the column AB) is exactly vertical and the axis of the telescope horizontal. For this adjustment, the following *levelling* procedure is adopted.

(i) Turn the column AB till the axis of the telescope is parallel to the line joining any two of the base levelling screws (say S_1 and S_2) (Some instruments may have two levelling screws, the third leg resting on a pin of fixed height. In such case, the telescope axis should be made parallel to the line joining any of the two levelling screws (say S_1) and the fixed pin). Bring the bubble of the spirit level *half-way* back to the centre by turning the screw Q and the other half by turning the two base screws S_1 and S_2 by equal amounts in *opposite directions* simultaneously. (In the case of a fixed pin instrument, turn the screw Q half-way and the screw S_1 other half-way).

(ii) Now turn the column AB through 180°. If the bubble be not at the centre, bring it half-way back by the screw Q and the other half-way by S_1 and S_2 (or by S_1 only in the case of a fixed pin).

(iii) Next turn the column AB so that the axis of the telescope is at right angles to the previous line joining the base screws S_1 and S_2 (or S_1 and the fixed pin). Bring the bubble back to the centre, if it is found displaced, by turning the screw S_3 only (or S_2 in the case of fixed pin).

Repeat the observations (i), (ii) and (iii) several times. The bubble will finally remain at the centre whatever may be the direction of the telescope axis.

The telescope is then focussed on one end of the height to be measured and its cross-wire made to coincide with it. The position of the telescope is read from the vertical scale. The telescope is then moved up or down along the column AB and its cross-wire is made to coincide with the other end of the height to be measured and the scale reading is again taken. The difference between the scale readings for the two positions of the telescope gives the height to be measured.

1·8. Determination of the Young's modulus of the material of a bar by the method of flexure :

Apparatus : A bar of uniform rectangular cross-section and of length about 1 metre, made of a material whose Young's modulus is required (say, brass), two stout iron stands with levelling screws at the base and a sharp knife-edge fixed at the top, a rectangular stirrup with a knife-edge and a vertical pointer, a travelling microscope (or a cathetometer), standard weights, spirit level, a hanger, metre scale, a pair of slide callipers etc.

Theory : Suppose, a uniform rectangular bar supported *symmetrically* on two horizontal and parallel knife-edges be depressed by a load of mass m gm, suspended from the beam *midway* between the knife-edges. Then the depression x of the *middle* of the bar is given by $x=\dfrac{m.gl^3}{4b.d^3Y}$ or $Y=\dfrac{m.gl^3}{4b.d^3.x}=\dfrac{g.l^3}{4b.d^3}\cdot\left(\dfrac{m}{x}\right)$

where Y=Young's modulus, l=length of the bar between the two knife-edges, b=breadth, d=depth of the bar and g=acceleration due to gravity.

Experiment procedure : (1) Draw a line with the help of a pencil exactly at the centre of the flat face of the bar parallel to breadth. Draw two more similar lines *A* and *B* at equal distances from the extremity (say 5 cm) so that the distance between *A* and *B* may be about 90 cm. Now place the stands *C* and *D* at such distance away that the marks *A* and *B* of the bar may rest on the knife-edges of the stands [Fig 7]. The bar, at first, should rest on the knife-edges with its depth vertical as shown in the figure.

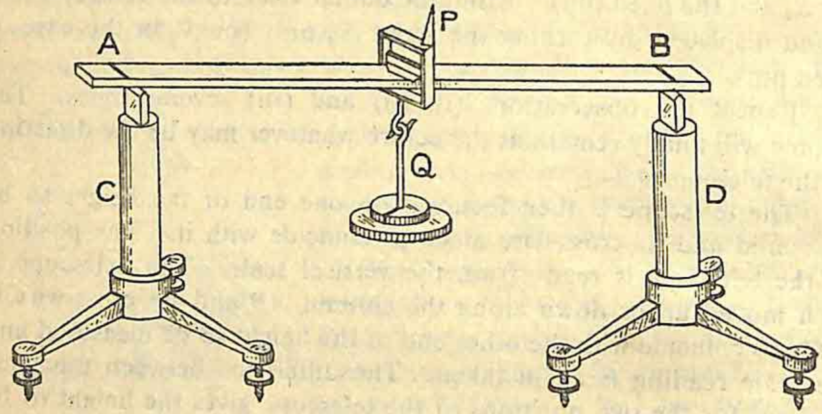

Fig. 7

(2) Put a spirit level on the central pencil mark of the bar and make the bar exactly horizontal by turning the levelling screws at the base of the stands.

[Note : In some apparatus, levelling screws are not provided with. In such cases, levelling is not necessary.]

Remove the spirit level and place the stirrup with the pointer *P* in its place, the knife-edge of the stirrup exactly coinciding with the pencil mark. Suspend the hanger Q from the hook of the stirrup. Make the pointer vertical.

(3) Place a travelling microscope (or a cathetometer) on the table in front of the pointer at such a distance away that the pointer may be well-focussed. By turning the levelling screws make the vertical scale of the microscope (or the cathetometer) vertical and the axis of the microscope (or the cathetometer) exactly horizontal. By adopting usual procedure find the vernier constant of the vertical scale of the microscope (or the cathetometer). Focus the eye-piece of the microscope (or the cathetometer) sharply on the cross-wires, one of which should be horizontal. Now focus the microscope (or the cathetometer) on the pointer *P*. An inverted and magnified image of the pointer will be visible through the microscope (or the cathetometer).

(4) Keeping the hanger empty, raise or lower the microscope (or the cathetometer) along the vertical scale till the pointed end of the pointer just touches the horizontal wire of the cross-wires [Fig 7(*a*)] This coincidence should be done avoiding parallax error. Read the main vertical scale and the vernier. This is the initial reading.

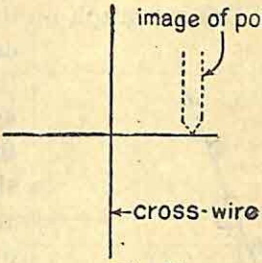

Fig. 7(*a*)

(5) Put a load of ½ kg (or 1 kg if convenient) on the hanger. The bar will be slightly depressed and the tip of the pointer will go a little below the horizontal cross wire. Now adjust the position of the microscope (or the cathetometer) so that tip of the pointer again touches the horizontal wire. The tangent screw provided with the microscope (or the cathetometer) may be used, if necessary, for fine adjustment of the microscope (or the cathetometer). Again read the vertical scale and the vernier.

(6) Increase the load on the hanger by equal steps of ½ kg (or 1 kg as the case may be). Adjust the position of the microscope (or the cathetometer) at each step to bring the tip of the pointer just in touch with the horizontal cross-wire and take the main scale and the vernier scale readings. The observations are to be repeated using six or eight ½ kg loads.

(7) Now remove the loads from the hanger by the same step of ½ kg (or 1 kg) and adjusting the position of the miscroscope (or the cathetometer) so that the tip of the pointer just touches the horizontal cross-wire at each step, take the readings of the main scale and the vernier scale. This is to be repeated till the hanger is again empty. This will give us two readings for each load—one when the load was increasing and the other when the load was decreasing. Find the mean of these two readings. From these mean readings, different loads and corresponding depressions can be found out.

(8) Without disturbing the positions of the stands, slowly turn the rod upside down so that upper face goes below and the lower face comes up. Check whether the distance between *A* and *B* remains the same. Now repeat the observations No. 4, 5, 6 and 7.

(9) Remove the bar taking care not to disturb the positions of the stands. With the help of a metre scale, measure the distance (l) between the knife-edges to the nearest millimeter. Using different parts of the scale, repeat the measurement several times and get the mean value. Using slide callipers, measure the breadth (b) and depth

(*d*) at several places of the bar and find the mean breadth and mean depth.

(10) Draw a graph plotting load m (in kg) along the X-axis and depression x (in cm) along the Y-axis, the origin being (0, 0). Two graphs are to be drawn from the readings of the two surfaces of the bar. The graph should be a straight line passing through the origin [Fig 7(*b*)]. Take a point P on the straight line and draw a perpendicular PM on the X-axis. OM represents a certain load and PM the corresponding depression of the bar. Substituting these values in the equation stated in the theory calculate the value of Y for the material.

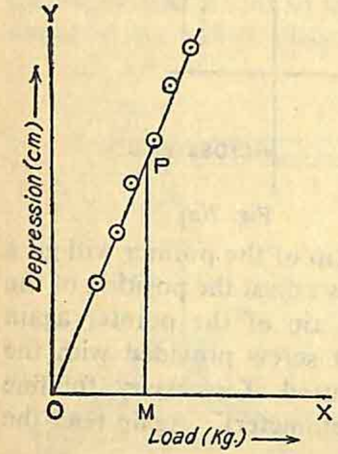

Fig. 7(*b*)

(11) The mean of the two value of Y obtained from the two graphs gives the correct value of the Young's modulus of the material of the bar.

(12) Time permitting, the whole procedure may be repeated, taking different values of l (say, 80 cm) or keeping the bar on its depth.

Measurements : (*a*) *Length of the bar between the knife-edges A and B.*

No. of Obs	Scale reading for the knife-edge A (x cm)	Scale reading for the knife-edge B (y cm)	Length $(y \sim x)$	Mean length (l cm)
1.	...	...	...	...
2.	...	...	...	
3.	...	...	...	

(*b*) *Load-depression table* :

The value of the smallest division of the main scale of the microscope (or the cathetometer)=... cm.

...Vernier divisions=... smallest divisions of main slace
=... cm.

∴ Vernier constant=... cm.

Surface of the rod	No. of Obs	Load on the hanger (Kg)	Readings when load increasing (cm)			Readings when load decreasing (cm)			Mean $\frac{1}{2}(a+b)$ (cm)	Depressions (cm) (x)
			Main scale	Vernier scale	Total reading (a)	Main scale	Vernier scale	Total reading (b)		
Upper	1.	0	..	..	..	..	..	..	..(i)	(i)-(i) = 0
	2.	½	..	..	..	..	..	..	..(ii)	(ii)-(i) = ..
	3.	1	..	..	..	..	..	..	..(iii)	(iii)-(i) = ..
	..	..	..	..	..	..	..	..	..(iv)	(iv)-(i) = ..
	6.	..							..(v)	(v)-(i) = ..
	..	..							..(vi)	(vi)-(i) = ..
	..	etc	..	..	..	..	..	..	etc.	..etc
Lower	1.	0	..	..	..	..	..	..	..	..
	2.	½	..	..	..	..	..	..		
		1	..	..	..	..	..	..	..	..
	3.	..								
	..	..								
	etc.	6								..etc
		..	..	..	..	..	..	..	..	

(c) *Measurement of breadth and depth of the bar* :

Vernier constant of slide-callipers= . . cm.

Quantity to be measured	No. of Obs	Main scale reading	Vernier scale reading	Total reading	Mean reading (cm)	Instrumental error	Corrected reading (cm)
Breadth (b)	1. 2. 3.	. . . . . .	. . . . . .	. . . . . .	. .	. .	. .
Depth (d)	1. 2. 3.	. . . . . .	. . . . . .	. . . . . .	. .	. .	. .

(d) *Table for drawing graph* [Data taken from load-depression table (b)] :

Quantity	1	2	3	4	5	6	7	8	9
Load (in Kg)→	½	1	1½	2	2½	3	3½	4	4½
Depression (in cm) →	. .	. .	. .	. .	. .	. .	. .	. .	. .

Calculations : (a) From the 1st graph, load (OM)= . . kg= . . gm(m)

Depression (PM)= . . cm (x)

$$\therefore \ \frac{m}{x} = \ldots \text{gm/cm}$$

(b) From the 2nd graph, load (OM)= . . kg= . . gm(m)

Depression (PM) = . . cm(x)

$$\therefore \ \frac{m}{x} = \,.\,.\,\text{gm/cm.}$$

From the 1st graph, $Y_1 = \frac{gl^3}{4b.d^3}\left(\frac{m}{x}\right) = \,.\,.$ dynes/cm^2 [$g=980$ cm/s^2]

,, ,, 2nd ,, $Y_2 = \,.\,.$ dynes/cm^2

$$\text{Mean } Y = \frac{Y_1 + Y_2}{2} = \,.\,.\ \text{dynes/cm}^2$$

Proportionate and Percentage error : (For Honours course only) :
See Ex 1, Page 4

Remarks : (1) As length and width of the bar occur in their third power in the formula, they should be measured very carefully. Slight error in the measurement of length or width will cause much error in the final result. (2) The load must be suspended from a point exactly mid-way between the knife-edges. (3) Instead of microscope or cathetometer, depression of the bar can also be measured by optical lever arrangement. (4) To secure symmetrical loading of the bar, the knife-edge of the stirrup should be parallel to the knife-edges of the stands.

Oral questions

1. Which quantity in this experiment should be measured with utmost care ?

Ans. Length and width of the bar. They occur in their third power in the formula.

2. Should you take a long length or a short length of the bar ?

Ans. A long length is preferable ; for depression is proportional to cube of the length. Hence, greater the length, greater is the depression and less the error in measuring the depression.

3. Will the Young's modulus be different if the breadth and width of the bar are altered ?

Ans. No ; Young's modulus is a property of the material ; As long as the material is not changed, Young's modulus will remain the same whatever may the diamension of the bar.

4. What is a cantilever ?

Ans. Consult any text book.

[**N.B.** Questions on Expt 1.5 are also applicable here.]

1·9. Determination of the modulus of rigidity of a wire by statical method :

Apparatus : Barton's rigidity apparatus, screw-guage, slide-callipers, weight-box, metre scale etc.

Description of the apparatus : Barton's vertical torsion apparatus (Fig 8) consists of a wire of the material under test fixed at the torsion head *T* at the top to a rigid support and at the bottom to a heavy cylinder *C* of brass. The cylinder being very heavy keeps the wire vertical. Two parallel flexible threads leave opposite sides of the cylinder tangentially. Passing over two small frictionless pulleys, they carry at the other ends scale-pans of equal weights. Equal loads are placed on the pans. Loads produce twist in the wire which is measured by means of pointer P and a circular scale S graduated in degrees. The pointer can move over the circular scale without friction. There are three levelling screws at the base of the frame.

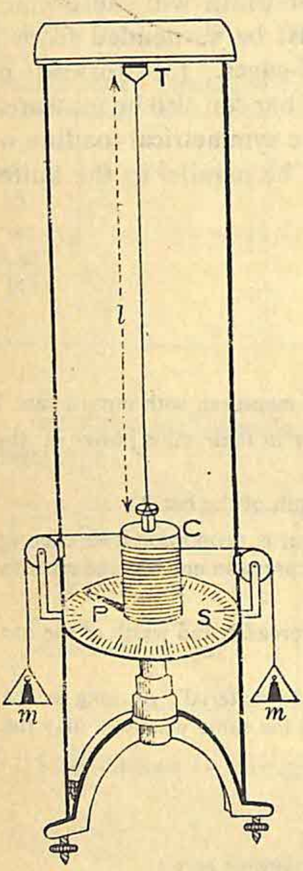

Fig. 8

Theory : Suppose, the length of the wire $= l$: radius of the wire $= r$: load placed on each of the pans $= m$: the diameter of the cylinder $C = D$. twist in the wire $= \theta$ radian.

In this case, the moment of torsional couple $= \frac{\pi n\theta r^4}{2l}$ where $n =$ modulus of rigidity of the material of the wire.

The moment of the external couple exerted by the loads on the pans $= m.g.D$.

For equilibrium, $\frac{\pi n\theta r^4}{2l} = m.g.D$.

$$\text{or} \quad n = \frac{m.g.D \times 2l}{\pi\theta.r^4}$$

If the angle of twist be expressed in degrees and if it be ϕ°,

then $\phi^\circ = \frac{\pi\phi}{180}$ radian $= \theta$.

$$\text{So, } n = \frac{m.g.D \times 2l \times 180}{\pi r^4 \times \pi\phi} = \frac{360.l.g.D}{\pi^2.r^4}\left(\frac{m}{\phi}\right).$$

Experimental procedure : (1) Level the base of the apparatus by means of the levelling screws, testing the levelling with a spirit level. Adjust the pulleys to be at the same level and see that the strings leaving the cylinder in the opposite directions are parallel and also tangential to the cylinder and the pulleys.

(2) Measure length (l) of the wire from the torsion head T upto the point where the wire is joined with the cylinder by a metre scale. Repeat the measurement several times and find the mean length.*

(3) With the help of a screw-gauge, measure the diameter of the wire at different points and at mutually perpendicular directions at

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***Note :** In some instruments, the lower end of the wire is fixed to the circular scale which is attached to the frame *above* the cylinder. In such case, the length of wire means the length from the torsion head to the scale.

every point. From these measurements, find the radius (r) of the wire.

(4) Measure the diameter (D) of the cylinder C at different places by slide callipers and calculate the mean value of the diameter.

(5) Keeping no load on the scale-pans, read the position of the pointer P against the circular scale. In some instruments, double-end pointers are provided. In such case, both the ends of the pointer are to be read off. This is the initial reading.

(6) Next place gently a load of, say 20 gm, on each pan. This will change the value of the applied couple and hence the twist in the wire. Again note the reading of the pointer. The change in the reading evidently gives the twist in the wire due to the additional couple applied on the wire.

(7) Now increase the load on the pans by equal amounts till maximum permissible load is reached and take down the reading of the pointer after addition of each load.

(8) Next remove the loads from the pans by the same equal amounts as were used previously till the pans become empty. Note down the reading of the pointer after the removal of each load. This gives us two readings for angle of twist for each load—one when the loads were increasing and the other when the loads were decreasing. Take the mean of the readings for each load. Subtracting the mean reading for zero load from the mean readings for other different loads, angles of twist for various loads can be found out.

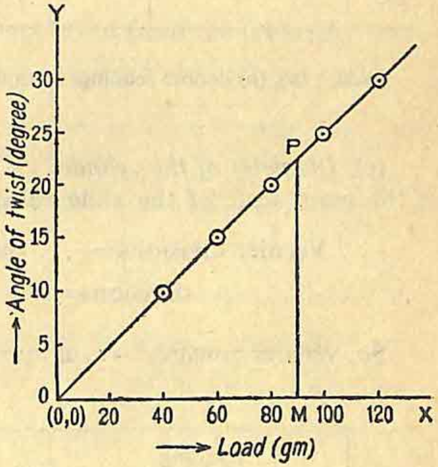

Fig. 8(a)

(9) Plot a graph with loads (in grammes) along the X-axis and angles of twist (in degrees) along the Y-axis with (0, 0) point at the origin. It will be a straight line passing through the origin [Fig 8(a)]. Taking a point P on the straight line draw a perpendicular PM on the X-axis. OM represents a certain load (m) and PM its corresponding angle of twist (ϕ). Find the value of $\frac{m}{\phi}$. Substituting the value in the equation stated in the formula, n can be found out.

Measurements : (*a*) *Length of the wire* :

(i) .... cm (ii) .... cm (iii) ....cm. Mean length (l)= .... cm.

(*b*) *Radius of the wire* : Value of the smallest division of the linear scale of the screw-gauge=... mm. Screw-pitch=... mm. Total number of divisions in the circular scale (*S.C.*)=...

$$\therefore \text{ Least count}=\frac{\text{Screw-pitch}}{\text{Total no. of div. in } S.C.}=\ldots\text{mm.}$$

No. of Obs	Reading (mm)			Mean reading (mm)	Instrumental error (mm)	Corrected diameter (mm)	Radius (r) (cm)
	Linear scale	Circular scale	Total				
1. (*a*)	..	..	..				
(*b*)	..	..	..				
				..	..	..	..
2. (*a*)	..	..	..				
(*b*)	..	..	..		etc		
etc							

[N.B. (*a*), (*b*) denote readings at mutually perpendicular directions.]

(*c*) *Diameter of the cylinder* : The value of the smallest division of the main scale of the slide callipers=.... mm

... Vernier divisions=.... smallest divisions of the main scale
∴ 1 ,, division=.. ,, ,, ,, ,, ,, ,,

So, vernier constant=.. mm=....cm.

No. of Obs	Reading			Mean reading (cm)	Instrumental error (cm)	Corrected diameter (D) (cm)
	Main scale (cm)	Vernier scale	Total (cm)			
1.	..	..	..			
2.	..	..	..	..	..	..
3.	..	..	..			
etc						

(*d*) *Load-angle of twist table* :

No. of Obs	Load on each pan (m gm)	Pointer readings in degree		Mean reading $\frac{1}{2}(a+b)$	Angle of twist (ϕ) (degree)
		Load increasing (a)	Load decreasing (b)		
1	20	..	..	..(i)	(i)−(i)=0
2	40	..	..	..(ii)	(ii)−(i)=..
3	60	..	..	..(iii)	(iii)−(i)=..
4	80	..	..	..(iv)	(iv)−(i)=..
5	100	..	..	..(v)	(v)−(i)=..
..	..				
etc	etc	etc	etc	etc	etc

[N.B. If the pointer has two ends, the columns under the headings 'load increasing' and 'load-decreasing' should be divided into two parts—one for end I and the other for end II of the pointer. The mean of the four readings available for each load should be put in the column under the heading 'Mean reading'.]

(*e*) *Table for drawing graph* [Data taken from the table (*d*)]

Quantity	1	2	3	..	..	..	8
Load (gm) →	20	40	60	..	..	..	..
Angle (degree) →	..	..	..	..	..	..	..

Calculations : From the graph, load (OM)= .... gm (m)
,, ,, ,, angle of twist (PM)= .... °(ϕ)

$$\text{So, } n=\frac{360.l.g.D}{\pi^2.r^4}\left(\frac{m}{\phi}\right)=\ldots\text{dynes/cm}^2 \quad [g=980\ \text{cm/s}^2]$$

Remarks : (1) For all metals Hooke's law holds only for small shear and ceases to apply when the shear exceeds 1/200 radians or $\frac{1}{3}^\circ$. In practice, the shear should not exceed $\frac{1}{9}^\circ$. Hence angle of twist $\phi=\frac{l\theta}{r}<\frac{l}{9r}$ degrees. When $l=50$ cm, $r=0{\cdot}2$ cm, $\phi_{max}=28^\circ$

(2) As the radius of the wire occurs in its fourth power in the equation and is a very small quantity, the diameter should be measured accurately
(3) The loads should be placed or removed from the pan gently

(4) A two-end pointer is preferable because the mean reading of both ends will be free from errors due to eccentricity of the axis of the wire with respect to the circular scale.

1·9. Alternative Method

The method described earlier suffers from an error due to the uncertainty in the exact position of the wire where it is clamped. The error can be eliminated by noting down twists for two lengths of the wire. The details of the method are as follows :

Theory : We have seen that $n=\frac{360.l.g.d}{\pi^2.r^4}.\left(\frac{m}{\phi}\right).$

If ϕ_1 and ϕ_2 are the angles of twist for two lengths l_1 and l_2 of the wire, for the same mass m then twist for the length (l_2-l_1) of the wire will be $(\phi_2-\phi_1)$ and hence the above expression for n becomes,

$$n=\frac{360.m.g.d}{\pi^2 r^4}.\frac{(l_2-l_1)}{(\phi_2-\phi_1)}.$$

Description of the apparatus : For the determination of the quantities involved in the above equation, Barton's apparatus is modified to suit the purpose. Fig 9 shows a modified form of Barton's apparatus. Here two or three circular scales graduated in degrees are attached to the frame at two or three different lengths of the wire. The circular scales are provided with pointers (P_1, P_2 etc) either single-ended or double-ended. The cylinder C is kept below all the circular scales. In this arrangement, for a particular load, different lengths will have different angle of twist which can be measured by different circular scales with its pointer.

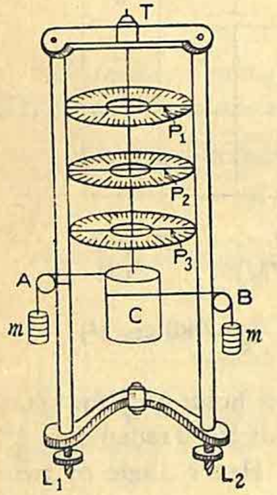

Fig. 9

Experimental procedure : (1) Levelling, measurment of radius of the wire and diameter of the cylinder are done as before.

(2) Measure the lengths of the wire from the torsion head T to the pointers of the three circular scales by a metre scale. This gives three different lengths of the wire.

(3) As before, gradually increase the loads on the pans from zero value to the maximum permissible value, by equal instalments (of 20 gm, say) and note the readings of the three pointers against their respective circular scales, after addition of each load.

(4) Now, remove the loads, one by one, and again read the pointers against the respective circular scales after the removal of each load.

(5) Plot a graph as shown in fig 8(*a*) [Page43] between the load (m) and twist (ϕ) for each value of the length of the wire. From the graph find the best value of ϕ/m for each value of l.

(6) Next plot a graph between ϕ/m and l. The graph will again be a straight line, passing through the origin [Fig 9(*a*)]. Take two convenient points P_1 and P_2 on this line and draw perpendiculars P_1M_1 and P_2M_2 on the X-axis. Now, $OM_2 - OM_1 = l_2 - l_1$ and $P_2M_2 - P_1M_1 = \phi_2 - \phi_1$. The values can be easily obtained from the graph and putting these values in the equation mentioned in the theory, n can be calculated.

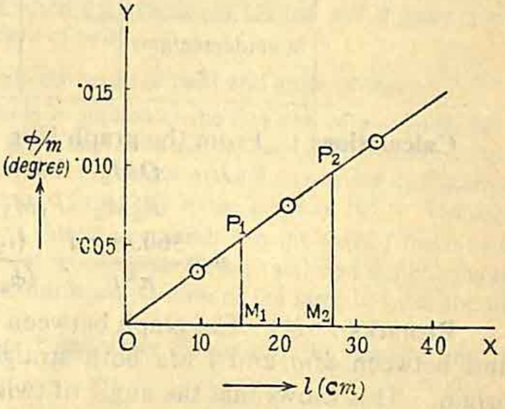

Fig. 9(*a*)

Measurements : (*a*) Tabulate the data for the measurements of diameters of the wire and the cylinder as the tables (*b*) and (*c*) in Expt no. 1·8.

(*b*) *Lengths of the wire :*

No of Obs	l_1 (cm)		l_2 (cm)		l_3 (cm)	
	Readings	Mean	Readings	Mean	Readings	Mean
1.	..		..		..	
2.	..	..	..	..	..	..
3.	..		..		..	

(*c*) *Load-angle of twist table :*

Make a table like the table (*d*) in Expt 1·8 for each length. In a single graph, plot three straight lines for $(m-\phi)$ for three separate lengths. From the straight lines, find three best values of $\dfrac{\phi}{m}=\dfrac{PM}{OM}$ [Fig 8(a)] for three separate lengths.

(*d*) *Table for drawing* $(l-\phi/m)$ *graph* :

Quantities	1	2	3
Length (cm) →	..	..	..
ϕ/m (degree/gm) →	..	..	..

Calculations : From the graph [Fig 9(a)] :

$$OM_2-OM_1=l_2-l_1=..\ \text{cm.}$$

$$P_2M_2-P_1M_1=(\phi_2-\phi_1)/m=..\ \text{degree/gm.}$$

$$\therefore\ n=\frac{360.m.g.d}{\pi^2.r^4}\cdot\frac{(l_2-l_1)}{(\phi_2-\phi_1)}=..\ \ ..\text{dynes/cm}^2$$

Remarks : (1) The graph between m and ϕ for a given value of l and between ϕ/m and l are both straight lines passing through the origin. This shows that the angle of twist is proportional to the value of the couple applied and that if l is changed, the value of twist for a given couple is proportional to the length of the wire. (2) With this apparatus, comparatively thin specimens can be tested.

Oral questions

1. What is modulus of rigidity ? What is its unit ?

Ans. For definition, consult any text book. In C.G.S. system, its unit is dynes/cm^2

2. What other elastic constants do you know of ? What are their definitions ?

Ans. Young's modulus, Bulk modulus and Poisson's ratio are other elastic constants. For their definitions, consult any text book.

3. In the equation of the modulus of rigidity, the length and radius of the wire are involved. What change in the value of the modulus would you find when these quantities are changed ?

Ans. No change ; modulus of rigidity depends on the material and not on the length or radius.

4. For the same load, will the angle of twist increase or decrease for thinner wire ?

Ans. $\phi\propto\frac{1}{r^4}$; so, angle of twist will increase for thinner wire.

5. In this experiment, which quantity would you measure with utmost care ?

Ans. The radius of the wire should be measured with utmost care. See remark no. 2 in Expt 1·8.

6. Has temperature any effect on the modulus of rigidity of a wire ?

Ans. Modulus of rigidity decreases with the rise of temperature.

7. Which one is better—one-end pointer or double-end pointer. Why ?

Ans. Double-end pointer. See remark no. 4 in Expt 1·8.

8. What is the harm if the wire is not rigidly fixed to the cylinder ?

Ans. If the wire is not rigidly fixed to the cylinder, the wire will not undergo the extent of twist that it should when the cylinder is twisted and thereby it will cause some error in the measurement of twist.

9. What is the difference between angle of twist and angle of shear ?

Ans. When a torsional couple is applied at the free end of a cylinder (or a wire), the other end being rigidly fixed, any point on the edge of the cross-sectional area of the free end moves along the arc of a circle making an angular displacement (ϕ) at the centre of the cross-section. This angle is the angle of twist. The angle of shear (θ) is the angle between a line drawn parallel to the axis of the cylinder connecting two points at the edges at its two ends before twist and the line joining this point at the fixed end with the displaced position of the same point at the free end. Mathematically, $\theta=\frac{r.\phi}{l}$ where r and l are respectively the radius and the length of the cylinder.

10. How is the maximum permissible twist in the experiment to be calculated ?

Ans. See remark no. 1 in Expt 1·8.

1·10. Determination of modulus of rigidity of the material of wire by dynamical method :

Apparatus : Torsional pendulum, stop-watch, screw-gauge, slide-callipers, balance etc.

Theory : If a solid cylinder be suspended by a long wire from a torsion head, forming a torsional pendulum and if the pendulum be set into torsional oscillations, the time-period of such oscillation is given by $T=2\pi\sqrt{\frac{I}{\tau}}$ or $\tau=\frac{4\pi^2 I}{T^2}$ where I is the moment of inertia of the cylinder about the suspension wire as axis and τ=shearing couple for one radian.

Now, if the axis of the cylinder coincides with axis of rotation, then $I=\frac{M.R^2}{2}$ where M=mass of the cylinder and R=radius of the cylinder.*

Again, for the suspension wire, $\tau=\frac{\pi.n.r^4}{2l}$ where l=length, r= radius and n=modulus of rigidity of the material of the suspension wire.

$$\therefore \quad \frac{\pi.nr^4}{2l}=\frac{4\pi^2.I}{T^2} \text{ or } n=\frac{8\pi.I}{r^4.T^2}\times\tfrac{1}{2}MR^2$$

Description of the apparatus : The torsion pendulum is illustrated in fig 10. It consists of a solid cylinder A suspended by a fairly thin and long wire. One end of the wire is rigidly fixed to a torsion head T and the other end is connected with the centre of the cylinder by means of a detachable pin P. With the help of the pin P, the cylinder A can be detached from the experimental wire. A pointer O is fixed at the bottom of the cylinder. When the cylinder undergoes torsional oscillations the pointer moves over a circular scale (S), graduated in degrees. The suspension wire coincides with the axis of the cylinder.

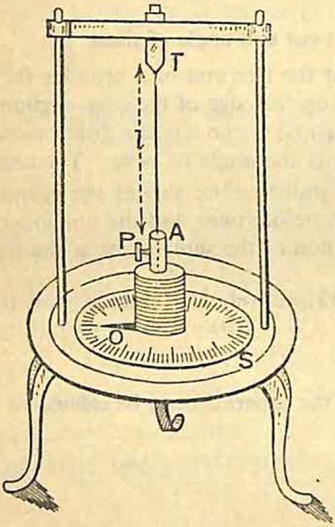

Fig. 10

Experimental procedure : (1) With the help of a screw-gauge, find the diameter of the wire at different places and at mutually perpendicular directions at every place. Find, from these observations, the mean diameter and hence the radius (r) of the wire.

(2) Now measure with the help of a metre-scale the length of the wire between the torsion head T and the point where it is connected with the cylinder. Repeat the observations several times and find the mean length (l) of the wire.

---

*[(i) If the bob of the pendulum be a disc, instead of a cylinder, $I=\frac{1}{2}MR^2$ (ii) If the bob of the pendulum be a solid cylinder with horizontal axis and the suspension wire passes through the centre of the cylinder. $I=M\left(\frac{L^2}{12}+\frac{R^2}{4}\right)$; L=length of the cylinder.

(3) Put a chalk-mark on the circular scale just below the pointer when the pendulum is at rest. Now give the cylinder a twist of about 60° and release it. The cylinder will execute torsional oscillations. When undesirable motions have subsided, say after two or three oscillations, start the stop-watch as soon as the pointer, moving from left to right, (say) crosses the chalk-mark. When the pointer again crosses the chalk-mark, moving from left to right, it executes one complete oscillation. With the help of the stop-watch, find the time taken by the pendulum for, 20 such complete oscillations. Repeat the observations, at least, thrice and find the mean time taken for 20 complete oscillations. From this determine the time-period (T).

(4) By taking out the pin P, detach the cylinder A from the wire. Find the mass (M) of the cylinder by a balance. [In some instruments, the wire is rigidly fixed to the cylinder. In such a case, the mass of the cylinder is to be supplied.]

(5) With the help of slide callipers, measure the diameter of the cylinder at various places and at mutually perpendicular directions at each place. Find, from these observations, the mean diameter and hence the radius (R) of the cylinder.*

Measurements : (*a*) *Diameter of the wire* :

The value of the smallest division of the linear scale of the screw-gauge=...mm. Screw-pitch=...mm ; Total no. of divisions on the circular scale=...

∴ Least count=....mm.

No. of Obs	Linear scale reading (mm)	Circular scale reading	Total reading (mm)	Instrumental error (mm)	Corrected diameter (mm)	Corrected diameter (cm)	Mean diameter (D) (cm)
1. (*a*)	..	..	..		..	..	
(*b*)	..	..	..		..	..	
2. (*a*)	..	..	..	..	..	..	
(*b*)	..	..	..		..	..	..
etc.	etc.	etc.			etc.	etc	
5. (*a*)	..	..	..		..	..	
(*b*)	..	..	..				

[N.B. (*a*), (*b*) denote mutually perpendicular readings at a particular position.]

∴ Mean rådius of the wire (r)=$D/2$=... cm.

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*(i) For a circular disc, its diameter and (ii) for a suspended cylinder with its axis horizontal the length and diameter of the cylinder need be measured.

(b) Length of the wire :

(i) .. cm (ii) .. cm (iii) .. cm

∴ Mean length (l) = .. cm.

(c) Measurement of time-period :

No. of Obs	Total time for 20 complete oscillations	Mean time for 20 oscillations (t)	Time-period $T=t/20$
1.	..min..sec	..min..sec	..sec
2.	..min..sec		
3.	..min..sec		

(d) Measurement of mass of the cylinder :

Mass (M) = .. gm.

(e) Diameter of the cylinder :

The value of the smallest division of the main scale of the slide-callipers = .. cm.

...vernier divisions = ..smallest divisions of the main scale

= ..cm.

or 1 ,, division = ..cm.

∴ Vernier constant = ... cm.

No. of Obs		Main scale reading (cm)	Vernier scale reading	Total reading (cm)	Mean Diameter (cm)	Instrumental error (cm)	Corrected Diameter (D) (cm)
1.	(a)	..	..	..	..	..	..
	(b)	..	..	..			
2.	(a)	..	..	..			
	(b)	..	..	..			
etc		etc	etc.	etc			
5.	(a)	..	..	..			
	(b)	..	..	..			

[N.B. (*a*), (*b*) denote readings at mutually perpendicular directions at a particular place.]

∴ Mean radius of the cylinder $(R)=\frac{D}{2}=$ ...cm.

Calculations : $n=\frac{8\pi l}{r^4.T^2}\times\frac{1}{2}.M.R^2=$ .. dynes/cm^2.

Remarks : (1) The axis of the cylinder and the suspension wire must remain in the same vertical line. (2) The motion of the torsion pendulum should be purely rotational in a horizontal plane. Up and down or pendulus motion should be completely checked. (3) The wire should not be twisted beyond elastic limit ; otherwise the torsional couple will not be proportional to the twist. (4) The suspension wire should be without kinks and be fairly thin and long, say about 75 cm long and 10 mm thick so that the torsional rigidity may be small and hence the time-period large. (5) As the radius of the wire occurs in its fourth power and time-period in its second power in the expression for *n*, they must be measured very accurately. A stop-watch reading upto $\frac{1}{5}$th of a second is preferable.

Oral questions

1. Of all the quantities involved in the experiment, which one should be measured with greatest care and caution ?

Ans. Radius of the wire as it enters into the expression for *n* in its fourth power. A small error in the measurement of radius will increase the error in the final result fourfold.

2. Should you take a large number of oscillations or a small number ?

Ans. If a smaller number of oscillations is taken, time involved will be smaller and the proportionate error in determining the time will be greater. On the other hand, if a large number of oscillations is taken, time of completion of the experiment will be unnecessarily lengthened. Number of oscillation should preferably be restricted somewhere between 20 and 30.

3. What is moment of inertia ?

Ans. Consult any standard text book.

4. Is it necessary that the oscillations should have small amplitude like a simple pendulum ? If not, why ?

Ans. There is no restriction on the amplitude of oscillation, particularly when the wire is fairly thin and long, provided the elastic limit is not exceeded.

5. Will the modulus of rigidity increase if a thicker wire is taken ?

Ans. No, modulus of rigidity will remain same whatever may be the thickness. Modulus of rigidity depends on the material of the wire.

6. Will the time-period of oscillation change if the cylinder is made heavier ?

Ans. The moment of inertia of the cylinder increases as the cylinder is made heavier. Again as time-period is directly proportional to moment of inertia, the time-period increases if the cylinder is made heavier.

7. What is the harm if the suspension wire does not coincide with the axis of the cylinder ?

Ans. In that case, the rotation of the cylinder will not take place with the suspension wire as axis and the formula ($I=MR^2/2$) for the determination of its moment of inertia will not apply.

8. Should the wire be long ?

Ans. It should be fairly long ; in that case the time-period will be longer.

9. What should be the change in time-period if (i) l were doubled (ii) r were doubled (iii) I were doubled ?

Ans. $T=2\pi\sqrt{\dfrac{I}{\tau}}$; So (i) No change in T when l is doubled (ii) No change in T when r is doubled (iii) T is doubled when I is doubled.

10. Do you get the same value for n from statical and dynamical methods ?

Ans. For the same material, the value will be same from the two methods.

1·11. Determination of the moment of inertia of a body about an axis passing through its centre of gravity and perpendicular to its length :

Apparatus : A cradle inside a glass box, a metallic bar of rectangular cross-section, stop-watch, slide-callipers, balance, weight box, an auxiliary bar of regular geometric shape (say, a rectangular bar) etc.

Theory : If a body be suspended by a wire(which can undergo twisting)in such a manner that the wire passes through the C.G. of the body and if the body is set into torsional oscillation, the time-period of such oscillations is given by, $T=2\pi\sqrt{\dfrac{I}{\tau}}$

where I=moment of inertia of the body about the suspension wire as axis and τ=torsional rigidity of the material of the suspension wire. Now suppose,

T_1=Time-period of torsional oscillation of the empty craddle about the suspension wire as axis.

T_2=Time-period of torsional oscillation of the craddle and a body of known moment of inertia about the same axis.

T=Time-period of torsional oscillations of the craddle and the body under test about the same axis.

Then it may be shown* $I=I_2.\dfrac{T^2-T_1^2}{T_2^2-T_1^2}$, where I=Moment of inertia of the body under test and I_2=that of the known body.

Description of the apparatus : Fig 11 shows a torsional cradle kept in a glass box. The cradle *R* is suspended by a wire *W*, the upper end of which is rigidly fixed to a torsion-head *T*. It is enclosed in a box (*A*) with glass windows. This prevents the air-current from disturbing the oscillations of the cradle. The cradle is so suspended that the wire passes through its centre of gravity—*i.e.* when the cradle executes torsional oscillations, oscillations take place in a horizontal plane with the wire *W* as axis. On the upper surface of the box, two long and thin slots are cut (*S*, *S*), through which oscillations of the cradle can be watched.

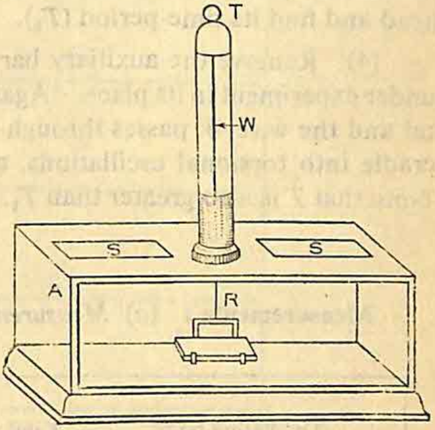

Fig. 11

Experimental procedure : (1) With the help of a balance, find the mass (*M*) of the auxiliary bar supplied. Also measure the length (*l*) and breadth (*b*) of the bar (if it is a rectangular bar) by a slide callipers. Find its moment of inertia from the formula

$$I_2=M(l^2+b^2)/12.$$

[**N.B.** If the auxiliary bar is a cylinder of circular cross-section, measure the length and radius (r). Then apply the formula $I_2=M\left(\frac{l^2}{12}+\frac{r^2}{4}\right)$.]

(2) Set the empty cradle into torsional oscillations by slightly turning the torsional head *T*. Count, by means of a stop-watch, the time taken for 20 complete oscillations. Repeat the observations, at

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*If I_1 be the moment of inertia of the empty craddle about the suspension wire as axis then,

$T_1=2\pi\sqrt{\frac{I_1}{\tau}}$. Also $T_2=2\pi\sqrt{\frac{I_1+I_2}{\tau}}$ and $T=2\pi\sqrt{\frac{I_1+I}{\tau}}$

$\therefore\ T_1^2=\frac{4\pi^2\times I_1}{\tau}$; $T_2^2=\frac{4\pi^2\times(I_1+I_2)}{\tau}$ and $T^2=\frac{4\pi^2(I_1+I)}{\tau}$

Hence, $\frac{T^2-T_1^2}{T_2^2-T_1^2}=\frac{4\pi^2.I.\tau}{4\pi.^2I_2\tau}$ or $I=I_2.\frac{T^2-T_1^2}{T_2^2-T_1^2}$

least, thrice and find the mean time for 20 complete oscillations. Hence find the time-period (T_1) of the empty cradle.

(3) Put the auxiliary bar on the cradle. Take care that the bar is horizontal and that the wire W passes through the centre of the bar. As before, set the cradle into oscillation by slightly turning the torsion head and find its time-period (T_2). Note that T_2 is greater than T_1.

(4) Remove the auxiliary bar from the cradle and place the rod under experiment in its place. Again check whether the rod is horizontal and the wire W passes through the centre of the rod. Setting the cradle into torsional oscillations, as before, find the time-period (T). Note that T is also greater than T_1.

Measurements : (*a*) *Measurement of Time-periods* :

Oscillating body	Total time for 20 complete oscillations (t)	Mean time (t)	Time-period ($t/20$)
1. Empty cradle	(i) . . min . . sec (ii) . . min . . sec (iii) . . min . . sec	. . min . . sec	. . sec (T_1)
2. Cradle + Auxiliary bar	(i) . . min . . sec (ii) . . min . . sec (iii) . . min . . sec	. . min . . sec	. . sec (T_2)
3. Cradle + Rod under test	(i) . . min . . sec (ii) . . min . . sec (iii) . . min . . sec	. . min . . sec	. . sec (T)

(*b*) *Mass of the auxiliary rod* :

$M = \dots$ gm.

(*c*) *Length and breadth of the auxiliary rod* :

Vernier constant of slide-callipers = . . . cm.

No. of Obs	Length (*l*) in cm						Breadth(*b*) in cm					
	Main scale (cm)	Vernier scale	Total reading (cm)	Mean reading (cm)	Instrumental error (cm)	Corrected length (cm)	Main scale (cm)	Vernier scale	Total reading (cm)	Mean reading (cm)	Instrumental error (cm)	Corrected breadth (cm)
1.	..	..	..				..	..	..			
2.	..	..	..	..	..	..	..	..	..	..	..	..
3.	..	..	..				..	..	..			

Calculations : Moment of inertia of the auxiliary bar

$$I_2 = M(l^2+b^2)/12 = \ldots \text{ gm}-\text{cm}^2$$

$$I = I_2 . \frac{T^2 - T_1^2}{T_2^2 - T_1^2} = \ldots \text{gm}-\text{cm}^2$$

[**Note :** If the rod under test is rectangular, find its length, breadth and mass and determine its moment of inertia theoretically from the equation $I=M(l^2+b^2)/12$ and compare the theoretical value and the experimental value.]

Remarks : (1) The cradle should always remain horizontal so that its moment of inertia I_1 about the axis of rotation remains unaltered throughout the whole experiment. (2) The bodies should be so placed on the cradle that the axis about which the moment of inertia is known or to be determined coincides with the axis of the suspension wire without disturbing the horizontality of the cradle. (3) The motion of the cradle should be purely rotational in a horizontal plane. Up and down and pendulus motion must be completely avoided. (4) As time-periods occur in the 2nd power in the final formula, they must be accurately determined. (5) The suspension wire should not be twisted beyond elastic limit, otherwise the restoring couple due to torsional reaction will not be proportional to the angle of twist.

1·12. Alternative method (Using a moment of inertia table)

Apparatus : All the apparatus of the previous experiment except the glass box containing the cradle. In its place, a moment of inertia table is used.

Theory : As in expt 1·11.

Description of the apparatus : The moment of inertia table consists of a circular aluminium table (*R*) about 6″ in diameter suspended by means of a wire (*W*) fixed at the top to a torsion head *T* fitted to a cross-bar between two pillars which stand vertically on a heavy iron base *A* provided with three levelling secrews (S_1, S_2 etc). The table is made of aluminium because it will be light so that the time-period of oscillation of the empty table will differ much from the time period of oscillation when some other body is kept on the table. The upper face of the table has a few concentric circles described on it (one such circle is shown by dotted line) which helps symmetrical placing of other bodies on the table. On the table a concentric groove is cut in which four balancing masses (m, m etc) may be placed in order to set the table horizontal. Two spirit levels (L_1 and L_2) are permanently fixed on the table at right angles to each other to test the horizontality of the table. The whole apparatus is kept enclosed in a glass case to protect it from air-current disturbances.

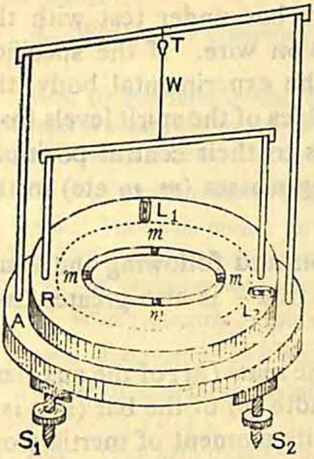

Fig. 11(*a*)

Experimental procedure : (1) Level the heavy iron base *A* by the levelling screws and test the levelling with a spirit level. Adjust the positions of the masses (m, m etc) in the groove so that the bubbles of the spirit levels (L_1 and L_2) are brought at the centre. This ensures that the table *R* is horizontal.

(2) Put a chalk-mark on the edge of the disc *R* and set up a vertical pointer just opposite to the chalk-mark on the iron base *A*. Now set the table oscillating by slightly rotating it in a horizontal plane and then releasing it. Take care that the table has no up and down motion or pendulus motion. Start the stop-watch as soon as the chalk-mark crosses the pointer, moving from ,say left to right. When the chalk-mark will again cross the pointer, moving from left to right, one complete oscillation will be over. Determine thrice the time taken for, say 20 such complete oscillations, by means of an accurate stop-watch (reading $\frac{1}{5}$th of a second) and from it find the time-period (T_1).

(3) Next, taking the help of concentric circles, place centrally on the table *R* the auxiliary bar. Move it to and fro till the bubbles of the spirit levels L_1 and L_2 are at their central positions. *Do not disturb the balancing masses.* This ensures the horizontality of the table with the auxiliary bar and the passage of the suspension wire through the C.G. of the bar.

(4) Having done this find the time-period (T_2) of the system by setting it into torsional oscillation. (Note that T_2 is greater than T_1).

(5) Replace the auxiliary bar by the bar under test with the specified axis coinciding with the suspension wire. If the specified axis does not pass through the C.G. of the experimental body, the table might be tilted, displacing the air-bubbles of the spirit levels from their central positions. Bring the bubbles to their central positions by adjusting the positions of the balancing masses (*m*, *m* etc) in the grove.*

(6) Now set the system in oscillation and following the usual procedure, find its time-period (T). Note that T is also greater than T_1.

(7) With the help of a balance, find the mass (M) of the auxiliary bar. Also measure the length (l) and breadth (b) of the bar (if it is a rectangular bar) by slide-callipers. Find its moment of inertia from the formula $I_2=M(l^2+b^2)/12$.

Measurements : Make tables like (*a*), (*b*) and (*c*) of Expt No. 1.11. In table (*c*), however, the word 'cradle' is to be replaced by 'table'.

Calculations : Same as in Expt 1.11.

Remarks : (1) The spirit levels L_1 and L_2 should be permanently fixed on the table. If detached single spirit level is used for levelling the table, then the removal of the spirit level after levelling is done, may leave the table in a slightly tilted position. (2) The auxiliary body must be of uniform density throughout. (3) This method yields more accurate result than the previous cradle method.

Oral questions

1. What is moment of inertia ? What do you mean by radius of gyration ?

Ans. Consult any standard text book.

2. Does moment of inertia depend on the axis of rotation ? What is the unit of moment of inertia ?

Ans. Moment of inertia depends on the axis of rotation. Moment of inertia of the same body is found to be different for diffierent axes of rotation. Unit of moment of inertia is gm$-$cm^2.

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*The displacement of the balancing masses in the groove will not alter the moment of inertia of the table as long as the horizontality of the table is maintained. The displacement of the masses only shifts the C.G. of the table. It may, however, be clearly noted that if the specified axis *passes through* the C.G. of the experimental body, its position on the table should be adjusted without disturbing the initial positions of the balancing masses.

3. Why is the apparatus enclosed in a glass box ?

Ans. To protect the oscillations of the cradle or the table from the disturbance of air draught.

4. What is the difference between torsional oscillation and pendulus oscillation ?

Ans. (i) Pendulus oscillation is controlled by force of gravity while torsional oscillation is controlled by the torsional couple set up in the suspension wire (ii) Pendulus oscillation is linear but the torsional oscillation is angular.

5. Upon which factors does the time-period of torsional oscillations depend ?

Ans. The time-period of torsional oscillations depends on (i) torsional rigidity of the suspension wire and (ii) moment of inertia of the oscillating body about the suspension wire as axis.

6. Which quantity in this experiment would you measure with utmost care ?

Ans. Time-period as it occurs in its second power in the final formula.

7. What is the function of the balancing masses in the moment of inertia table ?

Ans. They help to alter the position of C.G. of the table.

8. Is the time-period of oscillation of the empty cradle or empty table greater than that when the auxiliary bar or the experimental bar is placed on the cradle or the table ?

Ans. No ; it is less. When the auxiliary bar or the experimental bar is placed on the cradle or the table, its moment of inertia increases due to additional mass and hence the time-period increases.

9. Instead of a circular groove for placing the balancing masses, can there be four straight grooves meeting at the centre and equally inclined to one another ?

Ans. Yes ; Four straight grooves of above nature will allow displacement of balancing masses in the same manner as in the circular groove.

10. Why is the inertia table made very light ?

Ans. The table is made light because in that case when a body is placed on it there may be considerable difference between the periods of oscillation of the combination and that of the table alone.

1·13. Determination of surface tension of water by capillary tubes :

Apparatus : Travelling microscope, 3 or 4 glass capillary tubes, a long pointed needle, a glass plate, suitable clamp and stand, metre scale etc.

Theory : If a glass capillary tube of uniform circular bore be dipped vertically into water, it is found that water rises into the tube to a certain height. If h be the height of the level of water in the tube measured from the level outside and r the internal radius of the tube then the surface tension of water is given by $T=\frac{1}{2}r.g\left(h+\frac{r}{3}\right)$ where g is the acceleration due to gravity.

Experimental procedure : (1) Select a well-lit place in the laboratory, say a table near a window. Clean the capillary tubes with some dilute caustic soda and wash out repeatedly with tap water. Dry the tubes by passing hot air. Fix the capillary tubes on a strip of glass plate *P* with soft wax so that the tubes are all parallel to one another. Now hold the glass plate with the help of a suitable stand and a clamp so that the tubes are all vertical* [Fig 12].

(2) Take some tap-water** in a clean glass vessel *A* and keep it below the capillary tubes so that each tube dips in the water. Due to capillary action, water will rise in each tube, height being inversely proportional to the radius of the tube. Raise the vessel *A* a little and then lower it to its original position. Column of water in each tube will rise to some extent and then fall back to the original levels. By this, the inside of the tubes will be moistened by water and the liquid columns will attain proper heights in the tubes.

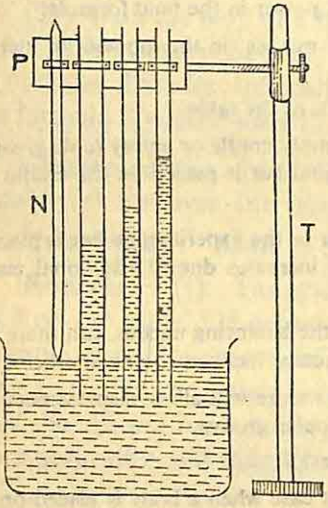

Fig. 12

(3) Fix a clean, long needle (*N*) with pointed end to the glass plate so that it is parallel to the capillary tubes and that its pointed end just touches the water surface in the vessel *A*. Keep some distance

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* Students may have to draw capillary tubes themselves. For this purpose, the following procedure may be adopted.

Take a piece of glass tube of about 10 inches long and clean the tube with some dilute caustic soda and wash out repeatedly with tap water. Dry the tube by passing hot air. Hold the tube horizontally with two hands and apply heat at the middle of the tube by a fish tail burner. Keep the tube in slow rotation so that all sides of the tube are heated equally. After some time when the glass has become soft, pull the two ends away horizontally, until a capillary tube is made. Note that a slow pull makes the bore wide while a quick pull a narrow bore. A very narrow bore is not preferable because relative error in the value of *r* is greater than that in *h*. Tubes of diameter between 0.5 mm and 2 mm should be made.

**Water distilled in a modern electric distilling plant may also be taken. It is not contaminated with grease.

between the needle and the group of tubes. Note the temperature of water by means of a thermometer.

(4) Keeping a travelling microscope in front of the tubes, level it with the help of a spirit level so that the axis of the microscope is horizontal. Focus the cross-wires of the microscope distinctly.

(5) Move the microscope slowly towards left and bring it in front of the needle. Adjust the height of the microscope such that the top-end of the needle is well-focussed† and is clearly visible through the microscope. Set the horizontal cross-wire tangential to the curved end of the needle. For this adjustment, the tangent screw of the instrument may be used if necessary. Read the vertical main scale and its vernier. Suppose, the reading is x_1.

(6) Now move the microscope slowly towards right and place it in front of the first capillary tube. Adjust the height of the microscope so that the water meniscus in the tube may be focussed. In reality, the surface of water in the tube is a concave meniscus but when focussed it will appear as a convex meniscus through the microscope. Setting the horizontal cross-wire tangential to the convex meniscus, read the main scale and the vernier. Let this reading be x_2.

Now, difference of these two readings *i.e.* $x_1 \sim x_2 = x$ (say) gives the height of water level in the capillary tube from the top-end of the neddle. If l be the length of the needle, then the height of the water column standing in the capillary tube is given by $h = l \pm x$. [+ sign is to be taken when the top-end of the needle is *below* the meniscus in the tube and − sign when the top-end is *above* the meniscus].

(7) In the same way, moving the microscope slowly towards right, bring the convex water meniscii of other capillary tubes into focus and read the main scale and the vertical scale after setting the horizontal cross-wire tangential to each meniscus. Having finished the observations on the height of water columns, note the temperature of water again.

(8) Now mark the position of water level in each tube by a small ink dot on the outer surface of the tube. Removing the first tube from the glass plate, cut the tube neatly at the position of ink-mark by a sharp glass cutting file. Take another strip of glass plate and hold it in a horizontal plane with suitable clamp. Place the capillary tube on the glass plate so that the cut-end of the tube may face the microscope.

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†If there is any difficulty in focussing the microscope, hold a piece of paper behind the needle and focus it on the paper first, as a guide.

(9) Bring the microscope infront of the cut-end and focus it. A magnified image of the circular cross-section of the tube will be seen through the microscope. Move the microscope slowly with the help of the tangent screw leftward and set the *vertical cross-wire* tangential at the left edge of the inner bore [Fig 12(*a*)]. Read the horizontal main scale and its vernier. Let this reading be y_1. Now move the microscope towards right and set the vertical cross-wire tangential at the right edge of the inner bore. Again read the horizontal scale and its vernier. Let this reading be y_2. The horizontal diameter of the inner bore of the capillary tube is $D_1=(y_1\sim y_2)$.

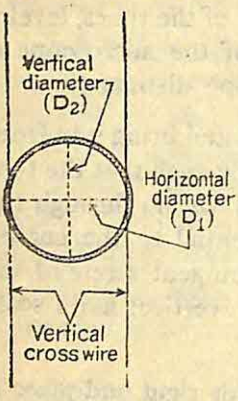

Fig. 12(*a*)

(10) Now raise the microscope along the vertical scale and set it in such a position that the *horizontal cross-wire* is now tangential at the top-end of the inner bore. Read the vertical main scale and its vernier. Let this reading be y_3 ; similarly set the horizontal cross-wire tangential at the lower edge of the inner bore and take the reading. Let this reading be y_4. Then, the vertical diameter of the inner bore is $D_2=(y_3\sim y_4)$. If the cross-section of the bore is exactly circular, the horizontal diameter (D_1) will be equal to the vertical diameter (D_2).* Find the mean value of the diameter and hence the radius of the inner bore of the tube. In the same way, the diameters of the other tubes are to be found out.

(11) Find the length (l) of the needle by a metre scale. Repeat the observations several times using different parts of the scale and determine the mean length.

Measurements : (*a*) *Length of the needle* :

(i) . . . cm (ii) . . . cm (iii) . . . cm.

Mean length (l)= . . cm.

(*b*) *Temperature of water* :

Initial temperature= . . . °*C*

Final temperature= . . . °*C*

Mean temperature during experiment= . . . °*C*

---

*If the tur diameters differ greatly in any case, it is to be understood that the cross-section of the tube is not circular. The tube has got to be rejected.

(*c*) *Radius of the internal bore of the capillary tubes* :

The value of the smallest division of the main scale of the microscope=..cm

...Vernier divisions=.. smallest divisions of the main scale =... cm.

∴ Vernier constant=.. cm.

Serial no. of the capillary tubes	Readings of horizontal scale (cm)						Horizontal diameter (D_1) $y_1 \sim y_2$ cm	Readings of the vertical scale (cm)						Vertical diameter (D_2) $(y_3 \sim y_4)$ cm	Mean diameter $\frac{(D_1+D_2)}{2}$ cm	Mean radius (r) cm
	Left edge of the bore			Right edge of the bore				Top end of the bore			Bottom of the bore					
	Main scale	Vern scale	Total reading (y_1)	Main scale	Vern scale	Total reading (y_2)		Main scale	Vern scale	Total (y_3)	Main scale	Vern scale	Total (y_4)			
1.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
2.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
3.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
etc	etc	etc						etc				etc			etc	etc

(*d*) *Heights of water columns* :

Serial no. of capillary tubes	Water meniscus reading (cm)			Needle head reading (cm)			$x=x_1\sim x_2$ (cm)	Height of water column $h=(l\pm x)$ (cm)
	Main scale	Vernier scale	Total reading (x_2)	Main scale	Vernier scale	Total reading (x_1)		
1.	..	..	..				..	..
2.	..	..	..	..	..	..	..	..
3.	..	..	..				..	..
etc	..	..	..				..	..

(*e*) *Consolidated table* :

Serial no. of capillary tubes	Height of water column (*h* cm)	Radius of bore (*r* cm)	Surface tension *T* (dynes/cm)
1.	..	..	..
2.	..	..	..
3.	..	..	..
etc			

∴ Mean surface tension= . . dynes/cm at . . °*C*.

Calculations : $T=\frac{1}{2}.r.g\,(h+\frac{1}{3}\,r)=..$ dynes/cm.

[**Notes :** (i) If the surface tension of a solution like copper sulphate solution is to be found out, the density of the solution is to be determined by a specific gravity bottle. For this the following tabular form may be used :

Temperature of water= . . °*C* (*t*)
Density of water at *t*°*C*= . . (ρ_t) [available from the standard table]

Mass of empty bottle $=W$	Mass of bottle+ water filling the the bottle $=W_1$	Mass of the bottle+liquid filling the bottle $=W_2$	Density of liquid $\rho=\frac{W_2-W}{W_1-W}\times\rho_t$
. . gm+ . . gm + . . mg+ . . mg = . . gm	. . gm+ . . gm + . . mg+ . . mg = . . gm	. . gm+ . . gm + . . mg = . . gm	. .

Calculations : $T=\frac{1}{2}\rho.g.\ r\,(h+\frac{1}{3}\,r)=..$ dynes/cm.

(ii) *Verification of Jurin's law :*

If h be the height of liquid column in a capillary tube dipped vertically in the liquid, then neglecting meniscus effect, we have,

$T=\frac{1}{2}.r.h.\rho.g$ or $r.h=\dfrac{2T}{\rho.g}$ =a constant, because for a given liquid the surface tension at a given temperature is constant. This relation between the radius (r) of the capillary tube and height (h) of the liquid column in it is referred to as Jurin's law.

For verification of the law, a graph is drawn between h and $1/r$. The relation mentioned above shows that if the law is true, the graph will be a straight line passing through the origin.

Taking values of h and r from the table(e), page 66, we can tabulate as follows :

Quantities	1	2	..	4
h (cm)	..	..	..	..
$1/r$ (cm^{-1})	..	..	..	..

Plotting $1/r$ along X-axis and h along Y-axis, a straight line graph is obtained, which passes through the origin O [Fig 12(b)]. This verifies Jurin's law.]

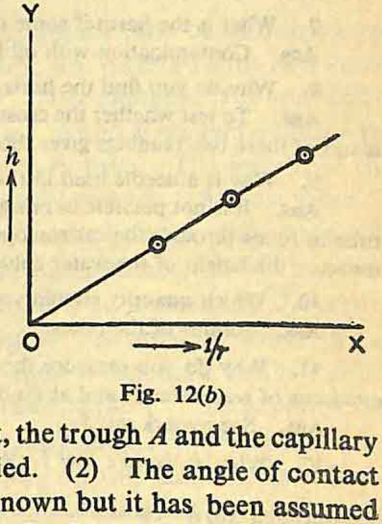

Fig. 12(b)

Proportional and Percentage error :

See Ex 2, Page 6.

Remarks : (1) Surface tension of water alters appreciably if the water is contaminated with impurities—specially of greasy or oily nature. So, special care should be taken against such contamination. Before the commencement of experiment, the trough A and the capillary tubes should be properly cleaned and dried. (2) The angle of contact between water and glass is not exactly known but it has been assumed zero. This brings in some error. (3) Surface tension of a liquid depends on temperature ; hence temperature at the time of experiment should be mentioned. (4) The capillary tubes and the needle should remain vertical. (5) If the tube is not of uniform radius it should be cut at the place corresponding to the meniscus at the top of the water column and the diameter there should be measured.

Oral questions

1. What is surface tension ? What is its unit ?

Ans. Consult any text book. Its unit is dynes/cm in C.G.S. system.

2. What is angle of contact ? What is the angle of contact between (i) glass and water and (ii) glass and mercury ?

Ans. Consult any standard text book. (i) nearly 0° (ii) nearly 140°.

3. Is it that all liquids ascend in a capillary tube like water ?

Ans. Liquids which have angle of contact with glass less than 90° (*i.e.* acute) ascend in a glass capillary tube. For example, water, alcohol, copper sulphate solution etc. On the other hand, liquids which have angle of contact with glass more than 90° (*i.e.* obtuse) descend in a glass capillary tube. For example, mercury.

4. What is the relation between the radius of a capillary tube and the height of water column in it ?

Ans. $h \propto 1/r$ *i.e.* less the radius, more is the height of liquid column.

5. Which one is better—a wide tube or a narrow tube ?

Ans. Comparatively wider tube is preferable ; because the relative error in the value of r is greater than that in h. Tubes of diameter between 0.5 mm and 2 mm are preferred.

6. How does surface tension of water depend on temperature ?

Ans. Surface tension of water diminishes with the rise of temperature.

7. What is the harm if some oil gets mixed up with water ?

Ans. Contamination with oil lowers the surface tension appreciably.

8. Why do you find the horizontal and the vertical diameters of the tube ?

Ans. To test whether the cross-section of the bore is circular or not. Further, mean of these two readings gives the correct diameter.

9. Why is a needle used in this experiment ?

Ans. It is not possible to bring the bottom of the water column in the capillary tube in focus through the microscope. To avoid this difficulty, a needle is used to measure the height of the water column by the method described in the experiment.

10. Which quantity should you measure with the greatest care ?

Ans. Radius of the tubes.

11. Why do you measure the diameter of the tube at the position where the meniscus of water stands and at no other position ?

Ans. See remark no. 5.

12. What is Jurin's law ? What is the nature of the graph when (i) $r.h=$ constant and (ii) $h=\text{const}\times\frac{1}{r}$.

Ans. See note (ii). The graph in the case (i) is a rectangular hyperbola and in the case (ii) a straight line.

1·14. Determination of the coefficient of viscosity of water by Poiseuille's method :

Apparatus : Viscosity apparatus, travelling microscope, a capillary tube (about 50 cm long and about 1 mm in diameter), beaker, balance, stop-watch, metre-scale, thermometer etc.

Theory : When water is allowed to flow in a stream-line manner through a uniform capillary tube of length l and of radius r, the volume V of water flowing out per second is given by, $V=\frac{\pi P.r^4}{8l\eta}$ or $\eta=\frac{\pi.P.r^4}{8l.V}$

where P is the pressure-difference between the two ends of the capillary tube and η the co-efficient of viscosity of water.

If the pressure difference P be measured by an U-type water manometer, then $P=h.g$ where h=difference of water levels in the two arms of the U-tube. Hence, $\eta=\frac{\pi.r^4.h.g}{8l.V}$ [Density of water is taken as 1]

(i) Knowing h and V, the co-efficient of viscosity η can be found out.

(ii) From the above relation, it is clear that the graph between V and h will be a straight line whose slope$=\frac{\pi r^4.g}{8.l.\eta}$. So knowing the slope, η can be calculated.

Description of apparatus : T is a capillary tube of uniform bore [Fig 13]. Two ends A and B of the tube are inserted into two small brass chambers. V is a water-reservoir. The chamber A is connected to the water-reservoir by a rubber tubing. E and F are two manometer tubes connected to the ends of the capillary tube. The mouth of the water-reservoir is closed air-tight by a rubber-cork. A glass tube C, open at both ends, is introduced almost to the bottom of the reservoir through a hole in the rubber-cork. As water comes out of the reservoir, air is drawn in through the tube C, so that pressure at D remains equal to atmospheric pressure as long as the level of water in the reservoir remains above D. A pinch-cock G attached to the rubber tubing controls the exit of water from the reservoir. Water comes out of the reservoir, flows through the capillary tube T and collects in the beaker P. The difference of water levels in the manometer tubes E and F measures the pressure-difference between the ends of the capillary tube. If the water level in the tube F is not visible, a pinch-cock may be fixed (not shown in the figure) to the rubber-tubing, coming out of the chamber *B*. Controlling the pinch-cock *G*, water is allowed

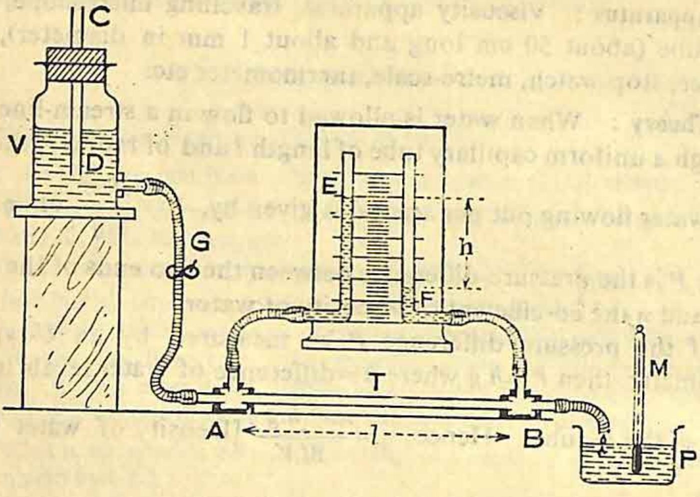

Fig. 13

to drip at a slow rate from the open end *B*. If water runs back beneath the exit tube, smear the underside of the tube near the end with a small spot of vaseline. A thermometer *M* may be inserted into the beaker *P* to note the temperature of water.

Experimental procedure : (1) Take out the capillary tube *T* from the chambers and measure its length by a metre-scale. Repeat the observations thrice and find the mean length (*l*).

(2) To measure the diameter of the bore of capillary, take a specimen tube of the same capillary of small length. Find the horizontal and vertical diameters of the bore by adopting the procedure mentioned in Expt 1·4. Find the mean diameter and hence the radius (*r*).

[*Alternative method of measuring r* :

Clean the capillary tube *T* with dil. caustic soda and wash out repeatedly with tap water. Dry the tube by passing hot air. When the tube is perfectly dry, suck some mercury in it. Keep the tube in a horizontal position so that mercury pellet covers some length of the tube. Measure the length of the mercury pellet by means of a metre scale. Repeat the observations several times after shifting the mercury pellet to different portions of the tube *T*. If the lengths of mercury pellet so measured come out to be very nearly equal, the tube is of uniform bore. Now, find the mean length of the pellet. Weigh an empty crucible in a balance correct to a centigram. Carefully pour the mercury pellet into the crucible and weigh again. From these two weighings mass of the mercury pellet can be found out.

If M and L be the mass and length of the pellet then,

$$M=\pi r^2\times L\times \rho_m \quad \text{or} \quad r^2=\frac{M}{\pi . L . \rho_m} \quad \text{where } \rho_m=\text{density of mercury.]}$$

(3) Put the capillary tube T in the chambers A and B and restore its connection with the water-reservoir through the rubber tubing as shown in fig 13. Control the pinch-cock G so that water flows through the capillary at a slow rate and collects in the beaker, drop by drop. See whether the columns of water in the manometer tubes E and F are steady. From the scale attached by the side of manometer tubes, note the levels of water in E and F.*

(4) Take a clean and dry glass beaker and weigh it empty in a balance. When the flow of water through the capillary tube T has been steady, put the beaker below the exit tube at the end B. As soon as water starts collecting in the beaker, start a stop-watch. Allow the water to collect a little more than half the volume of the beaker. When the collection is over, remove the beaker and stop the stop-watch. Find the time of collection of water to the nearest second.

(5) Weigh the beaker with water collected in it and find the mass of water collected. Note the temperature of water with the help of the thermometer M.

(6) By regulating the flow of water through the capillary tube T by means of the pinch-cock G, repeat the observations (3), (4) and (5) at least twice.

(7) Plot various values of h obtained from table (*d*) along the X-axis and corresponding values of V obtained from table (*c*) along the Y-axis. From the straight line so obtained find the slope *i.e.* the ratio of PM/NM [Fig 13(*a*)].

---

*(i) In some instruments, scale is not provided by the sides of the manometer tubes. In such cases, measure the heights of the water columns in E and F from the surface of the table by a metre scale and get the difference.

(ii) Level difference can also be measured by a cathetometer. For this purpose, adopt the following procedure :

Level the cathetometer (Fig 6) with the help of levelling screws so that the pillar is vertical and the axis of the telescope is exactly horizontal. Focus the cross-wire without any parallax. Set the telescope in front of the level E and focus the meniscus. It will appear as a convex meniscus. Raise or lower the telescope along the vertical scale and set the horizontal cross-wire tangential to the convex meniscus. Read the main scale and the vernier. Similarly focus the other level at F and take the reading. Difference of these two readings gives h.

Measurements : (*a*) *Length of the capillary tube :*

(i) . . . cm (ii) . . . cm (iii) . . . cm.
Mean length (l)=. . . cm.

(*b*) *Diameter of the bore of the capillary :*

The value of the smallest division of the main scale of the microscope=. . . cm.

. . . Vernier divisions=. . . smallest divisions of the main scale
∴ 1 ,, ,, =. . ,, ,, ,, ,, ,, ,,
or, Vernier constant=. . . cm.

No. of Obs	Horizontal scale readings (cm)						Horizontal diameter (D_1) ($y_1 \sim y_2$)	Vertical scale readings (cm)						Vertical diameter (D_2) ($y_3 \sim y_4$)	Mean diameter $\frac{(D_1+D_2)}{2}$ (cm)	Mean radius (r) cm
	Left end of the bore			Right end of the bore				Top of the bore			Bottom of the bore					
	Main scale	Vern scale	Total reading (y_1)	Main scale	Vern scale	Total reading (y_2)		Main scale	Vern scale	Total reading (y_3)	Main scale	Vern scale	Total reading (y_4)			
1.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
2.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..

[**Alternative table when mercury pellet is used :**

Length of the pellet : (i) ..cm (ii) .. cm (iii) .. cm (iv).. cm.

Mean length (L)= .. cm.

Mass of empty crucible = ..gm+..gm+..gm+..gm+..mg

= ..gm (M_1)

,, ,, crucible+mercury = ..gm+..gm+..gm+..mg

= ..gm (M_2)

Mass of mercury pellet $= M_2 - M_1 =$ ..gm − ..gm = ..gm (M)]

(c) *Rate of flow of water :*

Temperature of water = ..°C

Density of water at that temperature (ρ) = ..gm/c.c.

(From the standard table)

No. of Obs.	Wt. of empty beaker (m_1)	Wt. of beaker +collected water (m_2)	Wt. of water collected $(m=m_2-m_1)$	Time for collection of water (t)	Rate of flow of water $(V=m/\rho.t)$
1.	..gm+..gm +..mg=..gm	..gm+..gm +..mg=..gm	..gm	..sec	..c.c./sec
2.	..	..	..	..	..
3.	..	..	..	..	..

(d) *Pressure-difference from manometer readings :*

$(g=980 \text{ cm}/s^2)$

No. of Obs	Height of water level in the arm E (h_1 cm)	Height of water level in the arm F (h_2 cm)	Difference of heights $h=(h_1-h_2)$cm	Pressure diff. $(P=h.g)$
1.	..	..	..	..dynes/cm^2
2.	..	..	..	..
3.	..	..	..	..

Alternative table when cathetometer is used :

Value of one small division of the main scale of the cathetometer
=. .cm.
. . Vernier divisions = . . small divisions of the main scale
= . . cm.
∴ 1 ,, ,, = . . main scale div.
∴ Vernier constant = . cm.

No. of Obs	Readings corresponding to level E (cm)			Readings corresponding to level F (cm)			Difference ($h=h_1-h_2$) (cm)	Pressure diff. ($P=hg$) dynes/cm^2
	Main scale	Vernier scale	Total (h_1)	Main scale	Vernier scale	Total (h_2)		
1.	..	..	..	..	..	..	..	..
2.	..	..	..	..	..	..	..	..
3.	..	..	..	..	..	..	..	..

(*e*) *Consolidated table* :

No. of Obs	l (cm)	V (c.c./sec)	r^4	P dynes/cm^2	$\eta=\dfrac{\pi P r^4}{8lV}$	Mean η
1.	..	..		..	..	
2.	..	..	..	..	..	..
3.	..	..	..	..	..	

(*f*) *Table for drawing graph between h and V* [Obtained from tables (*c*) and (*d*)]

Quantity	1	2	3	4
V (c.c./sec)→	..	..	..	..
h→ (cm)	..	..	..	..

Calculations : *(a) Radius from mercury pellet expt :*

$r^2=\frac{M}{\pi L\rho_m}=..$ sq cm $[\rho_m=13{\cdot}59$ gm/c.c.] $\therefore\ r^4=....$

(*b*) $\eta=\frac{\pi Pr^4}{8lV}=.$ Poise at ..°*C.*

(*c*) *Calculation from the graph :*

$$\text{Slope}=\frac{PM}{NM}=..$$

$$\therefore\ \frac{PM}{NM}=\frac{\pi r^4.g}{8l.\eta}$$

or, $\eta=..$Poise at..°*C.*

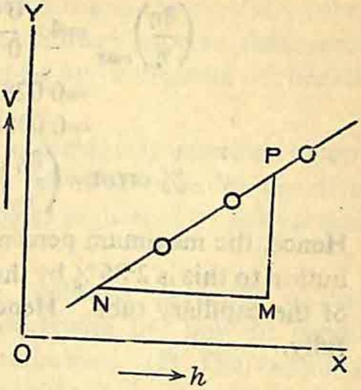

Fig. 13(*a*)

Proportional and Percentage error

$$\eta=\frac{\pi r^4hg}{8.lV}=\frac{\pi d^4.h.g.t.\rho}{128\ l.m}\qquad\left[\because\ V=\frac{m}{\rho.t}\right]$$

where *d* is the diameter of the tube.

Assuming ρ and *g* to be known to a high degree of accuracy we have, for the maximum proportional error in η given by,

$$\left(\frac{\delta\eta}{\eta}\right)_{max}=4.\ \frac{\delta d}{d}+\frac{\delta h}{h}+\frac{\delta t}{t}+\frac{\delta l}{l}+\frac{\delta m}{m}.$$

Let us take the data of a set of observations as follows :

$d=0{\cdot}14$ cm	$\delta d=0{\cdot}001$ cm (L.C. of microscope)*
$h=25{\cdot}5$ cm	$\delta h=0{\cdot}1$ cm (least count for a metre scale ; in the case of cathetometer the L.C. is 0·001 cm.)
$t=130$ sec.	$\delta t=0{\cdot}5$ sec (minimum measure of the stop watch)
$l=40$ cm	$\delta l=0{\cdot}1$ cm (metre scale)
$m=50$ gm	$\delta m=0{\cdot}001$ gm (1 mg)

---

*In the case of mercury pellet method the proportional error in *d* may be calculated in the following way : $d=2\sqrt{\frac{M}{\pi L\rho_m}}$. Assuming ρ_m is known to a high degree of accuracy, we have, $\left(\frac{\delta d}{d}\right)_{max}=\left(\frac{\delta M}{M}+\frac{\delta L}{L}\right)$. In one case, say, $M=5$ gm and $\delta M=0.001$ gm (1 mg) ; $L=5$ cm and $\delta L=0.1$ cm (metre scale)

$$\therefore\left(\frac{\delta d}{d}\right)_{max}=\left(\frac{0{\cdot}001}{5}+\frac{0{\cdot}1}{5}\right)=0{\cdot}0202$$

So the percentage error in diameter measurement is 2%. In microscope method the error is about 3%.

Substituting these values in the above equation,

$$\left(\frac{\delta\eta}{\eta}\right)_{max} = 4\times\frac{0{\cdot}001}{0{\cdot}14}+\frac{0{\cdot}1}{25{\cdot}5}+\frac{0{\cdot}5}{130}+\frac{0{\cdot}1}{40}+\frac{{\cdot}001}{50}$$

$$=0{\cdot}0285+0{\cdot}004+0{\cdot}00384+0{\cdot}0025+0{\cdot}000005$$

$$=0{\cdot}0388$$

$$\therefore\ \%\ \text{error}=\left(\frac{\delta\eta}{\eta}\right)_{max}\times 100\,\% = 3{\cdot}88\,\%$$

Hence, the maximum percentage error is nearly 4% and major contribution to this is 2·85% by the error in the measurement of the diameter of the capillary tube. Hence diameter needs be measured very carefully.

Alternative simpler arrangement

Description of the arrangement : There is another arrangement of viscosity apparatus simpler than the one described on page 70. Fig 13(*b*) shows the arrangement. In this arrangement water is allowed to flow through the capillary tube *T* from a constant level water tank *R*. The tank consists of a brass cylinder of about 7.5 cm length and about 6 cm internal diameter and is provided with one outlet tube *A* near the side and a waste tube which maintains a constant level in the tank. The ends of the tube are corrugated for a tight-fit for rubber tubing. The height of the tank from the capillary tube can be varied by sliding it along the supporting rod. The end of the outlet tube *A* of the tank is connected through an inverted *T*-piece to the capillary tube, the tail of *T* being connected to a vertical glass tube *B* which measures the head of water. A metre scale *S* stands by the side of the tube *B* for measuring the height of liquid column in *B*. From a tap above the tank, water is poured into the tank and a beaker *P* at the end of the capillary tube collects the outflowing liquid.

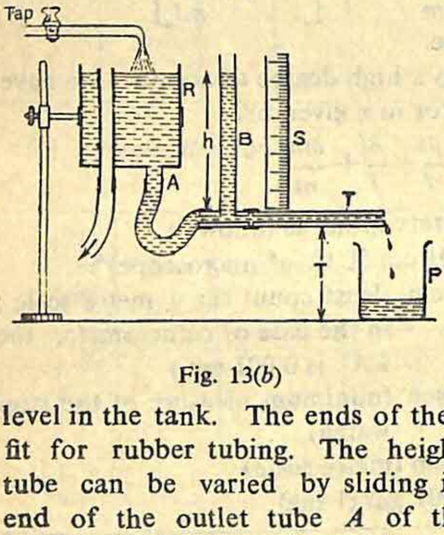

Fig. 13(*b*)

Experimental procedure : Allow the water from the tap to run into the tank *R* and adjust its flow at such a rate that the level of water in the tank is maintained constant, excess water being flown out through the waste. Then watch the flow of water through the capillary tube. Adjust the height of the tank from the capillary tube so that water flows through the capillary tube at a slow rate and collects in the beaker *P*, almost drop by drop.

The rest of the procedure is the same as previously described except that *h* is measured by the height of the level of water in the constant level water tank *R* from the capillary tube as registered by the vertical glass tube *B* [Fig 13(*b*)].

Remarks : (1) Pressure-difference between the ends of the capillary tube should not be high as otherwise the flow of liquid through it will not be stream-line but turbulent. (2) The capillary tube should be horizontal ; its diameter should not be more that 2 mm ; otherwise the flow of water will not be stream-line. (3) As viscosity of water depends on the temperature, the temperature of water during the experiment should be noted. (4) The radius of the capillary tube occurs in its fourth power in the expression of η and hence it demands special care during measurement. (5) For the measurement of volume of water collected, a graduated cylinder of small least count may be used. (6) The alternative arrangement, though rather simple, suffers from an error in respect of the kinetic energy of the moving water.

Oral questions

1. What is co-efficient of viscosity ? What is stream-line motion ? What is the unit of co-efficient of viscosity ?

Ans. Consult any standard text book. Unit of co-efficient of viscosity is 'Poise'.

2. What is the harm if the pressure-difference between the ends of the capillary tube is high ?

Ans. See remark no. 1.

3. How does the co-efficient of viscosity of water depend on the temperature of water ?

Ans. Co-efficient of viscosity decreases with the increase of temperature.

4. In measuring the radius of capillary tube, extra precaution is necessary. Why ?

Ans. Radius occurs in its fourth power in the final formula. A slight error in its measurement will be magnified much and increase the percentage of error in the final result to a high degree.

5. Which method is preferable for the measurement of radius—microscope method or the mercury pellet method ?

Ans. Mercury pellet method. See foot note on page 71.

6. What is the harm if the tube is of wider bore ?

Ans. In a tube of wider bore, a slight difference of pressure between the ends of the tube makes the flow of water turbulent. It is very difficult to get stream-line motion in wide bore tubes.

7. What is Reynold's number ?

Ans. Consult any standard text book.

8. What factors are conducive to stream-line motion ?

Ans. Fine bore, high viscosity and low density.

9. How can the uniformity of the bore of the capillary tube be tested ?

Ans. See mercury pellet method on page 70.

10. Is the method suited to determine the viscosity of all liquids ?

Ans. Not for very viscous liquid.

2. HEAT

2·1. Determination of the co-efficient of linear expansion of a metal by travelling microscope :

Apparatus : A rod (about 1 metre long) of the metal whose co-efficient of linear expansion is to be measured (say, brass), travelling microscope, thermometer, jacket, boiler etc.

Theory : The co-efficient of linear expansion of a substance is the increase in length per unit original length of the substance (in the form of a rod) per degree rise in temperature. If α be the co-efficient of linear expansion, then,

$$\alpha=\frac{l_2-l_1}{l_1(t_2-t_1)}=\frac{x}{l_1(t_2-t_1)}$$

where l_1=length of the rod at t_1°C ; l_2=length of the rod at t_2°C $(t_2>t_1)$ and x=increase in length.

Description of the apparatus : The arrangement (Fig 1) consists of a metallic rod AB kept within a jacket through which steam can pass. Steam enters through the tube P_1 and leaves through the tube P_2. Through a hole at the middle of the jacket tube, a thermometer is inserted which records the temperature of the space within the jacket. There are two marks C and D on the rod almost at the two ends of the rod. The two ends along with the marks project out of the jacket tube. Two travelling microscopes M_1 and M_2 are so placed that they can easily focus the marks C and D respectively.

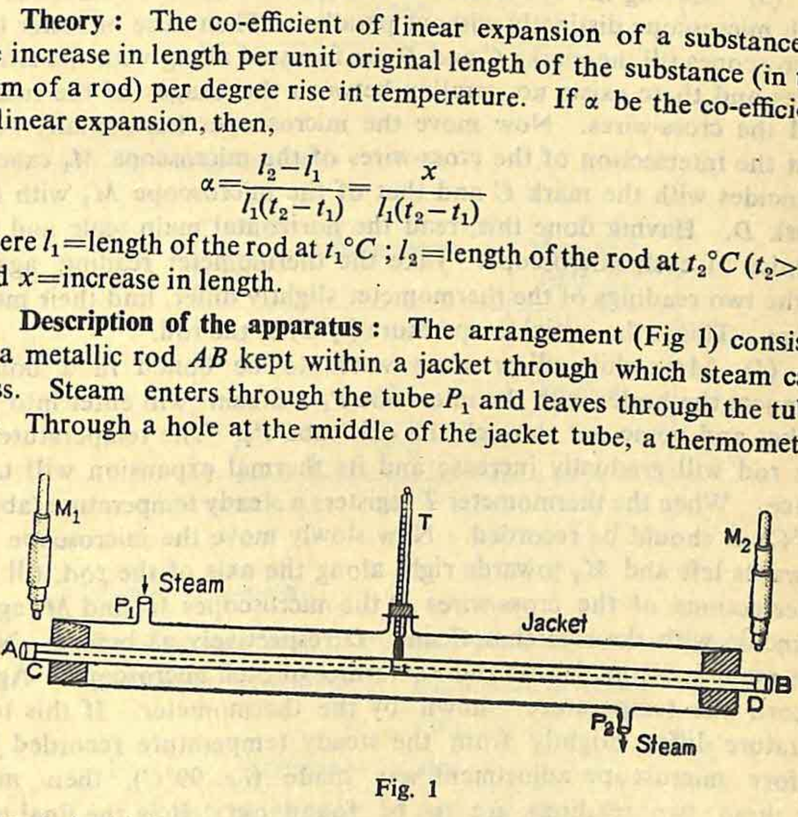

Fig. 1

Experimental procedure : (1) Taking the rod out of the jacket tube, measure the length (l_1) between the marks C and D by a metre scale. Repeat the observations several times using different parts of the scale and find the mean length. [If the arrangement does not permit the rod to be taken out of the jacket tube, measure the length with the help of a beam compass and a metre scale.]

(2) Put the rod within the jacket tube and insert a thermometer *T* (1/10 th of a degree division) through the hole in the rubber cork which closes the opening. Wait for some time and see whether the temperature indicated by the thermometer is steady. When the temperature becomes steady, record it.

(3) Moving the eye-lens back and forth focus the cross-wires of each microscope distinctly without parallax. Then raise or lower the microscopes till the marks *C* and *D* are focussed along with the cross-wires and there exists no parallax between the images of the marks and the cross-wires. Now move the microscopes horizontally such that the intersection of the cross-wires of the microscope M_1 exactly coincides with the mark *C* and that of the microscope M_2 with the mark *D*. Having done this, read the horizontal main scale and the vernier of each microscope. Take the thermometer reading again. If the two readings of the thermometer slightly differ, find their mean value. This is the initial temperature (t_1°*C*) of the rod.

(4) Meanwhile allow some water to be boiled in a boiler. Connect the boiler with the inlet tube P_1. Steam will enter into the jacket and come out through the exit tube P_2. The temperature of the rod will gradually increase and its thermal expansion will take place. When the thermometer *T* registers a steady temperature (about 99°*C*), it should be recorded. Now slowly move the microscope M_1 towards left and M_2 towards right along the axis of the rod, till the intersections of the cross-wires of the microscopes M_1 and M_2 again coincide with the scratches *C* and *D* respectively as before. Note the main horizontal scale and its vernier of each microscope. Again record the temperature shown by the thermometer. If this temperature differs slightly from the steady temperature recorded just before microscope adjustment was made (*i.e.* 99°*C*), then mean of these two readings are to be found out. It is the final temperature (t_2°*C*).

(5) If the difference between the initial and final readings of the microscope M_1 be x_1 and that for the microscope M_2 be x_2, then, the total linear expansion of the rod $x = x_1 + x_2$.

Measurements : (*a*) *Length between the marks C and D* :

(i) . . . cm (ii) . . . cm (iii) . . . cm.

Mean length (l_1) = . . . cm.

(*b*) *Initial temperature* :

(Data are for illustration)

No. of Obs.	Temperature	Mean initial temp. (t_1)
1.	35·3°C	35·25°C
2.	35·2°C	
3.	35·2°C	

(*c*) *Final temperature* :

No. of Obs.	Temperature	Mean final temp. (t_2)
1.	99·1°C	99·05°C
2.	99°C	
3.	99°C	

(*d*) *Linear expansion* :

Value of one division of the main scale of the microscope=..cm.

... Vernier divisions=... main scale divisions.

=... mm.

∴ 1 ,, division=... mm.

∴ Vernier constant=... mm=... cm.

[N.B. Generally two microscopes of same vernier constant are taken. If the microscopes happen to have different vernier constants, they should be shown seperately.]

Stage of observation	Steady temperature	Readings of microscope M_1			Readings of microscope M_2			Expansion on the left ($x_1=a\sim c$)	Expansion on the right ($x_2=b\sim d$)
		Main scale (cm)	Vernier scale	Total reading (cm)	Main scale (cm)	Vernier scale	Total reading (cm)		
Before steam is passed	t_1°C	..	..	.. (*a*)	..	..	.. (*b*)		
After steam is passed	t_2°C	..	..	.. (*c*)	..	..	.. (*d*)		

∴ Total linear expansion of the rod $x=(x_1+x_2)=\ldots$ cm.

Calculations : $\alpha = \dfrac{x}{l_1(t_2 - t_1)} = \ldots$ per °C.

Remarks : (1) The scratches C and D should be as near to the jacket as possible. If they happen to be far away from the jacket the temperature of the exposed portion of the rod will not be same as that of the covered portion. (2) The horizontal displacement of the miscroscopes should be parallel to the axis of the rod ; otherwise measurement of linear expansion will be erroneous . (3) While focussing the cross-wire of the eye-piece, parallax should be avoided. (4) While working the screws of the microscope back-lash error should be avoided. (5) Instead of one thermometer at the middle, in some arrangements, there may be two thermometers at the two ends of the rod. In that case, the mean of the two thermometer readings should be taken.

2.2. Alternative method :

Apparatus : A hollow tube (about 1 metre long) of the metal whose coefficient of linear expansion is to be found out, a heat insulating jacket, boiler, thermometer, a micrometer screw gauge with linear scale, battery, torch bulb, metre scale etc.

Theory : As in Expt no. 2·1.

Description of the arrangement : T is a hollow tube of the metal under investigation and is clamped rigidly at one end A which is joined by a rubber tubing R to a boiler (not shown). The tube is surrounded by insulating jacket J which extends as far as a stout metal strip P, fixed rigidly to the other end of the tube. A micrometer screw gauge S, with a horizontal millimetre scale M is fixed to the base of the apparatus. The screw S can move freely along the millimetre scale. Contact between the screw S and the metal strip P is indicated when the torch bulb L lights up. The bulb is in series with a battery E and the gap

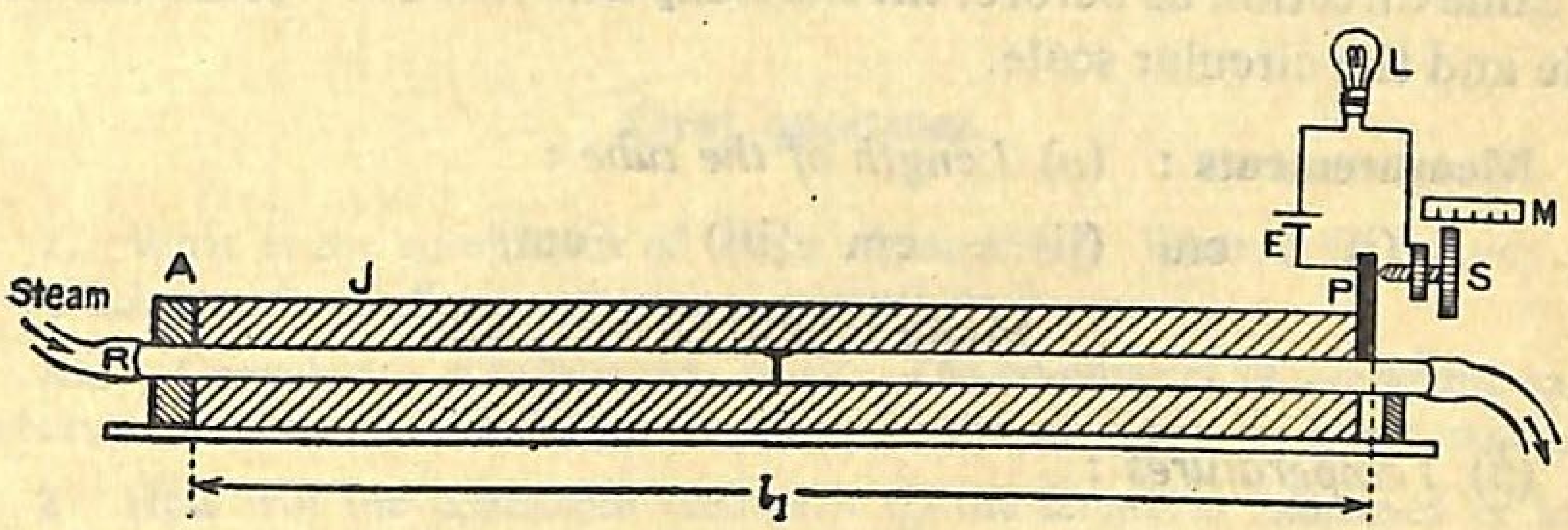

Fig. 2

between the screw point of S and the strip P. Steam or water can enter the tube through the end A and pass out through the exit rubber tubing at the other end of the tube.

Experimental procedure : (1) Measure the length (l_1) of the tube from the inner face of the clamp A to the metal strip P with a metre scale. Repeat the observation thrice and find the mean length. Measure the least count of the screw-gauge in the usual way.

(2) Boil some water in a boiler and pass a steady flow of steam through the tube T. Just before passing steam, the screw S is moved well back to avoid any strain on the metal strip P due to expansion of the tube. With the passage of steam, the temperature of the tube will rise and it will expand in length. Push a thermometer well down the tube through the exit tube and note the rise of temperature.

(3) When the thermometer registers a steady temperature t_2 (about 99°C), slowly turn the screw S *always in one direction* so that the lamp L is just on. Read the linear scale M and the circular scale S.

(4) Disconnect the boiler and pass cold water through the tube T. After some time, again push a thermometer well down the tube through the exit tube. When the thermometer registers a steady temperature (t_1) contraction of the tube will be complete and a gap will be left between the screw-head and the strip P. As a result, the lamp will be put out. Note the steady temperature.

(5) Continuing the flow of cold water, again turn the screw S, in the same direction as before, till the lamp L is just on. Read the linear scale and the circular scale.

Measurements : (*a*) *Length of the tube* :

(i) . . . cm (ii) . . . cm (iii) . . . cm.

∴ Mean length (l_1)= . . cm.

(*b*) *Temperatures* :

Condition	1	2	..	Final steady value
Steam	..	..	..	t_2°C
Cold water	..	..	..	t_1°C

(*c*) *Linear expansion* :

The value of the smallest division of the linear scale=.. mm.

Pitch of the circular scale=... mm.

Total no. of circular divisions in the scale=...

∴ least count (*l.c.*)=... mm.

Temperature	Scale readings (mm)				Expansion $(x=x_2 \sim x_1)$ mm.	x (cm)
	Linear scale	Circular scale	Cir. scale $\times l.c$	Total (mm)		
t_2°C	..	..	..	.. (x_2)	...	...
t_1°C	..	..	..	.. (x_1)		

Calculations : Linear expansion coefficient $\alpha=\dfrac{x}{l_1(t_2-t_1)}$=..per °C

Remarks : (1) Micrometer screw should always be turned in the same direction to avoid back-lash error. (2) If the metal strip is flexible some error comes in setting and reading the micrometer because the screw may be turned back through some distance while the lamp just remains on. (3) The result gives the coefficient of linear expansion of the metal within the temperature range of t_2 and t_1.

Oral questions

1. What is the co-efficient of linear expansion ? What is its relation with the coefficients of superficial and cubical expansions ?

Ans. Consult any standard text book. The co-efficient of superficial expansion is twice and that of cubical expansion thrice the coefficient of linear expansion.

2. How will the coefficient change if (i) the length is expressed in inches instead of centimetre (ii) the temperature is expressed in Fahrenheit instead of Celsius ?

Ans. (i) No change ; Coefficient of linear expansion does not depend on the unit of length (ii) Since $1°F=\frac{5}{9}°C$, $\alpha_F=\frac{5}{9}.\alpha_C$.

3. Will the coefficient of linear expansion be doubled if the length is doubled ?

Ans. No ; Coefficient of linear expansion is a property of the material.

4. Will the coefficient of linear expansion of a material change if the range of temperature is changed ?

Ans. Yes ; co-efficient of linear expansion of a material slightly changes when the range of temperature is changed. See remark no. 3.

5. What is the harm if the scratches on the rod are away from the jacket in the first experiment ?

Ans. See remark no. 1.

6. What is the harm if the rod is slightly disturbed after the initial reading has been taken in the first experiment ?

Ans. Correct expansion of the rod will not be obtained and the result becomes erroneous.

7. Is the first experiment possible if one end of the rod is rigidly fixed and a microscope is fitted at the other end ?

Ans. Yes ; The expansion will take place only in the direction of the free end. Microscope will measure this expansion.

8. Is the alternative arrangement more accurate than the first ?

Ans. No ; there are many sources of error in the alternative arrangement.

2.3. Optical lever and its principle :

Fig 3 shows an optical lever. It consists of a metallic tripod stand *ABC*, having two arms *AB* and *AC* of equal length and three pointed legs *A*, *B* and *C*. A plane mirror *M* is hinged at the back at right angles to the line *BC*. The plane mirror *M* can be turned about a horizontal axis and its plane can be set vertical. Very small linear expansion can be accurately measured with the help of an optical lever. The substance whose linear expansion is to be measured is placed below the front leg *A* and the lever is made horizontal and its mirror vertical. Due to expansion in length, the point *A* will be raised a little, tilting the mirror *M*. The angle through which the mirror *M* is tilted is measured and from that, the linear expansion is calculated. The perpendicular distance (*AD*) of the vertex *A* of the tripod from the base *BC* is called the arm of the lever. In fig 3, *y* is the length of the lever arm.

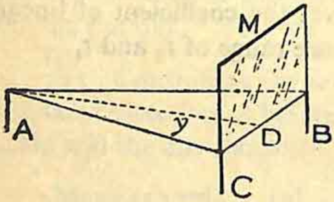

Fig. 3

Suppose, the linear expansion of the rod *PQ* is to be measured with the help of an optical lever [Fig 3(*a*)]. Set the mirror vertical after putting the front leg *A* of the lever on the top of the rod *PQ*. Suppose,

M_1 is the initial vertical position of the mirror. When the rod PQ increases in length, the mirror M is tilted. Let x be the increase in length of the rod and θ, the inclination of the mirror. Let M_2 be the inclined position of the mirror. The inclination of the lever arm is also θ.

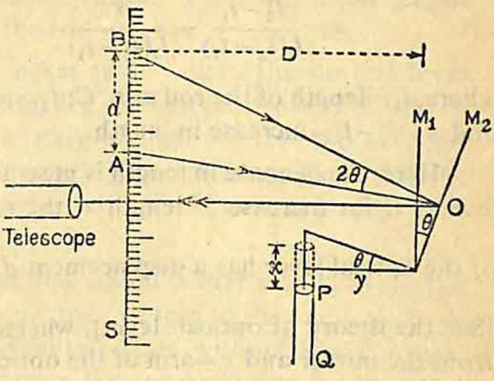

Fig. 3(a)

A vertical metre scale (S) is set up at a distance of one or two metres from the mirror and its image in the mirror is viewed though a telescope. Suppose in the initial position of the mirror (*i.e.* at M_1), the mark A of the scale coincides with the horizontal wire of the cross-wires of the telescope. When the mirror is tilted to the position M_2, let the mark B of the scale coincides with the horizontal cross-wire. Hence, the displacement of the scale-divisions $=AB=d$ (say). It is clear that $\angle AOB=2\theta$, because the reflected ray turns twice the angle through which the mirror turns, provided the incident ray remains unchanged.

$\therefore\ 2\theta=\frac{d}{D}$, where D is the perpendicular distance between the mirror and the scale. Hence $\theta=\frac{d}{2D}$

Again, considering the inclination of the lever arm, we can write,

$$\theta=\frac{x}{y} \quad \text{or} \quad x=y\theta=\frac{d}{2D}\cdot y.$$

So, knowing d, y and $2D$, the linear expansion x of the rod can be found out.

2.4. Determination of the co-efficient of linear expansion of a metal rod, using an optical lever :

Apparatus : Pullinger's apparatus, thermometers, scale and telescope, optical lever, metre scale etc.

Theory : The co-efficient of linear expansion of a substance is the increase in length per unit original length of the substance (in the form of a rod) per degree rise in temperature. If α be the co-efficient of linear expansion, then,

$$\alpha=\frac{l_2-l_1}{l_1(t_2-t_1)}=\frac{x}{l_1(t_2-t_1)}$$

where, l_1=length of the rod at t_1°C: l_2=length of the rod at t_2°$C(t_2>t_1)$ and $x=l_2-l_1$=increase in length.

Here, the increase in length is measured with the help of an optical lever. If for increase of length x, the reflected light from the mirror of the optical lever has a displacement d on the scale, then $x=\frac{d}{2D}.y$ [See the theory of optical lever], where D= the distance of the scale from the mirror and y=arm of the optical lever.

$$\therefore\quad \alpha=\frac{d.y}{2Dl_1(t_2-t_1)}$$

Description of the arrangement : The arrangement of the experiment has been shown in fig 4. A uniform rod AB of the metal whose coefficient of linear expansion is to be determined is placed in a metal jacket J, with its lower end B resting on a marble plate. There is, therefore, no room for the expansion of the lower end of the rod. The upper end of the rod projects slightly above the cork closing the mouth of the jacket and can expand upward. Two thermometers T_1 and T_2 are inserted into the jacket through two openings—one almost at the top and the other almost at the bottom of the jacket. Steam enters into the jacket through the inlet tube C and goes out through the exit tube D. The front leg of an optical lever rests on the end A of the rod and the hind legs on a glass plate G. At about 1 metre away from the mirror of the optical lever, are placed a vertical metre scale and a telescope (not shown in the diagram). Looking through the telescope an inverted image of the scale is visible.

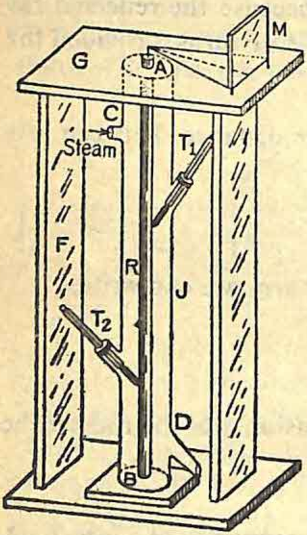

Fig. 4

An optical lever consists of a metallic triangular frame (a tripod) with two equal arms and three pointed legs. There is a vertical plane mirror at right angles to the line joining the two hind legs.

Experimental procedure : (1) Take the rod out of the jacket and find its length, by means of a metre scale. Using different parts of the scale, repeat the observations several times. Find the mean length (l_1). This is the initial length of the rod.

(2) Put the rod inside the jacket tube. Place the optical lever on the glass plate such that the front leg rests on the projecting top (A) of the rod and the mirror M is exactly vertical. Insert two thermometers T_1 and T_2 into the jacket tube. Take their readings. If the thermometers give slightly different readings, take their mean. It is the initial temperature of the rod (t_1°C).

(3) Keep the scale and telescope about a metre away from the mirror of the optical lever. Moving the eye-lens of the telescope back and forth, focuss the cross-wires of the telescope without any parallax. Next turn the focussing screw of the telescope and bring the image of the scale formed by the mirror within sharp focuss. In this condition, both the cross-wires and the scale will be clearly visible through the telescope. Remove parallax between the two, if there be any.

(4) Find the scale division which coincides with the horizontal cross-wire of the telescope. Suppose it is d_1 cm. [Eye-piece may be rotated, if necessary, to make one of the cross-wires exactly horizontal.]

(5) Allow some water to boil in a boiler and pass the steam in the jacket through the inlet tube C. The steam comes out of the exit tube D. The temperature of the rod will increase as evidenced by the upward movement of the mercury thread in the thermometers. When the thermometers T_1 and T_2 give steady temperatures (nearly 99°C) read them and get their mean value, if they differ slightly. This is the final temperature (t_2°C). In this condition, find the scale division coincident with the horizontal cross-wire of the telescope. Avoid parallax while reading the scale division. Suppose the reading is d_2 cm. Here, the displacement of light spot on the scale $d = d_1 \sim d_2$.

(6) Ascertain the distance between the mirror of the optical lever and the scale by a piece of thread and then find the length of the thread with the help of a metre scale. This gives D.

(7) Placing the optical lever on a piece of white paper, apply some pressure on it. Three pointed legs will leave impressions on the paper. Joining the impressions, draw the isosceles

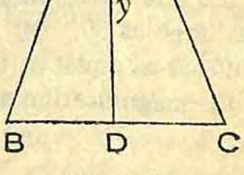

Fig. 4(a)

triangle ABC [Fig 4(*a*)] by a pencil. BC is the line obtained by joining the two back legs. Draw a perpendicular AD on BC. Find the length of the perpendicular AD by a metre scale. This gives the length of the lever arm (y).

Measurements : (*a*) *Initial length of the rod :*

(i) ... cm (ii) ... cm (iii) ... cm.

Mean length (l_1)=... cm.

(*b*) *Temperatures :*

Thermometers	Initial reading	Mean reading (t_1)	Final reading	Mean reading (t_2)
T_1	..°C	..°C	..°C	..°C
T_2	..°C		..°C	

(*c*) *Scale reading :* Initial scale reading (d_1)=... cm.

Final scale reading (d_2)=... cm

Displacement of the light spot (d)

$=d_1 \sim d_2=$...cm.

(*d*) *Distance of the scale from the mirror :*

$D=$... cm

(*e*) *Length of lever arm :*

$y=$... cm.

Calculations : $\alpha=\dfrac{d.y}{2D.l_1(t_2-t_1)}=$..../°C.

Remarks : (1) Displacement of the light spot and hence the value of d will be large if the distance between the metre scale and the mirror is large. It should be more than 1 metre. (2) Both the metre scale and the mirror of the optical lever should be vertical. (3) $2D/d$ is called the multiplying factor of the optical lever and may be as large as 50. (4) The distance of the telescope from the scale should be as small as the focussing of the telescope allows in order that the magnification may be as great as possible.

Oral questions

1. You are given two optical levers—one with a long arm and the other with a short arm ? Which one is better ?

Ans. Greater the value of y, greater is the angle of tilt and hence greater is the displacement of the scale division past the cross-wire.

2. Why should you keep the metre scale at a long distance from the mirror ?

Ans. See remark no. 1.

[Questions of Expt 2.3 are also applicable here.]

2.5. Determination of the pressure co-efficient of air at constant volume by Jolly's apparatus :

Apparatus : Jolly's apparatus, a thermometer, a large beaker, Bunsen burner etc.

Theory : The pressure co-efficient of expansion of a gas at constant volume is defined as the fraction of its pressure at 0°C, by which the pressure of a fixed mass of gas increases per degree celsius rise in temperature.

Suppose, P_0=pressure of a fixed mass of gas at 0°C

P_1= ,, ,, ,, ,, ,, ,, t_1°C

P_2= ,, ,, ,, ,, ,, ,, t_2°C

Then $P_1=P_0(1+\gamma_v.t_1)$ and $P_2=P_0(1+\gamma_v t_2)$

$$\text{(i)} \quad \therefore \quad \gamma_v=\frac{P_1-P_0}{P_0.t_1}.$$

$$\text{(ii) Also, } \frac{P_1}{P_2}=\frac{1+\gamma_v.t_1}{1+\gamma_v.t_2} \quad \text{or,} \quad \gamma_v=\frac{P_2-P_1}{P_1.t_2-P_2t_1}.$$

Thus the determination of γ_v involves the measurement of the pressures exerted by the given mass of the gas under constant volume at two known temperatures t_1 and t_2 or t_1 and 0°C. To do this, the pressures exerted by a given mass of gas at constant volume are measured at different temperatures. Then a graph is drawn with temperatures plotted along X-axis and pressure along Y-axis, 0°C temperature being coincident with the origin. By extrapolation of the graph upto Y-axis, P_0, the pressure at 0°C is found out. Taking any two points on the graph, the values of P_1 and P_2 and the corresponding temperatures t_1 and t_2 may also be found out.

Description of Jolly's apparatus : Fig. 5 shows a form of Jolly's apparatus. It consists of a glass bulb E which can be immersed in a bath contained in a large beaker. The bulb is in communication with mercury manometer through a glass tubing of fine bore. The manometer tube is quite wide and consists of two limbs *AB* and *CD* containing mercury and connected by a thick pressure tubing. The level of mercury in *CD* can be adjusted by raising or lowering the limb *AB* till the meniscus of the mercury level in *CD* just touches a mark *F*. The function of this mark is to ensure constancy of the enclosed volume of air or gas in the bulb *E*. The pressure exerted by the enclosed gas can be determined by noting the difference in the levels of mercury in the two limbs and adding this to or subtracting it from the atmospheric pressure as the case may be. The atmospheric pressure at the time of recording observations is available from Fortin's barometer. For accurate measurement of the difference in levels of the mercury meniscuses, the apparatus is so arranged that the two limbs lie close together beside a paper scale ($M-M$) attached to the frame. Keeping the glass bulb immersed in water contained in a beaker, the water is heated by a burner and always stirred by a stirrer *S*. To read the temperature of air in the bulb, a thermometer *T* is kept close to the bulb. In some instruments, levelling screws are provided at the base for levelling the instrument.

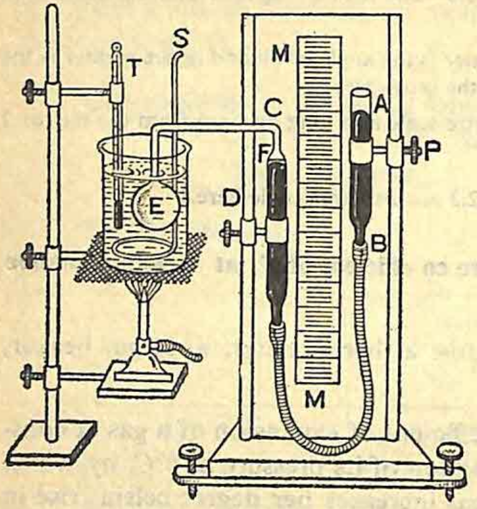

Fig. 5

Experimental procedure : (1) With the help of the levelling screws, make the scale and the manometer tubes vertical.

(2) Read the atmospheric pressure from a barometer. Fill the beaker with water so that the glass bulb *E* is completely submerged. Stir the water for some time and see whether the thermometer gives a steady temperature. Read the temperature when it is steady. Now raise or lower the tube *AB* so that the mercury meniscus in the tube *CD* just touches the mark *F*. Read the positions of the mercury levels in both the limbs on the paper scale *MM*. This is the initial reading of pressure at the room temperature.

(3) Heat the water in the beaker by the burner and slowly stir it. When the temperature of water rises by 5°*C* (or 10°*C*), keep it constant for at least five minutes by controlling the heat. [Remove the burner when the temperature rises by 4°*C*. The heat of water will raise the temperature to some extent. If necessary, again apply some heat. In this way application of heat is to be regulated]. The air in the bulb *E* will be heated and will expand in volume, pushing down the mercury column in the limb *CD*. The level of mercury in the other limb *AB*, as a result, will go up. During the interval when the temperature of air is kept constant, the position of the tube *AB* is to be adjusted by raising or lowering so that the mercury meniscus in *CD* just touches the mark *F*. In this condition, read the position of the mercury level in the limb *AB* on the scale and the temperature as recorded by the thermometer *T*.

(4) In this way, by regulating the heat, raise the temperature of water-bath by steps of 5°*C* (or 10°*C* whichever is convenient) and repeat the observation no. 3 at each step. Take seven or eight observations till the temperature of water-bath is near about 80°*C*.

(5) At the end of the observations of pressures of enclosed air, take the barometer reading again. The mean of the initial and final barometer readings gives the atmospheric pressure (*H*) at the time of experiment.

(6) If the observed difference between the mercury levels (*h*) in the manometer *AB* and *CD* in each case be added or subtracted from the atmospheric pressure ($H \pm h$), the pressures of air enclosed in the bulb *E*, at constant volume, will be obtained. If the mercury level in *AB* is higher than that in *CD*, plus sign *i.e.* ($H+h$) is to be taken and if the level in *AB* is lower than that in *CD*, negative sign *i.e.* ($H-h$) is to be taken.

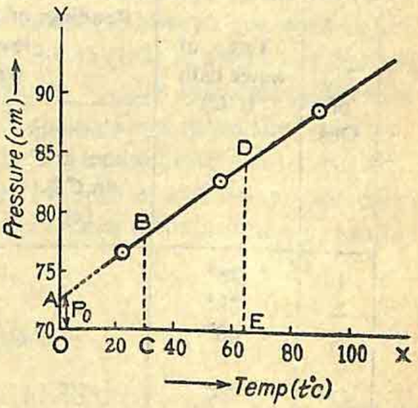

Fig. 5(a)

(7) A graph is then plotted between different temperatures as abscissa and the corresponding pressures in cm. of mercury as ordinate. The temperature should start with 0°*C* located at the origin while a suitable minimum value of pressure may be considered as the origin. A straight line graph will be obtained [Fig. 5(*a*)]. The graph can be employed

to get the value of P_0, the pressure of the enclosed air at 0°C by extrapolation. Produce the straight line DB to cut the pressure axis at A. Pressure corresponding to the point A gives the value of P_0. Now, take any two points B and D on the straight line and draw perpendiculars BC and DE on the temperature axis. Then $AO=P_0$; $BC=P_1$; $DE=P_2$; $OC=t_1$ and $OE=t_2$.

Measurements : (*a*) *Atmospheric pressure from barometer ;*

Value of the smallest division of the main scale=...cm
... Vernier divisions=... main scale divisions
=... cm.
$\therefore$ 1 ,, ,, =... cm.
Vernier constant=... cm.

Time of measurement	Barometric height			
	Main scale (cm) (x)	Vernier scale (y)	Total reading $=x+y\times v.c.$ (cm)	Mean (H) (cm)
Before the start of the expt.	..	..	..	..
After finishing the expt	..	..	..	

(*b*) *Pressures at different temperatures :*

No. of Obs.	Temp. of water bath (t°C)	Readings of mercury levels (cm)		Diff. of levels ($h=a\sim b$) (cm)	Total pressure ($P=H\pm h$) (cm)
		Constant level at F in CD (a)	Level in AB (b)		
1.	28°	.	..	..	..
2.	38°	,,	..	..	..
3.	48°	,,	..	..	..
etc	etc	etc	etc	etc	etc
6.	78°	,,	..	..	..
7.	88°	,,	..	..	..

(*c*) *Table for drawing graph* : [Data taken from table (b)]

Temp. (t°C)	28°	38°	48°	58°	68°	78°	88°
Pressure (P *cm*)	..	..	..	..	..	..	..

Calculations : (i) From the graph, $P_0=OA=$..cm of Hg

$P_1=BC=$.. ,,

$t_1=OC=$.. °C

$$\therefore \gamma_v=\frac{P_1-P_0}{P_0.t_1}=..$$

(ii) From the graph, $P_1=BC=$.. ; $P_2=DE=$.. ; $t_1=OC=$.. °C ; and $t_2=OE=$.. °C.

$$\gamma_v=\frac{P_2-P_1}{P_1.t_2-P_2.t_1}=\ldots.$$

Mean value of $\gamma_v=$....

Remarks : (1) The bulb E containing air should be completely immersed in the bath and the latter should be constantly stirred. (2) The air enclosed in the bulb should be completely dry. As water vapour does not obey gas laws, slight traces of moisture in the enclosed air will affect the result. (3) Mercury used in the manometer tubes should be pure. Impure mercury stricks to the wall of the glass tube. For this reason, manometer limbs should be tapped gently before taking the final observation. (4) When the temperature of the bath increases, the volume of the glass bulb E also increases to some extent. So, the volume of the enclosed air remains always constant—this statement is not exactly tenable. (5) Since glass is a bad conductor of heat, it takes time for the enclosed air to acquire the temperature of the bath. The readings should, therefore, be taken when the temperature of the bath has remained steady for some time. (6) As the portion of the glass tubing connecting the bulb E and the manometer limb CD remains outside the bath, temperature of enclosed air cannot be taken same throughout.

Oral questions

1. What is the coefficient of expansion of gas ? Is it same for all gases ? What is its value ?

Ans. Consult any standard text book. It is same for all gases and its value is 0·00365.

2. Can an unknown temperature be found out by this arrangement ?

Ans. Yes. Knowing γ_v, and determining the pressure exerted by the enclosed gas at the unknown temperature, the equation mentioned in the theory helps us to determine the unknown temperature.

3. Is the coefficient of volume expansion of a gas equal to its pressure coefficient ?

Ans. Yes, their value is 0·00365.

4. What is the harm if the enclosed air in the experiment is moist ?

Ans. See remark no. 2.

5. Can any liquid other than mercury be used for the measurement of pressure ?

Ans. Mercury has the advantage that the level differences in this case, when directly added to or subtracted from barometric reading, gives the air-pressure because barometers also employ mercury. For any other liquid, the pressure difference is to be calculated from the formula $p=h.\ \rho.g$ and then converting it in terms of mercury height it is to be added to or subtracted from the barometric height.

6. Why should you keep the temperature of water-bath steady for some time ?

Ans. See remark no. 5.

7. Why should you use a stirrer ?

Ans. To keep the temperature of the bath uniform throughout, it should be constantly stirred. The stirrer is used for that purpose.

8. How many coefficients of expansion does a gas possess ? What is their relation ?

Ans. Two ; one is pressure coefficient at constant volume and the other is volume co-efficient at constant pressure. They are equal to each other.

*2.6. Determination of the volume co-efficient of air at constant pressure by Regnault's apparatus :

Apparatus : Regnault's apparatus, thermometer, beaker, burner etc.

Theory : The volume coefficient of expansion of a gas at constant pressure is defined as the fraction of its volume at 0°C, by which the volume of a fixed mass of gas increases per degree celsius rise in temperature.

Suppose, V_0=volume of a fixed mass of gas at 0°C
V_1= „ „ „ „ „ „ „ t_1°C
V_2= „ „ „ „ „ „ „ „ t_2°C

Then, $V_1=V_0(1+\gamma_p.t_1)$ and $V_2=V_0(1+\gamma_p.t_2)$

(i) $\therefore\ \gamma_p=\dfrac{V_1-V_0}{V_0.t_1}$

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*For North Bengal University only.

(ii) Also, $\frac{V_1}{V_2}=\frac{1+\gamma_p.t_1}{1+\gamma_p.t_2}$ or $\gamma_p=\frac{V_2-V_1}{V_1t_2-V_2t_1}$

Thus, the determination of γ_p involves the measurement of the volumes occupied by the given mass of the gas under constant pressure at two known temperatures t_1 and t_2 or t_1 and 0°*C*. To do this, the volumes occupied by a given mass of gas, at constant pressure, are measured at different temperatures. Then a graph is drawn with temperatures plotted along X-axis and volumes along Y-axis, 0°*C* temperature being coincident with the origin. By extrapolation of the graph upto Y-axis, V_0, the volume at 0°*C* is found out. Taking any two points on the graph, the values of V_1 and V_2 and the corresponding temperatures t_1 and t_2 may also be found out.

Description of Regnault's apparatus : Fig. 6 shows a form of Regnault's apparatus. It consists of a glass tube *BC* bent almost in the form of a U, one end of which is open to air and the other end terminating in a bulb *A*. The bulb is full of dry air and is graduated in c.c. Some part of the bulb *A* and the U-tube *BC* contain sulphuric acid. At the bottom of the U-tube, a narrow glass tube with a stop cock *D* is fitted. A wide glass jacket cylinder containing water surrounds the U-tube. The lower end of the cylinder is closed by a rubber cork *EF*. The exit tube *D* projects through a hole at the middle of the rubber cork. A bent copper tube *MN* is inserted into the jacket through two holes in the rubber cork. Steam is sent through the copper tube *MN* which warms up the water in the jacket. A stirrer *S* to stir water and a thermometer *T* to read the temperature of air in the bulb *A* are also provided. By draining out some acid on opening the stop-cock *D* or by adding some through the open end of the U-tube, when the levels of acid in the two limbs *B* and *C* are made equal, the pressure of air in the bulb *A* will be equal to atmospheric pressure.

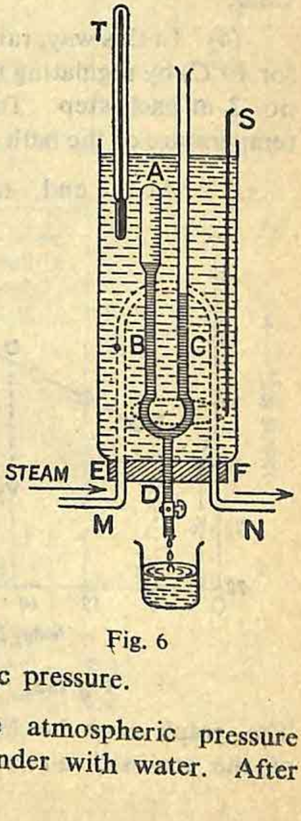


Fig. 6

Experimental procedure : (1) Get the atmospheric pressure from a barometer. Fill up the jacket cylinder with water. After stirring the water by the stirrer *S* for some time, record its temperature from the thermometer *T*.

(2) Put a beaker below the tube *D*. Drain out acid slowly by opening the stop-cock so that the meniscuses of the acid in the limbs *B* and *C* stand at the same level. When this is done, read the volume of air in the bulb *A* from the scale graduated on it. This will give the initial volume of air at the room temperature.

(3) Allow some water to boil in a boiler and pass the steam through the bent tube *MN*. The temperature of water in the jacket will slowly rise. By controlling the flow of steam and constant stirring, the temperature of the bath is made 5°*C* (or 10°*C* whichever is convenient) more than the room-temperature and is kept steady for a few minutes. Meanwhile, the acid levels in the limbs *B* and *C* are to be brought at the same height by slowly draining out the acid through the tube *D*. [If necessary, some acid may also be poured through the open end of the limb *C*]. Record the temperature of the bath from the thermometer and the volume of air in the bulb *A* from the scale.

(4) In this way, raise the temperature of the bath by steps of 5°*C* (or 10°*C*) by regulating the flow of steam and repeat the observation no. 3 at each step. Take seven or eight such observations till the temperature of the bath rises to about 80°*C*.

(5) At the end, take the barometer reading again. The mean of the initial and final barometer readings gives the atmospheric pressure at the time of experiment.

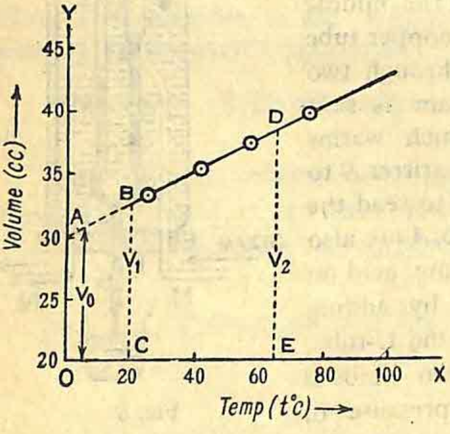


Fig. 6(a)

(6) A graph is then plotted between different temperatures as abscissa and the corresponding volumes in c.c. as ordinate. The temperature should start with 0°*C* located at the origin while a suitable minimum value of volume may be considered as the origin. A staright line graph will be obtained [Fig 6(*a*)]. The graph may be employed to get the value of V_0, the volume of the enclosed air at 0°*C* by extrapolation. Produce the straight

line DB to cut the volume axis at A. Volume corresponding to the point A gives the value of V_0. Now, take any two points B and D on the straight line and draw perpendiculars BC and DE on the temperature axis. Then, $AO=V_0$; $BC=V_1$; $DE=V_2$; $OC=t_1$ and $OE=t_2$.

Measurements : (*a*) *Atmospheric pressure from barometer.*

Value of the smallest division of the main scale=... cm.

... Vernier divisions=...main scale divisions.

=... cm.

$\therefore$ 1 ,, ,, =... cm.

$\therefore$ Vernier constant=... cm.

Time of measurement	Barometric height			
	Main scale (cm) (x)	Vernier scale (y)	Total reading ($x+y.\times v.c.$)	Mean (H) cm
Before the start of the expt.	..	..	..	..
After finishing the expt.	..	..	..	

(*b*) *Volumes at different temperatures :*

No. of Obs	Temp. of water bath (t°C)	Volume of air in the bulb A (V c.c.)
1.	28°C (room temp)	..
2.	33°	..
3.	38°	..
..	..	..
..	..	..
7.	58°	..
8.	63°	..
9.	68°	..
etc	etc	

Calculations : (i) From the graph, $V_0=OA=\ldots$ c.c.

$V_1=BC=\ldots$ c.c.

$t_1=OC=\ldots$ °C

$$\therefore \gamma_p=\frac{V_1-V_0}{V_0 . t_1}=..$$

(ii) From the graph, $V_1 = BC = \ldots$ c.c. ; $V_2 = DE = \ldots$ c.c. ; $t_1 = OC = \ldots$ °C and $t_2 = OE = \ldots$ °C

$$\gamma_p = \frac{V_2 - V_1}{V_1 t_2 - V_2 t_1} = \ldots$$

Mean value of $\gamma_p = \ldots$

Remarks : (1) For a good graph 9/10 points are necessary. For this reason, the temperature of the bath should be changed by steps of 5°C or 10°C. (2) While making the acid levels in the two limbs *B* and *C* equal, the draining out of acid may sometimes exceed the desired extent. In that case some acid may have to be poured through the open end of the limb *C*. (3) While reading the volume of enclosed air, parallax should be avoided. (4) The temperature of the water bath should remain steady when observation on volume of air is taken. (5) As glass is not a good conductor of heat, the air in the bulb *A* takes some time to acquire the temperature of the bath. For this reason, equalisation of acid levels and observation of air volume should be made some time after the water-bath has attained a steady temperature.

Oral questions

1. What is volume coefficient of expansion of a gas ? Is it same for all gases ? What is its value ?

Ans. See theory. It is same for all gases. Its value is 0·00366.

2. Can you find out an unknown temperature by this apparatus ?

Ans. Yes ; $t = \frac{V_1 - V_0}{\gamma_p . V_0}$; knowing V_1, V_0 and γ_p, temperature t can be found out.

3. Is volume coefficient of gas equal to its pressure co-efficient ?

Ans. Yes ; their value is 0·00366.

4. Usually mercury is used in a manometer. Why is sulphuric acid used here ?

Ans. Sulphuric acid is lighter than mercury. A slight difference of pressure will cause an appreciable difference in the levels of sulphuric acid in the two limbs. Further, as sulphuric acid is hygroscopic, it keeps the air in the bulb *A* perfectly dry.

5. Can you take water in place of sulphuric acid ?

Ans. No ; water will evaporate and make the air in the bulb moist. Moisture does not obey gas laws.

6. In this experiment you have kept the pressure of air constant. What is the value of the constant pressure ?

Ans. The constant pressure is equal to the atmospheric pressure.

7. Should you take reading as soon as the temperature of the bath becomes steady ?

Ans. See remark no. 5.

8. Does the volume coefficient of expansion of a gas depend on (i) the mass of the gas (ii) the scale of temperature ?

Ans. It does not depend on the mass of the gas but depends on the scale of temperature.

9. Can you find out the value of absolute zero from this experiment ?

Ans. Yes ; at absolute zero, the volume of a given mass of gas theoretically becomes zero. So, if the graph *DB* is extrapolated to intersect the temperature axis,, then the intersection point will give the value of absolute zero in celsius scale ($-273°C$).

2.7. Determination of the co-efficient of absolute expansion of mercury by Dulong and Petit's method :

Apparatus : Dulong and Petit's apparatus, travelling microscope, (or a cathetometer), thermometers etc.

Description of the apparatus : The apparatus is illustrated in Fig 7. It consists of a glass tube bent in the form of a rectangle *ABCD*.

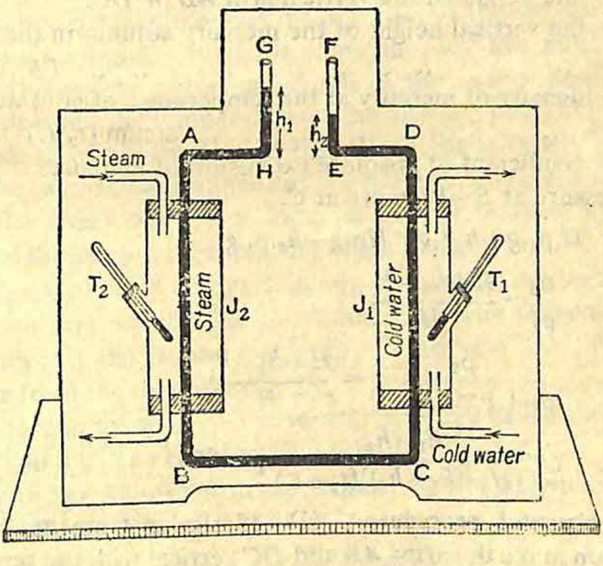


Fig. 7

The arms *AB* and *DC* are vertical while the arms *BC* and *AD* are horizontal. The portions of the tube after *A* and *C* have approached each other and then have gone up vertically along HG and EF respectively remaining close to each other. The arms DC and AB are enclosed in jacket tubes J_1 and J_2 respectively whose mouths are closed by corks. Through the jacket J_2 steam is passed and through the jacket J_1 cold water (at room temperature) is circulated. Two thermometers T_1 and T_2 are inserted into the jackets J_1 and J_2 respectively. The thermometer T_1 records the temperature of cold water while the thermometer T_2 records the temperature of steam. Dry and pure mercury is poured in the tube until mercury enters in the limbs HG and EF. The whole arrangement is fixed on a wooden board by suitable clamps so that AB and DC are vertical and BC and AD are horizontal. Levelling screws are provided at the base in some instruments. During an experiment, the arm EC is wrapped up with wet cloth or wet blotting papers so that heat may not be conducted through the arm.

Theory : The coefficient of absolute expansion of mercury is defined as the real increase in volume per unit volume of mercury at $0°C$ for $1°C$ rise in temperature. Suppose,

H=the height of the vertical arm AB or DC.
h_1=the vertical height of the mercury column in the tube HG
h_2= ,, ,, ,, ,, ,, ,, ,, ,, ,, ,, EF
ρ_1=density of mercury at the temperature of cold water ($t_1°C$)
ρ_2= ,, ,, ,, ,, ,, ,, ,, steam ($t_2°C$)
γ=coefficient of absolute expansion of mercury.

Now, Pressure at B=Pressure at C.

$$\text{or,}\quad H.\rho_2.g+h_1\rho_1 g=H\rho_1 g+h_2.\rho_1.g$$

$$\text{or,}\quad \frac{\rho_2}{\rho_1}=\frac{H+h_2-h_1}{H}$$

$$\text{or,}\quad \frac{\rho_2}{\rho_2\{1+\gamma(t_2-t_1)\}}=\frac{H+h_2-h_1}{H}$$

$$\therefore\quad \gamma=\frac{h_1-h_2}{\{H-(h_1-h_2)\}(t_2-t_1)}\text{ per }°C.$$

Experimental procedure : (1) If the instrument has levelling screws then make the arms AB and DC vertical with the screws. Allow the jackets J_1 and J_2 to be filled up by air so that the thermometers T_1 and T_2 register the same temperature (room temperature). The heights of mercury columns in the tubes EF and GH will also be equal . If the thermometers register different temperatures and the heights of mercury columns in EF and GH are not equal, their differences are to be noted.

(2) Wrap the horizontal arms AD and BC with wet cloth or wet blotting paper. Circulate water at room temperature through the jacket J_1 and note its temperature ($t_1°C$) by the thermometer T_1.

(3) Allow some water to boil in a boiler and pass the steam coming from the boiler through the jacket J_2. The thermometer T_2 will show a gradually increasing temperature and the mercury column in the tube HG will slowly rise up.

(4) Meanwhile find the vernier constant of a travelling microscope and focus its cross-wires. Make the axis of the microscope horizontal and bring it near the tube EF. Turning the focussing screw, focus the microscope on the mercury column. The meniscus of mercury column in the tube will appear convex. Raise or lower the microscope along its vertical scale so that the horizontal cross-wire is set tangential to the convex meniscus. In this conditi. read the vertical main scale and its vernier.

(5) By this time, the temperature of the jacket J_2 will attain a steady value. Read this steady temperature ($t_2°C$) from the thermometer T_2. Shift the microscope towards left and bring the mercury column in the tube GH in focus. Adjust the height of the microscope such that its horizontal cross-wire is tangential to the convex meniscus of the mercury column. Wait for sometime, say five minutes. See whether the cross-wire is still tangential to the convex meniscus. It may so happen that the temperature of the mercury column in HG has not yet attained steady value and that its height is slowly changing. If that be the case, the adjustment of the horizontal cross-wire is to be repeated after every 5 minutes' interval till the temperature of the jacket J_2 and the height of the mercury column in GH become constant. When the system finally attains a steady state, read the vertical scale and its vernier after setting the horizontal cross-wire tangential to the convex surface of the mercury column. Read also the final steady temperature from the thermometer T_2.

(6) The difference of the readings obtained from the observations no. 5 and 4 will be equal to (h_1-h_2).

(7) With the help of a metre scale, find the height of the arm AB or CD. This is equal to H.

Measurements : (*a*) *Microscope measurement* :

Value of one small division of the main scale = . . . cm.
. . Vernier divisions = . . . main scale divisions
= . . : cm.
∴ 1 ,, ,, = . . . cm.
Vernier constant = . . . cm.

(i) *When the jackets contain air at room temperature* :

Interval in mnts	Temperature (°C)		Meniscus readings (cm)						Difference (h_1-h_2)
			Column *GH* (h_1)			Column *EF* (h_2)			
	Jacket $J_1(t_1°)$	Jacket $J_2(t_2°)$	Main scale	Vernier scale	Total	Main scale	Vernier scale	Reading	
0	..	..	..	..	..	..	..	..	Nil
5	..	..	..	..	..	..	..	..	,,
10	..	..	..	..	..	..	..	..	,,

(ii) *When cold water and steam are circulated through the jackets* :

Interval in mnts	Temperature (°C)		Meniscus readings (cm)						Difference (h_1-h_2) cm
			Column *GH* (h_1)			Column *EF* (h_2)			
	Jacket $J_1(t_1°)$	Jacket $J_2(t_2°)$	Main scale	Vernier scale	Total	Main scale	Vernier scale	Total	
0	..	..	..	..	..	..	..	..	..
5	..	..	..	..	..	..	..	..	..
10	..	..	..	..	..	..	..	..	..
etc	..	..	..	..	..	..	..	..	..
..	.. (steady)	.. (steady)	..	..	..	..	..	..	.. (steady)

(*b*) *Height of the arm AB or DC* :

(i) ... cm (ii) ... cm (iii) ... cm.

$\therefore$ Mean height $(H)=$... cm.

Calculations :

$$\gamma=\frac{h_1-h_2}{\{H-(h_1-h_2)\}(t_2-t_1)}=\ldots \text{ per } °C.$$

Remarks : (1) While reading the position of mercury meniscus, there should be no parallax between the cross-wires and the image of the meniscus. (2) Final reading should be taken when the system has attained a steady state. (3) The heights of the tubes *AB* and *DC* are taken equal ; but due to difference of temperature, their heights will be slightly different. Hence, the final result is not faultless. (4) Although the horizontal arms *AD* and *BC* are wrapped up with wet cloth, yet some heat from steam will be conducted to water. As a result, hot mercury column will conduct some heat to cold column. (5) In some instruments, a scale graduated on a mirror is fixed by the side of the tubes *GH* and *EF*, from which the value of $(h_1 - h_2)$ may be directly obtained. (6) Since *H* is much larger than $(h_1 - h_2)$, it may be measured to a sufficient degree of accuracy with a metre scale.

Oral questions

1. What is the coefficient of absolute expansion of a liquid ? What is its relation with the coefficient of apparent expansion ?

Ans. See theory. Coefficient of absolute expansion $=\gamma$, that of apparent expansion $=\gamma'$ and the cubical expansion of the container $=\gamma_g$., then, $\gamma = \gamma' + \gamma_g$.

2. What is the harm if the cross-sections of the tubes *AB* and *DC* are different ?

Ans. No harm because pressure does not depend on the cross-section ; but if one of the tubes very narrow then, capillary action will cause a difference of heights of the mercury columns even when the temperature is same in both the columns.

3. Can you keep air instead of water in the jacket J_1 ?

Ans. Yes ; provided the temperature of air can be kept steady throughout.

2.8. Determination of the coefficient of apparent expansion of a liquid by a weight thermometer :

Apparatus : A specific gravity bottle (or a weight thermometer), thermometer (reading upto $\frac{1}{5}$ of a centigrade degree), experimental liquid (say paraffin or glycerine), burner, beaker, a balance with weight box, a stand, a piece of thread etc.

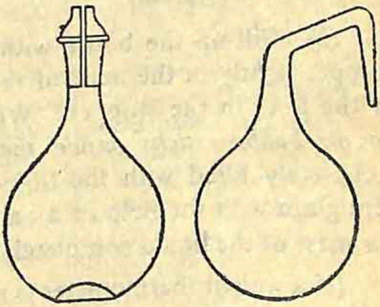


Fig. 8

Description of the weight thermometer : It consists of a glass bulb with a long narrow neck [Fig 8(b)]. The neck is usually bent twice at right angle and has a tapering end. Although its appearance differs very much from an ordinary Celsius or Fahrenheit thermometer, yet it is called a thermometer because an unknown temperature can be found out with its help. To fill up the thermometer with a liquid, alternate heating and cooling process is to be adopted. For this purpose, the bent neck is inserted in the liquid taken in a basin and the bulb is heated a little. The air in the bulb will expand and will go out bubbling through the liquid. If the bulb is now cooled, some liquid will enter the bulb. In this way, the bulb is to be alternately heated and cooled until it is completely filled up with the liquid.

An ordinary specific gravity bottle may also serve the purpose of a weight thermometer. It consists of a flat-bottomed glass bottle with a ground neck in which a ground glass stopper may fit in a water-tight condition. There is a long narrow hole in the stopper [Fig 8(a)]. When the bottle is filled up with a liquid and the stopper is placed in the neck, excess liquid will come out through the hole. As specific gravity bottles are easily available, in the description of the experiment below, a specific gravity bottle has been used instead of a weight thermometer.

Theory : Let the mass of liquid completely filling the bottle at $t_1°C = m_1$ gm

" " " " " " " " " $t_2°C = m_2$ gm

Then, the coefficient of apparent expansion of the liquid ($t_2 > t_1$)

$$\gamma' = \frac{m_1 - m_2}{m_2(t_2 - t_1)} = \frac{\text{mass of liquid expelled}}{\text{mass of liquid left} \times \text{diff of temp}}$$

Experimental procedure : (1) Find the weight (w_1) of the clean dry and empty specific gravity bottle with the help of a balance, upto the nearest centigram.

(2) Fill up the bottle with the liquid under test and place the stopper lightly in the neck of the bottle. Some liquid will come out of the hole in the stopper. Wipe the bottle clean. Take care that *no air bubble sticks inside the bottle.* The bottle, thus becomes completely filled with the liquid. Weigh the bottle to the nearest centigram with the help of a balance. Let the weight be w_2. Then the mass of the liquid completely filling the bottle $m_1 = (w_2 - w_1)$ gm.

[If a weight thermometer is used, it is to be filled up by the liquid by alternate heating and cooling as described earlier.]

(3) Take some water in a beaker at room temperature. Note its temperature with the help of a thermometer ($t_1°C$). This will also be the initial temperature of the liquid under test. Tie a piece of thread to the neck of the specific gravity bottle and immerse the bottle into

the water upto its neck as shown in the Fig 8(c). Keep the bottle hanging from a suitable stand and place the beaker on a piece of wire gauge over a tripod stand. Insert a thermometer *T* and a stirrer *S* into the beaker.

(4) With the help of a burner, apply heat gently to the beaker and stir water slowly. The temperature of water bath will gradually rise. When the temperature of the bath attains a specified value t_2°*C* (say 80°*C*), keep the temperature steady for at least 5 minutes. The liquid inside the bottle, in the mean time, expands and some of it comes out through the hole of the stopper. After keeping the bottle for about 5 minutes in the steady temperature bath, take it out of the bath, holding the thread. Note the steady temperature (t_2°*C*).

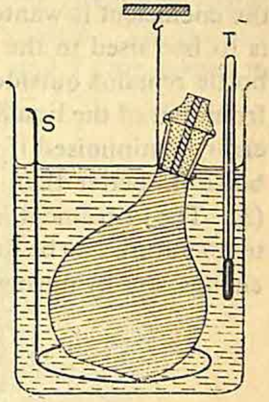


Fig. 8(c)

(5) Keep the bottle hanging in air. Wipe out carefully any water or liquid that may stick to the neck of the bottle by some clean cloth or blotting paper. When the bottle cools down to the room-temperature (*i.e.* t_1°*C*) again, carefully wipe clean the body of the bottle. Weigh the bottle again up to the nearest centigram. Let this weight be w_3.

Then the mass of the liquid completely filling the bottle at t_2°*C* $m_2=(w_3-w_1)$ gm.

Measurements : Initial temperature (t_1°*C*)=... °*C*
Final steady temperature (t_2°*C*)=... °*C*

Mass of empty bottle (w_1)	Mass of bottle +liquid filling the bottle at t_1°*C* (w_2)	Mass of liquid filling the bottle at t_1°*C* ($m_1=w_2-w_1$)	Mass of bottle+ liquid filling the bottle at t_2°*C* (w_3)	Mass of liquid filling the bottle at t_2°*C* ($m_2=w_3-w_1$)
..+..+..	..+..+..		+..+..+	
+..=..gm	..+..=..gm	..gm	..=..gm	..gm

Calculations :

$$\gamma'=\frac{m_1-m_2}{m_2(t_2-t_1)}=\ldots \text{ per } {}^\circ C.$$

Remarks : (1) The above experiment gives the coefficient of apparent expansion of the liquid within the range of t_1° and $t_2^\circ C$. If the coefficient is wanted for different range of temperatures, the bottle is to be raised to the corresponding temperatures. (2) Neck of the bottle remains outside the bath and hence its temperature is different from that of the liquid. This brings in slight error in the result. This error is minimised if the neck is short. In this respect, a sp. gravity bottle is better than a weight thermometer which has a long neck. (3) The instrument is called *weight* thermometer because an unknown temperature can be found out with its help, not by noting any thermal change, but by noting the weight of some liquid.

Oral questions

1. What is the coefficient of apparent expansion of a liquid ? What is its difference with the coefficient of real expansion ?

Ans. Consult any text book.

2. Which coefficient did you measure by this experiment ?

Ans. The coefficient of apparent expansion.

3. Can you find out the coefficient of real expansion from this experimental result ?

Ans. Yes, if the cubical expansion of the container is known. The relation is $\gamma=\gamma_1+\gamma_g$ where γ=coefficient of real expansion, γ_1=coefficient of apparent expansion, γ_g= coeffn of cubical expansion of the container.

4. Which are the sources of error in this experiment ?

Ans. Errors occur in weighing the bottle, full or empty. Error also occurs in measuring temperature, due to non-uniformity of temperature and due to incomplete drying of the bottle.

5. Why is the instrument called weight thermometer ?

Ans. See remark no. 3.

2.9. Determination of the thermal conductivity of a metal by Searle's method :

Apparatus : Searle's apparatus, four thermometers (two reading $\frac{1}{2}{}^\circ C$ and the other two $\frac{1}{10}{}^\circ C$), stop watch, measuring cylinder, beaker, slide-callipers etc.

Description of the apparatus : Fig. 9 shows the appearance of Searle's apparatus. *AB*, the specimen, is a polished cylindrical rod of copper of length about 20 cm and diameter about 5 cm. One end

A of the bar is inserted into a steam-chest *E*. A copper coil *C* is soldered at the other end. This end is cooled by circulating water through the coil from a constant-head water reservoir *R*. To measure the temperature gradient, two thermometers T_1 and T_2 are placed in two mercury-filled cups. The cups are made of the same metal as the bar and are soldered to the specimen at a certain distance apart. The temperatures of the inflowing and outflowing water are recorded respectively by two other thermometers T_4 and T_3. The whole apparatus is heavily lagged with felt.

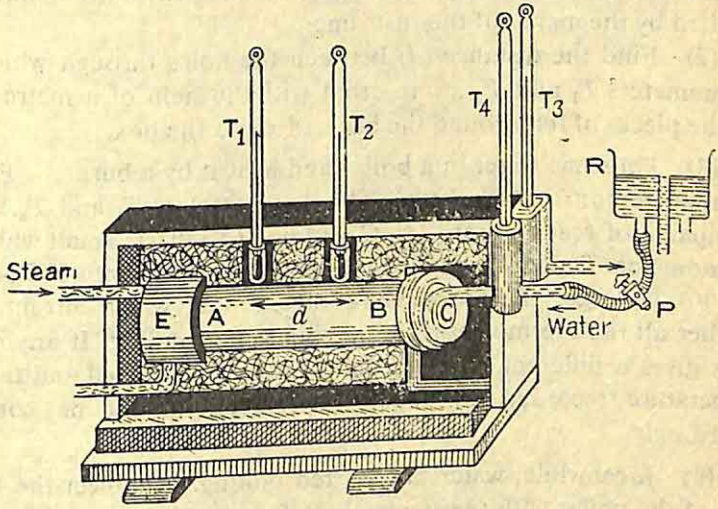


Fig. 9

Theory : If Q be the heat conducted through any section of the rod at the steady state, then $Q=\frac{K.A.(\theta_1-\theta_2)t}{d}$ where K=thermal conductivity of the metal ; A=cross-sectional area of the specimen ; θ_1 and θ_2=temperatures as recorded by the thermometers T_1 and T_2 ; t= time of flow of heat and d=the distance between the holes through which the thermometers T_1 and T_2 are inserted.

Again, $Q=m(\theta_3-\theta_4)$ where m=mass of water that circulated through the copper coil *C* during the interval *t*, and θ_3, θ_4=the temperatures of outflowing and inflowing water

$$\therefore\ K=\frac{d(\theta_3-\theta_4).m}{A(\theta_1-\theta_2).t}=\frac{d(\theta_3-\theta_4).m}{\pi r^2(\theta_1-\theta_2)t}\ [r=\text{radius of the rod } AB]$$

Experimental procedure : (1) Open the box containing the specimen rod and remove the pieces of felt surrounding the rod.

Determine the diameter of the bar at a number of places with the help of slide-callipers. At every place, make the determination along two mutually perpendicular diameters. Find the mean diameter and hence the radius (r) of the bar. [In some instruments, the diameter of the rod can not be directly measured. In such cases the diameter is supplied by the maker of the instrument.]

(2) Find the distance (d) between the holes through which the thermometers T_1 and T_2 are inserted with the help of a metre scale. Put the pieces of felt around the bar and close the box.

(3) Put some water in a boiler and heat it by a burner. Put the thermometers in their sockets. The thermometers T_1 and T_2 should be capable of reading upto $\frac{1}{10}$°C because $(\theta_1-\theta_2)$ is small while the thermometers T_3 and T_4 should be capable of reading upto $\frac{1}{2}$°C because $(\theta_3-\theta_4)$ is large. Before passing steam and water current, note whether all the thermometers are giving same reading. If any one of them gives a different reading, the error is to be noted and the final temperature recorded by that thermometer should be corrected accordingly.

(4) Meanwhile, water has started boiling. Connect the rubber tube of the boiler with the steam-chest E and pass a steady current of steam into the steam-chest to heat the end A of the copper bar. At the other end connect the copper coil C with the constant-head water reservoir R. Take care that the head of water in the reservoir remains always constant. To have a measurable difference of temperature (not more than 10°C) between the incoming and outgoing water through the copper spiral C, adjust the rate of flow of water low by regulating the pinch-cock P fitted with the tube of the water reservoir.

(5) Wait till the steady state is about to be reached ; it generally takes about half an hour or so for the bar to attain the steady state. In the steady state, take the readings of all the thermometers. The fact that the bar has attained the steady state is confirmed by noting the temperatures of four thermometers at every five minutes' interval and observing finally that there is no change in their respective readings. Consider the final observations as giving the steady temperatures of the four thermometers. Now place a measuring cylinder below the outflow tube and collect water for some time. Note the time of collection of water (t) by a stop-watch.

(6) Repeat the experimental observation no. 5 after changing the rate of flow of water through the copper coil C by the pinch-cock P. Interchange the positions of the thermometers also.

(7) When the experiment is over, disconnect the boiler and the water-reservoir from Searle's apparatus.

Measurements : (*a*) *Radius of the bar* :

Radius (r) of the copper bar = . . cm

[Give the table for the readings of the slide-callipers].

(*b*) *The distance* (d) *between the thermometers* T_1 *and* T_2 :

(i) ... cm (ii) ... cm (iii) ... cm.

Mean distance (d) = ... cm.

(*c*) *Thermometer readings before steam and water circulation* :

Interval in minites	Thermometer reading				$(\theta_1-\theta_2)$	$(\theta_3-\theta_4)$
	T_1	T_2	T_3	T_4		
0	..	..	..	..	Nil	Nil
2	..	..	..	..	,,	,,
4	..	..	..	..	,,	,,
6	..(θ_1)	..(θ_2)	..(θ_3)	..(θ_4)	,,	,,

(*d*) *Thermometer readings at steady state* :

No. of Obs	Interval in minutes	Thermometer readings				$(\theta_1-\theta_2)$	$(\theta_3-\theta_4)$
		T_1	T_2	T_3	T_4		
	0	..	..	..	..		
	2	..	..	..	..		
1.	4	..	..	..	..	..	.. (8°C)
	6	..(θ_1) (steady)	..(θ_2) (steady)	..(θ_3) (steady)	..(θ_4) (steady)		
	0	..	..	..	..		
2.	2	..	..	..	..	..	.. (10°C)
	4	..	..	..	..		
	4	..	..	..	..		
	6	..(θ_1) (steady)	..(θ_2) (steady)	..(θ_3) (steady)	..(θ_4) (steady		

(*e*) *Mass of water collected* :

No. of Obs	Time of collection (t)	Volume of water collected	Mass of water collected (m)
1.	..sec	..c.c.	..gm
2.	..sec	..c.c.	..gm

(*f*) *Consolidated table* :

No. of Obs	radius of the rod (r)	Mass of water (m)	Time of collection (t)	$(\theta_1-\theta_2)$	$(\theta_3-\theta_4)$	K	Mean K
1.	..cm	..gm	..sec	..°C	..°C	..	..
2.	..cm	..gm	..sec	..°C	..°C	..	

Calculations :

1. $K=\dfrac{d(\theta_3-\theta_4).m}{\pi r^2(\theta_1-\theta_2)t}=..$ *c.g.s.*

2. $K=..$ *c.g.s.*

Proportional and Percentage error

We have, $K=\dfrac{d(\theta_3-\theta_4).m}{\pi r^2(\theta_1-\theta_2).t}$

The maximum % error in K is given by,

$$\left(\frac{\delta K}{K}\right)_{max}\times 100\%=\left[\frac{\delta d}{d}+\frac{\delta(\theta_3-\theta_4)}{(\theta_3-\theta_4)}+\frac{\delta m}{m}+\frac{2.\delta r}{r}+\frac{\delta(\theta_1-\theta_2)}{(\theta_1-\theta_2)}+\frac{\delta t}{t}\right]\times 100\% \quad ..\quad \text{(i)}$$

In one experiment, the following data were obtained,

$d=10$ cm. $\delta d=0.01$ cm (Least count of slide callipers)

$\begin{cases}\theta_3=93^\circ C\\ \theta_4=84^\circ C\end{cases}$ $\delta\theta_3=\delta\theta_4=0{\cdot}1^\circ C$ ((Least count of T_3 and T_4 is $\frac{1}{10}{}^\circ C$)

$m=35$ gm $\delta m=0{\cdot}5$ gm (Volume collected was 35 c.c. in a measuring cylinder whose least count=0·5 c.c.)

$r=2{\cdot}50$ cm	$\delta r=0{\cdot}001$ cm	(Least count of screw gauge)
$\begin{cases}\theta_1=60^\circ C\\ \theta_2=33{\cdot}2^\circ C\end{cases}$	$\delta\theta_1=\delta\theta_2=0{\cdot}5^\circ C$	(Least count of T_1 and T_2 is $\frac{1}{2}^\circ C$)
$t=120$ sec.	$\delta t=0{\cdot}2$ sec.	

Substituting the values in eqn (i)

$$\left(\frac{\delta K}{K}\right)_{max}\times 100\%=\left[\frac{0{\cdot}01}{10}+\frac{0{\cdot}2}{(93-84)}+\frac{0{\cdot}5}{35}+\frac{2\times0{\cdot}001}{2{\cdot}5}+\frac{0{\cdot}5\times2}{(60-33{\cdot}2)}+\frac{0{\cdot}2}{120}\right]\times 100\%$$

$$=[0{\cdot}001+0{\cdot}022+0{\cdot}0143+0{\cdot}0008+0{\cdot}0039+{\cdot}0016]\times 100\%$$
$$=0{\cdot}0436\times 100\%=4{\cdot}36\%$$

Hence the proportionate error in K in the above measurement is 4·36%. Of this error, the contribution due to measurement of temperature of inflowing and outflowing water $(\theta_3-\theta_4)$ is 2·2% and that due to measurement of mass of water (m) is 1·4%. So, the thermometers employed (T_3 and T_4) to measure $(\theta_3-\theta_4)$ should be more sensitive and the measuring cylinder used to collect water should have graduation less than 0·5 c.c.

Remarks : (1) It is necessary to check whether all the thermometers read equally before steam and water circulations are started. (2) Thermometers should be read only when the rod has attained steady state which is confirmed by the steady reading of all the thermometers. (3) It is essential that the head of water in the reservoir should remain always constant to ensure a steady rate of flow of water. Water should not be allowed to flow direct from the tap. (4) The temperature difference between the inflowing and outflowing water should not exceed 10°C. (5) Such quantity of water should be collected as would fill up at least $\frac{3}{4}$th of the volume of the measuring cylinder.

Oral questions

1. What do you mean by the thermal conductivity of a metal ?

Ans. Consult any standard text book.

2. Why do you keep the head of water in the reservoir constant ?

Ans. If the head of water is not kept constant the rate of flow of water through the copper coil will vary. Consequently, the amount of heat absorbed by the outflowing water will also vary. In that case, the formula mentioned in the theory will not be applicable.

3. Does the thermal conductivity depend on the diameter or the length of the rod ?

Ans. No ; thermal conductivity depends on the material of the rod

4. The difference of temperature $(\theta_3-\theta_4)$ should not exceed 10°C. Why ?

Ans. If the difference exceeds 10°C, the error due to exposed portion of the thermometers will be high. The error will be minimum when $(\theta_3-\theta_4)$ is nearly equal to $(\theta_1-\theta_2)$. Generally the difference $(\theta_3-\theta_4)$ is kept within 10°C.

5. Why is there a copper coil at one end of the rod ?

Ans. As copper is a good conductor of heat, it absorbs and conducts heat to the water satisfactorily.

6. In which case the method is suitable ?

Ans. The method is suitable only for materials which are good conductors of heat and are available in the form of a bar or a cylinder.

7. How do you ensure good thermal contact between the thermometers T_1 and T_2 and the copper bar ?

Ans. Contact is ensured by mercury which is a good conductor of heat.

3. LIGHT

3·1. Determination of the refractive index of the material of a convex lens by using the lens and a plane mirror :

Apparatus : A convex lens, a plane mirror, a spherometer, a long pin, stand with clamp, metre scale etc.

Theory : If f be the focal length of the lens, μ the refractive index of its material and r_1 and r_2 are the radii of curvatures of the two surfaces of the lens, then we know,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

If the lens be double convex, f and r_1 are negative. Hence,

$$-\frac{1}{f}=(\mu-1)\left(-\frac{1}{r_1}-\frac{1}{r_2}\right) \quad \text{or} \quad \mu=1+\frac{r_1.r_2}{f(r_1+r_2)} \quad .. \quad \text{(i)}$$

If the convex lens be placed on a plane mirror and a pin be held vertically above it at such a distance that the pin and its image coincide, then the position of pin is the focus of the lens. If x and x_2 be the distances of the pin from the upper surface of the lens and the plane mirror respectively then,

$$f=\frac{x+x_2}{2} \quad .. \quad .. \quad \text{(ii)}$$

Again, if d be the mean length of the three arms of the equilateral triangle formed by the tripod stand of a spherometer and h be the displacement of the central screw-tip when it touches successively the given spherical surface and a plane surface, then the radius of curvature r of the spherical surface is given by,

$$r=\frac{d^2}{6h}+\frac{h}{2} \quad .. \quad .. \quad \text{(iii)}$$

From this equation r_1 and r_2 can be found out.

So, knowing f, r_1 and r_2, μ can be found out from eqn (i).

Experimental procedure : (1) Put the plane mirror M on a horizontal table and on it place the convex lens L [Fig 1].

(2) Fix a long pin P in a vertical stand in such a way that the pin is horizontal and its tip is vertically above the centre of the lens L. Make the tip of the pin white by some chalk powder.

(3) With the eye at least 25 cm above P, look for the inverted image of P. If the image is magnified, P is too close to the lens ; if it is diminished, P is too far from the lens.

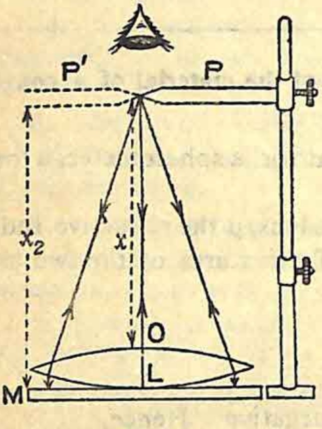


Fig. 1

(4) Move P up or down until there is no parallax between the tip of the pin P and its image P' as shown in fig 1. That the parallax has been avoided is confirmed if by slightly moving the eye to and fro, the pin and its image move together and do not get separated. As long as there is parallax, the two will not move together, but get separated when viewed in a to-and-fro manner. Having adjusted the position of the pin P, in the above-mentioned way, the tip of the pin may be said to have occupied the focus of the lens.

(5) Measure, with the help of a metre scale, the distance between the tip of the pin P and the centre O of the upper surface of the lens ($OP = x$). Also measure the distance of the tip of the pin from the surface of the plane mirror M ($MP' = x_2$). Find the focal length (f) of the lens using eqn (ii) mentioned in the theory.

(6) Repeat the setting of the no-parallax position having previously moved P downwards and upwards out of adjustment several times. Find from these readings, the mean focal length of the lens.

(7) Find the radii of curvature (r_1 and r_2) of the two surfaces of the lens by a spherometer following the procedure described in the expt no. 1·1.

Measurements : (*a*) *Focal length of the convex lens* :

No. of Obs	Distance of the tip of the pin P (cm)		$f = \frac{x + x_2}{2}$ (cm)	Mean f (cm)
	From the upper surface of the lens (x)	From the plane mirror M (x_2)		
1.	..	..	...	
2.	..	..	..	
3.	..	..	..	..
4.	..	..	..	
5.	..	..	..	

(*b*) *Radii of curvature of the lens by spherometer* :

The value of one small division of the linear scale=. . .mm
Total no. of divisions in the circular scale (N)=. . .
Screw pitch=. . .mm

$$\therefore \quad \text{Least count (L.C.)} = \frac{\text{Screw-pitch}}{N} = \ldots\text{mm} = \ldots\text{cm.}$$

(i) *Mean length of the three legs of the spherometer* :

(i) . .cm (ii) . .cm (iii) . .cm. Mean length (d)=. .cm.

Table for measurement of h

No. of obs	Surface of the lens	Circular scale reading when the screw touches the lens surface (p)	Circular scale reading when the central screw touches the glass plate			Total circular scale reading ($m=n \times N + n'$)	Value of h in circular scale ($m-p$)	Value of h in cm ($m-p$) $\times L.C.$
			No. of complete rotations (n)	Final circular scale reading (p')	Addl. no of circular scale division rotates (n')			
1.		25	3	82	(100−82)+25 =43	3×100 +43 =343	(343−25) =318	318× L.C. =. .cm
2.	Upper	..	..	..	..	..	..	..
3.		..	..	..	..	..	..	.. (h_1)
1.		..	..	..	..	..	..	..
2.	Lower	..	..	..	..	..	..	..
3.		..	..	..	..	..	..	(h_2)

Mean h_1= . . and mean h_2= . .

Calculations :

$$r_1 = \frac{d^2}{6h_1} + \frac{h_1}{2} = \ldots\text{cm.}$$

$$r_2 = \frac{d^2}{6h_2} + \frac{h_2}{2} = \ldots\text{cm.}$$

$$\mu = 1 + \frac{r_1 . r_2}{f(r_1 + r_2)} = \ldots$$

Remarks : (1) It is essential that parallax be avoided while measuring the focal length by a pin. (2) While the radii of curvature are measured by a spherometer, the central screw should always be rotated in the same direction to avoid back-lash error. (3) The up and down motion of the pin should take place along the axis of the lens. (4) From the value of the focal length, power of the lens can be derived from the formula $D=\frac{100}{f(\text{cm})}$ dioptre. (5) If the lens is equi-convex, $r_1=r_2$; in that case $\mu=1+\frac{r_1}{2f}$.

Oral questions

1. What is refractive index ? On what factors does it depend ?

Ans. Consult any standard text book. It depends on the material and colour of light.

2. Will refractive index increase if the medium is denser ?

Ans. Not always ; water is denser than turpentine (sp. gr of turpentine=0·87) but the refractive index of water is less than that of turpentine (μ of water=1·34 ; μ of turpentine=1·47).

3. Will the refractive index of a medium be greater for red light or for violet light ?

Ans. Refractive index is greater for violet light.

4. In measuring the focal length, why don't you take the height of the tip of the pin from the upper spherical surface of the lens ?

Ans. Focal length is the distance between the tip of the pin and the optical centre of the lens which is situated inside the material of the lens and equi-distant from the surfaces for an equi-convex lens. So, if the distance between the tip of the pin and the upper surface of the lens be taken as the focal length, the measured value will be slightly less than the correct focal length.

5. Can you perform the same experiment with a concave lens ?

Ans. No ; a concave lens produces a virtual image and not real image.

3.2. Determination of the refractive index of a liquid by a convex lens and a plane mirror :

Apparatus : Same as in the expt no. 3.1.

Theory : If, on a plane mirror kept on a horizontal plane, a few drops of a liquid are poured and on the liquid an equi-convex lens is placed, then the liquid sandwiched between the plane mirror and the lens will form a plano-concave liquid lens [Fig 2]. The convex lens and the liquid lens, remaining in contact, then form a combination of converging system. If F be the focal length of the combined lens, f_1 and f_2 those of the glass lens and the liquid lens respectively, then,

$$-\frac{1}{F}=-\frac{1}{f_1}+\frac{1}{f_2}$$

$$\text{or}\quad \frac{1}{f_2}=\frac{F-f_1}{f_1} \quad .. \quad \text{(i)}$$

If μ be the refractive index of the liquid, then, $\frac{1}{f_2}=(\mu-1)\frac{1}{r}$

or, $\mu=1+\frac{r}{f_2}$ .. (ii) where r is the radius of curvature of the *lower* surface of the lens which is in contact with the liquid.

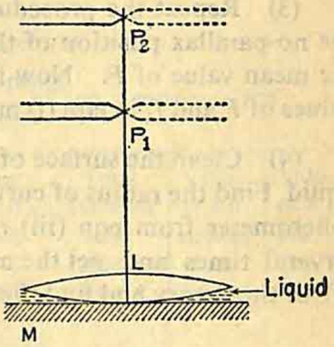


Fig. 2

Also $r=\frac{d^2}{6h}+\frac{h}{2}$ .. (iii), where d=mean length of the three arms of the equilateral triangle formed by the tripod stand of a spherometer and h=displacement of the central screw-tip when it touches consecutively the lower surface of the lens and a plane surface.

Experimental procedure : (1) Place the plane mirror M on a horizontal surface and put the convex lens L on the plane mirror. Following the procedure described in expt no. 3.1 find the position P_1 of the tip of the pin when it coincides with its image avoiding parallax and hence determine the focal length (f_1) of the convex lens.

(2) Removing the convex lens, pour a few drops of the liquid under test (say, water) on the mirror and then place the convex lens over the liquid. The liquid will form a plano-concave lens. The focal length of the combination formed by the glass convex lens and the liquid plano-concave lens in contact, is to be found out next.

For this purpose raise the pin higher to a position as P_2 [Fig 2]. With the eye at least 25 cm above P_2, look for the inverted image of P_2. Move the pin up and down until there is no parallax between the tip of the pin and its image. Let P_2 be the position of the pin when there is no parallax between its tip and its image. With the help of a metre scale, find the distance of P_2 from the upper surface of the lens (y)

and also from the plane mirror M (y_2). Find the focal length of the combined lens [$F=\frac{1}{2}(y+y_2)$].

(3) Repeat the procedure no. 2, at least, twice after disturbing the no-parallax position of the pin and from these observations, find the mean value of F. Now find the value of f_2 after substituting the values of F and f_1 in eqn (i) mentioned in the theory.

(4) Clean the surface of the lens L which was in contact with the liquid. Find the radius of curvature (r) of this surface with the help of a spherometer from eqn (iii) of the theory. Repeat the observations serveral times and get the mean value of r. Then apply the formula (ii) of the theory and find the value of μ.

Measurements : (*a*) *Focal length* (f_1) *of the convex lens* :

No. of Obs	Distance of the tip of the pin P_1 (cm)		$f_1=\frac{1}{2}(x+x_2)$ (cm)	Mean f (cm)
	From the upper surface of the lens (x)	From the plane mirror (x_2)		
1.	..	..	..	
2.	..	..	..	..
3.	..	..	..	

(*b*) *Focal length* (F) *of the combined lens* :

No of Obs	Distance of the tip P_2 of the pin (cm)		$F=\frac{1}{2}(y+y_2)$ (cm)	Mean F (cm)
	From the upper surface of the lens (y)	From the plane mirror (y_2)		
1.	..	..	..	
2.	..	..	..	..
3.	..	..	..	

(*c*) *Radius of curvature* (r) *of the spherical surface by spherometer :*

Value of the smallest division of the main scale=...mm

Screw-pitch=...mm.

Total number of circular scale divisions (N)=...

$$\text{Least count (L.C.)}=\frac{\text{screw-pitch}}{N}=\ldots\text{mm}=\ldots\text{cm.}$$

No. of obs	Surface of the lens	Circular scale reading when the screw touches the lens surface (p)	Circular scale reading when the central screw touches the glass plate			Total circular scale reading ($m=n\times N+n'$)	Value of h in circular scale ($m-p$)	Value of h in cm ($m-p$) $\times L.C.$
			No. of complete rotations (n)	Final circular scale reading (p')	Addl. no of circular scale division rotates (n')			
1.	..	..	..	..	..	..	..	..
2.	..	..	..	..	..	..	..	..
3.	..	..	..	..	..	..	..	..

(*d*) *Mean length between the three legs of the tripod :*

(i) ..cm (ii) ..cm (iii) ..cm

Mean distance (d)=..cm.

Calculations :

(*a*) $f_2=\frac{F.f_1}{F-f_1}=\ldots$cm (*b*) $r=\frac{d^2}{6h}+\frac{h}{2}=\ldots$cm.

(*c*) $\mu=1+\frac{r}{f_2}=\ldots$

Remarks : As in expt no. 3·1.

Oral questions

1. In which case is the above experiment suitable ?

Ans. The above experiment is suitable in the case when the liquid is available in a small quantity, say a few drops.

2. What are the other methods for determining the refractive index of a liquid?

Ans. There are many other methods—like travelling microscope method, total reflection method, hollow prism method etc.

3. What type of lens does liquid form in the above experiment ? Is the lens converging or diverging ?

Ans. It forms plano—concave lens. It is a diverging lens.

4. Is the combined lens converging or diverging ?

Ans. The combined lens is converging.

5. Can you find out the refractive index of the liquid by the above experiment without taking the help of a spherometer ?

Ans. Yes ; using a liquid of known refractive index ; we have for the known liquid $\mu' = 1 + \frac{r}{f_2'}$ and for the unknown liquid $\mu = 1 + \frac{r}{f_2}$; Hence $\frac{\mu' - 1}{\mu - 1} = \frac{f_2}{f_2'}$

3.3. Determination of the refractive index of a liquid by travelling microscope :

Apparatus : A travelling microscope, liquid (water or kerosene oil), a glass beaker (with a black scratch on the inside surface of the bottom), cork or lycopodium powder etc.

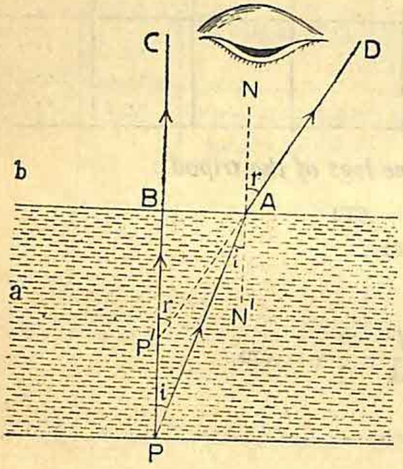


Fig. 3

Description of the microscope : See page 20.

Theory : If an object P be immersed in a liquid and if it is viewed from air vertically above, the object appears to be raised. The image of P seems to be situated at P'. The apparent displacement of the image depends on the real depth of the object in the liquid. Here, the refractive index of the liquid is given by,

$$\mu = \frac{\text{real depth of the object}}{\text{Apparent ,, ,, ,, ,,}} = \frac{BP}{BP'} \quad \text{[Fig 3]}$$

$\left[\frac{1}{\mu} = \frac{\sin i}{\sin r}\right.$: Now, $\sin i = \sin \angle PAN' = \sin \angle APB$ and $\sin r = \sin \angle DAN$

$= \sin \angle P'AN' = \sin \angle AP'B$

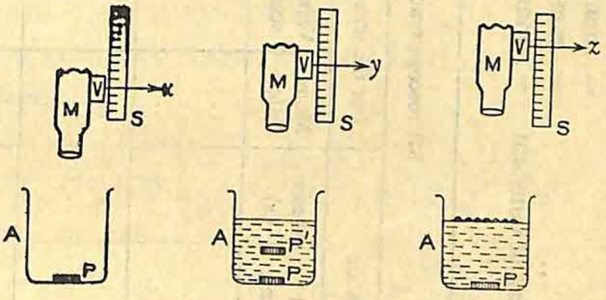
$$\therefore \frac{1}{\mu} = \frac{\sin \angle APB}{\sin \angle AP'B} = \frac{AB}{AP} \bigg/ \frac{AB}{AP'} = \frac{AP'}{AP}$$

If A and B are very close to each other (*i.e.* for vertical view) $AP'=BP'$ and $AP=BP$. $\therefore\ \mu=\frac{BP}{BP'}$]

Experimental procedure : (1) Determine the vernier constant of the vertical scale of the microscope and focus the eye-piece distinctly on the cross-wires.

(2) Place a spirit level along the line joining the two levelling screws L_1 and L_2 (Fig 3,page 20) at the ends of the linear scale. Bring the bubble of the spirit level at the centre by turning the screws L_1 and L_2 *equally in the opposite direction.* Now place the spirit level at right angles to the previous line. Turn the third levelling screw L_3 and bring the bubble of the spirit level at the centre. The instrument is now properly levelled. Now set the axis of the microscope vertical.

(3) Raise the microscope to a certain height and clamp it. Just vertically below the objective of the microscope M, put a piece of white paper on the platform of the instrument and on the paper keep a glass beaker (A) [Fig 4]. Bring down the microscope slowly until the scratch P on the bottom of the beaker is clearly visible through the microscope [Fig 4 (i)]. Fix the microscope by its fixing screw and slowly move the microscope up and down by the tangent screw till the scratch is well focussed. In the final adjustment, there should not be any parallax between the image of the scratch and the image of the cross-wires. Read the vertical main scale and its vernier. Repeat the observation twice and find the mean reading. Let the reading be x [Fig 4(i)].



(i) (ii) (iii)

Fig. 4

(4) Again raise the microscope and clamp it at a sufficient height. Without changing the position of the beaker, pour slowly some of the liquid under test so that the beaker is $\frac{2}{3}$rd full. If viewed through the

Measurements :

Value of the smallest division of the main scale (vertical)=.. mm.
..Vernier divisions=..main scale divisions
=..mm.
∴ 1 ,, ,, =..mm
Vernier constant =..mm=..cm.

No. of Obs		Microscope readings											
		For the scratch through air				For the scratch through liquid				For the upper surface of the liquid			
		Main scale	Vernier scale	Total reading	Mean (x)	Main scale	Vernier scale	Total reading	Mean (y)	Main scale	Vernier scale	Total reading	Mean (z)
First Depth	(i)	..	..	..	..	..	..	..	..	..	..	..	..
	(ii)	..	..	..	..	..	..	..	..	..	..	..	..
	(iii)	..	..	..	..	..	..	..	..	..	..	..	..
Second Depth	(i)	..	..	..	..	..	..	..	..	..	..	..	..
	(ii)	..	..	..	..	..	..	..	..	..	..	..	..
	(iii)	..	..	..	..	..	..	..	..	..	..	..	..
Third Depth	(i)	....	..	..	..	..	..	..	..	..	..	..	..
	(ii)	..	..	..	..	..	..	..	..	..	..	..	..
	(iii)	..	..	..	..	..	..	..	..	..	..	..	..

liquid, the image of the scratch P will appear to be be raised. Let P' be the image of P. Now slowly bring down the microscope and sharply focus the image P' without any parallax. After fixing the microscope by the fixing screw, read the position from the vertical scale and the vernier. Repeat the focussing process, at least, twice and get the mean reading. Let the reading be y [Fig 4(ii)].

(5) Again raise the microscope and clamp it at a sufficient height. Now the upper surface of the liquid is to be focussed. For this purpose, sprinkle some lycopodium powder or cork dust (a very thin layer) on the surface of the liquid. Slowly bring down the microscope and focus the powder sharply. Through the microscopes, the grains will appear magnified and shining. Read the position of the microscope from the main scale and the vernier. Repeat the focussing process of the powder and find the mean reading. Let the reading be z [Fig 4 (iii)].

(6) Here, the real depth of the scratch $=BP=z-x$ and the apparent depth $=z-y$. From this find the refractive index of the liquid.

(7) Repeat the whole procedure twice more by taking two other depths of liquid in the beaker. From the three values of refractive index so obtained, find the mean value.

[For measurements and Table, see page 124]

Calculations :

1. $BP=z-x=..\text{cm}$

$BP'=z-y=..\text{cm}$ $\qquad \therefore \quad \mu=\dfrac{z-x}{z-y}=..$

2. $BP=z-x=..\text{cm}$

$BP'=z-y=..\text{cm}$ $\qquad \therefore \quad \mu=\dfrac{z-x}{z-y}=..$

3. $BP=z-x=..\text{cm}$

$BP'=z-y=..\text{cm}$ $\qquad \therefore \quad \mu=\dfrac{z-x}{z-y}=..$

$$\text{Mean } \mu=\frac{..+..+..}{3}=..$$

Remarks : (1) While focussing the scratch or the lycopodium powder, it is essential that the parallax error be avoided. (2) A piece of white paper kept below the beaker will help the focussing process. (3) Lycopodium powder should be sprinkled thinly. (4) If the depth of the liquid exceeds the focal length of the microscope, focussing will not be possible.

Oral questions

1. What is the refractive index of a liquid? Does it depend on the colour of light ?

Ans. Consult any text book. Refractive index depends on the colour of light.

2. What is the necessity of a scratch at the bottom of the beaker ?

Ans. To get the real depth and the apparent depth of the liquid, the bottom of the beaker has to be focussed. As glass is transparent, the bottom as such can not be focussed. For this reason, a scratch is given at the bottom.

3. Should the depth of the liquid be greater or less ?

Ans. Depth should be as great as possible. Apparent elevation of the object is proportional to the depth of the liquid. Greater the depth, greater is the elevation and less is the error in measuring it. If the depth of the liquid be, however, greater than the focal length of the microscope, focussing will not be possible.

4. Why should the lycopodium powder be sprinkled in a thin layer ?

Ans. If the layer be thick, the level of liquid will not be focussed

5. Can you determine the refractive index of a solid material by a travelling microscope ?

Ans. If the solid material is available in a rectangular block form, its refactive index can be found out by a travelling microscope by the same process.

6. Is it proper to measure the refractive index of a volatile liquid like ether by the microscope method ?

Ans. No, if the liquid is volatile, it will evaporate quickly and the depth of the liquid will change during the experiment.

7. What is the advantage of keeping a piece of white paper below the beaker in this experiment ?

Ans. White paper will produce a white background against which the black scratch will be clearly visible. Focussing in that case will be accurate.

8. Will the refractive index vary if different depths of the liquid are taken ?

Ans. No ; refractive index depends on the material and not on the depth of the liquid.

3·4. Optical bench :

In several experiments on optics, optical bench will be necessary. It is, therefore, desirable that students should get acquainted with an optical bench at the very outset.

Fig. 5 shows an optical bench widely used in laboratories. It consists of a horizontal metallic bench about 2 metres in length. The bench is placed on four cast iron legs which are provided with levelling screws. A metre scale is engraved along the length of the bench. There are several carriages having rectangular metallic bases. The carriages carry the object, the lens and the screen. The bases can move along the bench smoothly and can be fixed at any position by screws. There is an index mark (↓) in each base, which gives the position of the object or the lens or the screen against the scale of the bench. For example, the object position on the bench is 20 cm while the screen position is 112 cm [Fig. 5]. The heights of the object, the lens and the screen are adjustable. The object stand has a small hole with two cross-fibres of silk or is covered with a piece of thin wire-gauage behind which a suitable lamp is arranged to illuminate it. The screen-stand has a piece of white paper pasted over it. The image of the silk-fibres or the wire-gauge is focussed on the paper screen by a lens placed between the two.

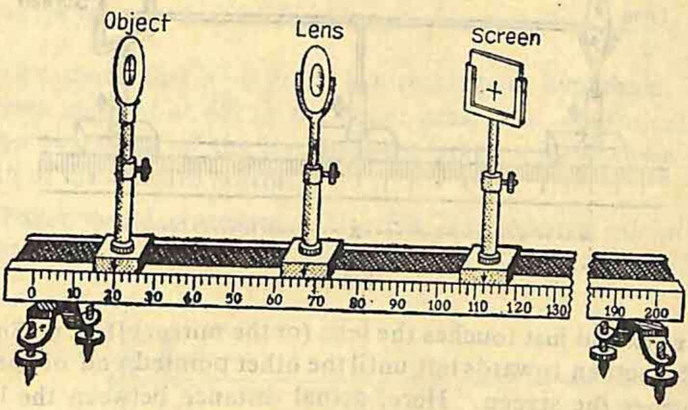


Fig. 5

3.5. Index error and its correction :

While determining the focal length of a lens or a mirror by an optical bench, we measure the object distance and the image distance with the aid of the index mark. But the index-marks on the bases of the carriages carrying the object, the lens and the screen may not coincide with the exact positions of the object and the screen or the optical centre of the lens. Here measurement of object distance or image distance with the aid of index marks are liable to certain errors, called the *index error*. The error may be corrected in the following way.

Suppose we want to find out the index error between the lens (or mirror) and the screen *i.e.* for image distance. To do this, we are to take the help of an *index rod*. It is usually a rod of brass about 20 cm long with two pointed ends. Find the exact length of the rod by means of a metre scale . Suppose the length is y ; Place the rod horizontally on a suitable carriage between the lens and the screen. Now move the lens (or the mirror) towards right so that one pointed end

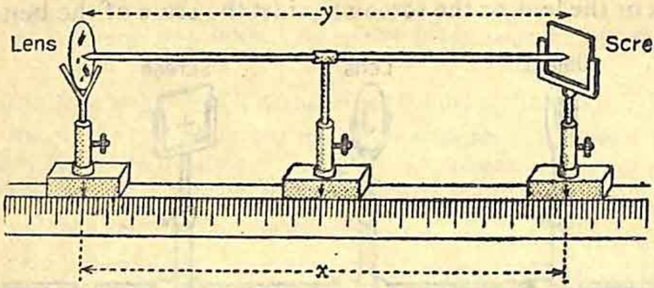


Fig. 6

of the index rod just touches the lens (or the mirror) [Fig. 6]. Similarly move the screen towards left until the other pointed end of the index rod touches the screen. Here, actual distance between the lens (or the mirror) and the screen $=y$.

Now, find, in this position, the distance between the lens (or the mirror) and the screen as given by the index marks. Let the distance be x. Here, the index error between the lens (or the mirror) and the screen $=y-x$.

If $y>x$, the difference $(y-x)$ is to be *added* to the apparent distance x shown by the index marks to get the correct value. If $y<x$, the difference $(x-y)$ is to be *subtracted* from the apparent distance x to get the correct values. If $y=x$, there is no error.

In the same way, the object distance is to be corrected for index-error. It is needless to mention that once the error is determined for the object distance or for the image distance, it remains constant for subsequent observations unless the position of the index mark on the bases is altered. Determination of error in each observation is therefore, not necessary.

3·6. Determination of the focal length of a concave mirror by graphical method :

Apparatus : A concave mirror, optical bench, three vertical carriages or stands (with screw arrangement), index rod, two brass pins etc.

Theory : If a beam of parallel rays, parallel to the axis of a concave mirror, be incident on the mirror, then after reflection, the rays are found to converge at a point on the axis of the mirror. The point is called the focus of the mirror and its distance from the pole of the mirror is called its focal length.

If u be the object distance and v, the image distance in the case of a concave mirror, then its focal length f is given by $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$; this equation shows that $u-v$ graph is a rectangular hyperbola. If a line is drawn inclined at 45° to the either axes, then co-ordinates of the intersection point of the line with the graph will be twice the focal length of the concave mirror.

Experimental procedure : (1) Fix the concave mirror M to a vertical carriage and place it at one end of the optical bench [Fig 7]

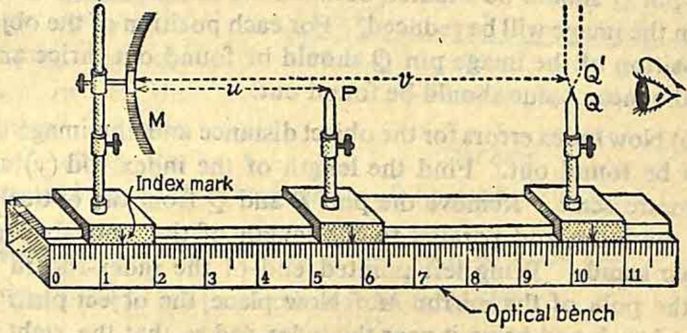


Fig. 7

Fix two pins P and Q to similar carriages and place them on the optical bench in front of the mirror. Adjust the heights of the pins such that their pointed ends lie on a straight line with the pole of the concave mirror M and that the straight line coincides with the axis of the mirror, which is again parallel to the length of the optical bench.

(2) Place the pin P, the object-pin (because it will act as an object) from the concave mirror at such a distance away that looking through the mirror, as shown in the figure, an inverted and magnified image Q' is visible (the image should not be too much magnified). Now bring the pin Q, the image pin (because it will act as an image) just below the magnified image so that their tips Q and Q coincide without any parallax between them. If the eye is moved right and left a little, the two tips will also move likewise together, if there is no parallax. If there is parallax, the two tips will be separated from each other. Having thus adjusted the position of the pin Q, read its position on the scale with the aid of its index-mark.

(3) Without disturbing the positions of the concave mirror and the object pin P, repeat the above observation, at least, thrice and find the mean value of the position of the pin Q. With the aid of respective index-marks, note the positions of the mirror M and the pin P.

(4) The difference of the readings corresponding to the mirror and the object pin P gives the apparent object distance (u') and the difference of the readings corresponding to the mirror and the image pin Q gives the apparent image distance (v'). These distances are called apparent because they need correction for index-error.

(5) Observations no. (2), (3) and (4) are to be repeated for six different positions of the object-pin P. Of these, in three positions, the pin P should be situated between the mirror M and the image pin Q when the image will be magnified and in other three positions, the image-pin Q should be situated between the mirror and the object-pin P when the image will be reduced. For each position of the object pin, the position of the image pin Q should be found out thrice and from these the mean value should be found out.

(6) Now index errors for the object distance and the image distance are to be found out. Find the length of the index rod (y) supplied by a metre scale. Remove the pins P and Q from the optical bench. Hold the index rod parallel to the length of the optical bench by suitable stand. Bring left pointed end of the index-rod in contact with the pole of the mirror M. Now place the object pin P on the optical bench and bring it near the index-rod so that the right pointed end of the rod comes in contact with the tip of the pin P. In this condition, find the distance between the mirror and the pin P with the aid of their index-marks and the bench scale (x_1). The index-error in the case of object distance is $e_1 = y - x_1$ [If $y > x_1$, their difference is to be *added* to the apparent object distance (u') ; if $y < x_1$ their difference is to be *subtracted* from the apparent object distance (u')].

Now put the image pin Q in place of the object pin. See that its tip touches the right pointed end of the index-rod. Measure the distance between the mirror and the image pin Q with the help of their index marks and the bench scale (x_2). The index error in the case of image distance is $e_2 = y - x_2$ [If $y > x_2$, their difference should be *added* to the apparent image distance (v') and if $y < x_2$, their difference is to be *subtracted* from the apparent image distance (v')].

(7) Adding or subtracting the index-error as the case may be, the apparent object distance and the apparent image distance in each observation are to be corrected and the correct values of object distance (u) and the image distance (v) are to be found out.

(8) Plotting the values of u along X-axis and those of v along Y-axis, a graph is to be drawn. Care should be taken so that *the scales along the two axes are equal and the origin coincides with the same value of u and v* [not necessarily (0,0) value]. The graph will be a rectangular hyperbola [Fig 7(*a*)]. Draw a line through the origin making an angle of 45° with X-axis. Find the co-ordinates of P, the intersection point. Half of either co-ordinates of P gives the focal length of the concave mirror.

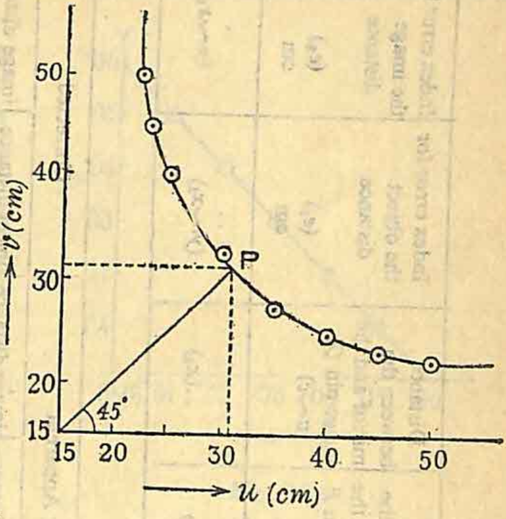


Fig. 7 (a)

Measurements : For tables (*a*) and (*b*), See Page 132

(*c*) *Table for graph* [data taken from table (*b*)] :

Corrected object distance (u)→	..	..	..	..	..	..	..
Corrected image distance (v)→	..	..	..	..	..	..	..

From the graph, the focal length of the concave mirror $f=\frac{31}{2}=$ 15·5 cm (as an illustration) (*a*) *Index error* :

Length of the Index rod (cm)	Position of the index mark			Distance between the mirror and the object pin P $(a \sim b)$ cm	Distance between the mirror and the image pin Q. $(a \sim c)$	Index error for the object distance (e_1) cm	Index error for the image distance (e_2) cm
	of the mirror M (a)	of the pin P (b)	of the pin Q (c)				
..(y)	..	..	..	..(x_1)	..(x_2)	.. $(y \sim x_1)$	.. $(y \sim x_2)$

(*b*) *Object distance* (u) *and image distance* (v) :

No. of. Obs.	Position of the index mark			Mean reading for the position of Q (c)	Apparent		Corrected	
	of the mirror M (a)	of the pin P (b)	of the pin Q		object distance $(u' = a \sim b)$	image distance $(v' = a \sim c)$	object distance $u = u' \pm e_1$	image distance $v = v' \pm e_2$
1.	..	..	.. .. .. }	..	..	..	..	..
2.	..	..	.. .. .. }	..	..	..	..	..
3.	..	..						
etc	etc	etc		etc				
6.	..	..	..	..	..	..	..	..

Alternative graphs : Instead of drawing graph between $u-v$ as described earlier, two alternative methods may be adopted. From each of these two methods the focal length of the concave mirror may be found out.

First method : Instead of plotting the values of u and v along X-axis and Y-axis respectively, their reciprocals *i.e.* $\frac{1}{u}$ and $\frac{1}{v}$ are to be plotted. The scales along the two axes must be equal and the origin should coincide with (0,0) values. The graph will be a straight line [Fig 7(*b*)].

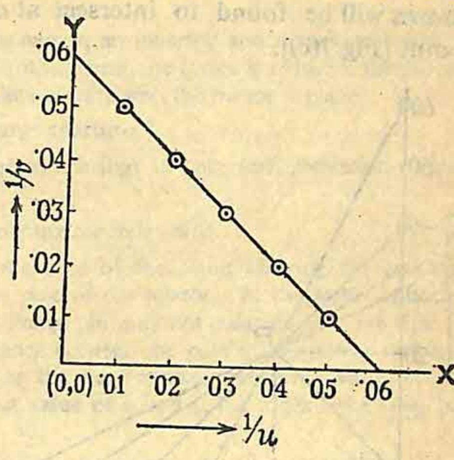


Fig. 7 (b)

Table for drawing the graph : [data obtained from the table (*b*)]

$\frac{1}{u}$ (cm^{-1})→	..	..	..	..	..	..
$\frac{1}{v}$ (cm^{-1})→	..	..	..	..	..	..

The intercept made by the straight line on X-axis or Y-axis [0.06 in Fig 7(*b*)] is equal to the reciprocal of the focal length of the mirror. From the graph, we get $\frac{1}{f}=0{\cdot}06$ cm or $f=\frac{1}{0{\cdot}06}=16.6$ cm (as an illustration).

Second method : In this method, scales along the two axes are taken equal and zero of each scale should start from the origin. Then, values of u taken from the table (*c*) are all plotted along the X-axis. Afterwards, the values of v taken from the same table are all plotted along the Y-axis. Now, first pair of points (60, 30, say) are joined by a straight line [Fig. 7(*c*)]. Similarly other pair of points are joined by drawing straight lines one after another. All the straight lines so drawn will be found to intersect at one point. *P* is the intersection point [Fig 7(*c*)].

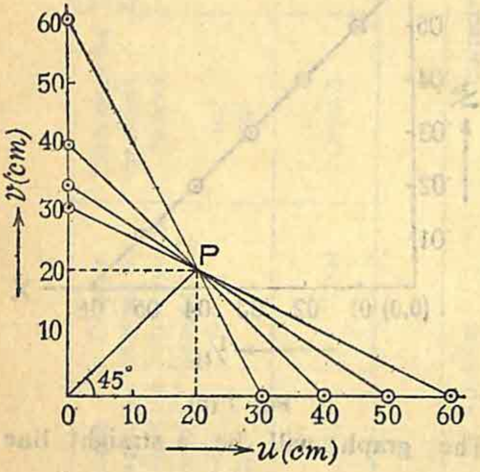


Fig. 7 (c)

From the graph, the co-ordinates of *P* are (20, 20). So, the focal length of the mirror =20 cm (as an illustration).

[**N.B.** Due to experimental error, all the straight lines may not pass through one point. In such cases, a line is to be drawn making an angle of 45° with the X-axis. The point lying on this line may be taken as the common intersection point and the co-ordinates of this point will give the focal length of the mirror.]

Remarks : (1) In adjusting the position of the image-pin, parallax error must be avoided. (2) Correct value of *f* is obtained when $u=v$. Hence the position of the object point should be so adjusted that the image is neither too magnified nor too reduced. If *r* be the radius of curvature of the mirror, the object distance should be either slightly greater than or slightly less than *r*. (3) If the aperture of the mirror is large, the image suffers from spherical aberration. It is very difficult to avoid parallax in such cases. (4) To avoid parallax, eye should be placed more than 20 cm away from the image pin. (5) In drawing a graph, the two axes should have same scale.

Oral questions

1. Define the following terms in relation to a concave mirror :— (i) focus (ii) radius of curvature (iii) aperture (iv) focal length.

Ans. Consult any text book on optics.

2. What type of image does a concave mirror form ?

Ans. Concave mirror forms real, virtual, magnified and reduced *i.e.* all kinds of images.

3. Where should you place the object pin so as to produce an image equal in size to the object ?

Ans. The object pin should be placed at the centre of curvature of the mirror.

4. How can you distinguish a concave mirror from a convex mirror or a plane mirror ?

Ans. If placing a finger near the mirror, an inverted and magnified image is seen, the mirror is concave. If, on the other hand, the image is reduced, the mirror is convex and if the image is equal to the object in size, the mirror is plane.

5. Should you take a mirror of large aparture ?

Ans. No ; a mirror of large aperture brings in spherical aberration in the image.

6. What is the necessity of determining the index-error ?

Ans. The index-mark given at the base of the stand carrying the concave mirror may not be concident with the pole of the mirror. In the same way, the index-marks of the object pin and the image pin may not coincide with the tips of these pins. So, in measuring the distance between the pole of the mirror and the object or the image if we depend only on the index-marks, some error will creep in. This is index-error. To get the correct value of u and v, the index-error need be measured.

7. Of all the possible values of u and v, which one will give correct value of f ?

Ans. Correct value of f will be obtained when $u=v$.

8. Can you find out the focal length of a concave mirror by the above experiment without drawing or graph ?

Ans. Yes ; By applying the formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$; the focal length for each observation [See table below] is to be found out and then the mean focal length is to be determined. The table should be drawn in the following way :

Focal length of the mirror [data obtained from table (*b*)]

No. of Obs.	Correct u	Correct v	$f=\frac{uv}{u+v}$	Mean f (cm)
1.	..	..	..	
2.	..	..	..	..
etc.	etc.	etc.	etc.	
6.	..	..	..	

[**N.B.** In the above case, don't take the mean value of u and the mean value of v and then find the value of f from the formula $f=\frac{uv}{u+v}$. This is erroneous. Each observation is independent and complete. Find f from each observation and then get the mean value of f.]

3·7. Determination of the focal length of a concave mirror without the help of an optical bench :

Apparatus : A concave mirror, three vertical stands with screw arrangement, metre scale, two pointed pins etc.

Theory : Same as in expt 3·6.

Experimental procedure : (1) Fix the concave mirror in a vertical stand and put it at the end of a horizontal table [Fig 8]. Fix two pins P and Q in similar stands and put them in a line in front of the mirror on the table. Adjust the heights of the pins and the mirror such that the tips of the pins P and Q, are in the same straight line with the pole M of the concave mirror and the straight line is parallel to the horizontal plane of the table.

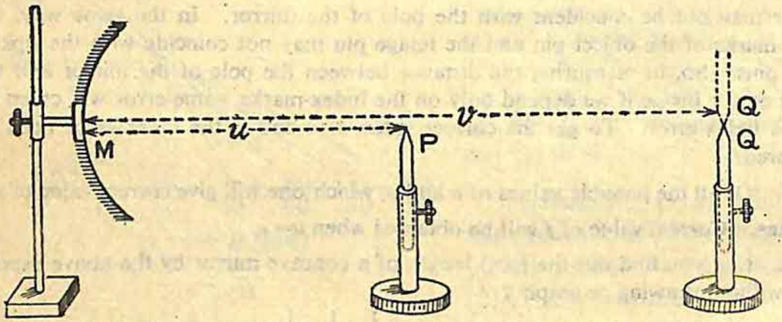


Fig. 8

(2) Proceed as described in operation no. 2 in the previous experiment. Having adjusted the position of the image-pin Q, find the distance between the pole M of the mirror and the tip Q of the pin with the help of a metre-scale held horizontally. This gives the image distance (v).

(3) Proceed as described in operation no. 3 in the previous experiment. Measure the distance between the pole M of the mirror and the tip of the object pin P by the metre scale. This gives the object distance (u).

(4) Repeat the above operations for six different positions of the object pin P. For three positions, the pin P should be placed between the mirror and the pin Q (the image is magnified) while for the other three positions, the pin Q should be placed between the mirror and the object pin P (the image is reduced).

Measurements : (a) *object distance* (u) *and image distance* (v) :

No. of Obs.	Distance from the pole M of the mirror (cm)		Mean v
	of the object pin P (u)	of the image pin Q (v)	(cm)
1.	..	.. ..	..
2.	..	.. ..	..
3.	..	.. ..	..
etc.	etc	etc	etc
6.	..	.. ..	..

(b) *Table for drawing graph* [data taken from table (*a*)].

Quantity	1	2	3	4	5	6
Object distance (u) in cm→	..	..	..	..	..	..
Image distance (v) in cm→	..	..	..	..	..	..

Methods of drawing graph : Same as before.

Result : Focal length of the concave mirror $f = ..$ cm.

Remarks : (1) In this method, index error need not be found out because distances have not been measured with reference to the index mark. (2) While adjusting the position of the image pin for no parallax, M, P and Q are very likely to move away from the axis of the mirror. Hence, the result obtained from this method is less accurate than that obtained from optical bench method.

3.8. Determination of the focal length of a concave lens by combination method :

Apparatus : Optical bench, a convex lens, a concave lens, stands, a paper screen, illuminated wire-gauge, index rod etc.

Theory : A converging lens of shorter focal length (f_1) and a diverging lens of longer focal length (f_2) in contact with each other form a combination which is converging in nature. If F be the focal length of the combined lens, then,

$$-\frac{1}{F}=-\frac{1}{f_1}+\frac{1}{f_2} \text{ or } \frac{1}{f_2}=\frac{1}{f_1}-\frac{1}{F} \quad \therefore\ f_2=\frac{F.\ f_1}{F-f_1} \quad \ldots \ldots \quad \text{(i)}$$

Again, for a fixed object and a fixed screen, there are, in general, two positions of a convex lens between them, for each of which a sharp image of the object is cast on the screen by the lens, provided the distance between the object and the screen is more than four times the focal length of the lens. If D be the distance between the fixed object and the fixed screen and x that between the two positions of the lens, it may be proved that,

$$f=\frac{D^2-x^2}{4D} \quad \ldots \quad \ldots \quad \text{(ii)}$$

where f=focal length of the lens.

Experimental procedure : (1) Fix a wire-gauge P in a stand and place it near one end of an optical bench. Place an electric bulb behind the wire-gauge and wrap the bulb with a piece of white tissue paper. The illuminated wire-gauge will serve the purpose of an object. Attach a piece of white paper Q in another stand and place it on the optical bench at sufficient distance apart from the wire-gauge.

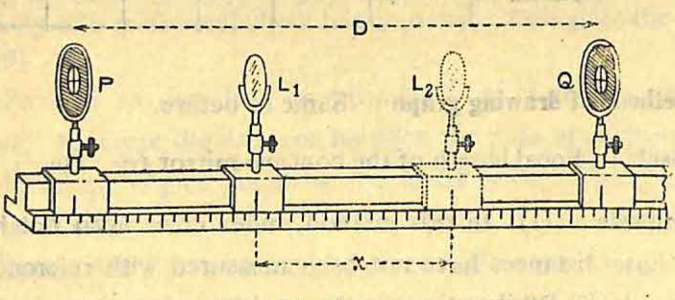


Fig. 9

In between P and Q place the convex lens L_1 held in suitable lens holder.*

(2) The heights of the wire-gauge, the paper screen and the lens are so adjusted that their centres lie on a straight line parallel to the length of the optical bench (Fig 9).

(3) Place the paper screen Q at such a distance from the lens that a sharp image of the wire-gauge P is formed on the paper screen. In this condition the distance between the object (*i.e.*, the wire-gauge) and the image (*i.e.*, the paper screen) may be taken, in the least, to be equal to four times the focal length of the lens. Now push the screen a little away to ensure that the distance between the object and the screen is more than four times the focal length of the convex lens. Note the positions of the object P and the screen Q with the aid of the index-marks against the scale of the bench. The difference of these two readings gives the *apparent* distance (D') between the object and the screen.

(4) Keeping the object (P) and the screen (Q) fixed in their positions, move the lens backward or forward along the bench until a sharp and magnified image of the cross-wires is formed on the paper screen. Suppose L_1 is the required position of the lens. Note its position with the help of the index-mark on its base and the scale of the bench. Repeat this adjustment twice and get the mean reading. This gives the first position of the lens.

(5) Now move the lens towards the screen. Adjust its position by shifting it to and fro along the bench so that a sharp image of the cross-wire is again formed on the screen. This image will be diminished. From the scale of the bench, note the position of the lens L_2. Repeat the adjustment twice and find the mean position. This gives the second position of the lens.

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*There is a convenient arrangement [Fig. 9(*a*)] for holding the convex lens alone as well as the combination of a convex and a concave lens in contact. It consists of a circular frame at the centre of a circular metallic disc. The convex lens is fitted in the circular frame while the concave lens is fitted in another similar circular frame which is hinged by the side of the convex lens. Like a window, the concave lens can be turned aside, leaving only the convex lens open to light; again it can be put in contact with the convex lens whenever required.

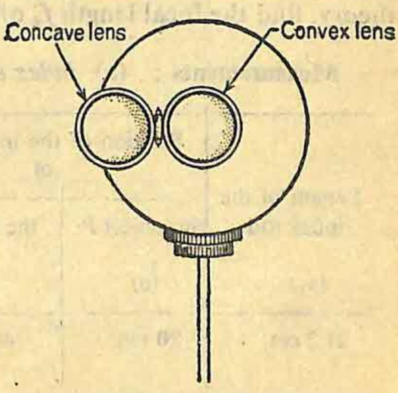

Fig. 9(*a*).

(6) Find the difference of the two readings corresponding to the two positions of the lens. It is x.

(7) Repeat the operations no. (3), (4) and (5) at least twice by changing the positions of the screen Q and the object P by 3 or 4 cm.

(8) Now index error is to be determined. Remove the lens from the optical bench. Find the length of the index-rod by a metre scale. Let it be y_1. Hold the index rod in a horizontal position on a suitable stand on the optical bench. Touch the left pointed end of the index rod with the wire gange. Now shift the paper screen Q towards the index rod so that the paper screen can just touch the right pointed end of the index rod. In this position, find the distance between the object and the screen from the index marks and the bench scale. Let it be y_2; then the index error $e=y_1-y_2$ [If $y_1>y_2$, their difference is to be added to D' to get the corrcet value ; if $y_1<y_2$ their difference is to be subtracted from D' to get the correct value.]

(9) Taking the correct value of D (*i.e.* $D=D'\pm e$) for each observation and the value of x, find the focal length f_1 of the convex lens with the help of eqn (ii) of the theory.

(10) Removing the index rod from the optical bench, replace the convex lens between the object and the screen. Now put the concave lens in contact with the convex lens. They will form a converging combination. Following the displacement method described earlier, find the focal length F of the combined lens.

(11) Substituting the values of f_1 and F in equation (i) of the theory, find the focal length f_2 of the concave lens.

Measurements : (*a*) *Index error between the object and the screen*

Length of the index rod (y_1)	Position of the index mark of		Dist. between P and Q according to index mark ($y_2=a\sim b$)	Index error $e=y_1-y_2$
	the object P (a)	the screen Q (b)		
21·2 cm.	20 cm.	40·0 cm.	20·0 cm.	1·2 cm. (+)

(*b*) *Focal length of the convex lens* :

No. of. Obs.	Position of the object P (a) cm.	Position of the screen Q (b) cm.	Apparent distance $(D'=a\sim b)$ cm.	Correct distance $(D=D'\pm e)$ cm.	Readings for convex lens				Displacement $(x=L_1\sim L_2)$ cm.	$f_1=\frac{D^2-x^2}{4D}$ cm.
					First position	Mean (L_1) cm.	Second position	Mean (L_2) cm.		
1.	20·1	83·9	63·8	63·8+1·2 =65	50·1 50·2 50·2	50·2	68·2 68·3 68·4	68·3	18·1	14·89
2.	..	..	..	..	.. .. ..	..	.. .. ..	..	..	..
3.	..	..	..	..	.. .. ..	..	.. .. ..	..	..	..

∴ Mean value of $f_1=$ .. cm.

(c) *Focal length of the combined lens :*

No. of. Obs.	Position of object P (a) cm	Position of screen Q (b) cm.	Apparent distance $(D'=a\sim b)$ cm.	Correct distance $(D_1=D'\pm e)$ cm.	Readings for combined lens				Displacement $x_1=L'_1\sim L'_2$ (cm.)	$F=\frac{D_1^2-x_1^2}{4D_1}$ cm.
					First position	Mean (L'_1)	2nd. position	Mean (L'_2)		
1.	20·1	143·9	123·8	123·8+ 1·2=125	55·2 55·3 55·4	55·3	67·3 67·4 67·5	67·4	12·1	30·95
2.	..	..	..	..	.. .. ..	..	.. .. ..	..	..	..
3.	..	..	..	..	.. .. ..	..	.. .. ..	..	..	..

$\therefore$ Mean value of $F=$ .. cm.

Calculations : $f_2 = \frac{F.f_1}{F-f_1} = ..$ cm.

Remarks : (1) To get real images in two positions of the lens, the distance between the fixed object and the fixed screen should be equal to or greater than four times the focal length of the lens. (2) To minimise the error in the determination of the focal length, the displacement of the lens (x) should be as small as possible. Hence, the magnitude of D is to be so adjusted as to make x as small as possible. (3) As x changes appreciably due to a small change in D, D should be changed by instalments of 2 cm or 3 cm. (4) The index error in lens displacement is automatically removed ; hence it need not be determined separately. (5) From the focal length of the concave lens, its power can be calculated from the formula $P = \frac{1}{f/100}$ dioptre.

Oral questions

1. What is focal length of a lens ? Is the focal length of a concave lens positive or negative ?

Ans. Consult any text book of optics. Focal length of a concave lens is reckoned as positive.

2. Why don't we adopt the usual $u-v$ method for determining the focal length of a concave lens ?

Ans. A concave lens produces a virtual image which can not be cast on a screen. So, $u-v$ method is not applicable in the case of a concave lens.

3. What should be the focal length of the convex lens in this experiment ?

Ans. The focal length of the convex lens should be smaller than that of the concave lens. In that case, the combined lens will behave like a converging system and will form a real image on a screen.

4. What is the harm if D is too large or too small ?

Ans. If D is too large, x will be large and the error in determining the value of f will increase. If D is too small, two positions of convex lens will not be available and the method can not be applied.

5. What should be the best value of D ?

Ans. The best value of D is a little more than four times the focal length of the convex lens.

6. Why don't you require to find the index error in reading the positions of the convex lens ?

Ans. Index error remains the same in the two positions of the lens and in finding the difference between the readings of the two positions, the index error becomes automatically eliminated. Hence index error is not required in reading the positions of the convex lens.

7. What are the sizes of the image in the two positions of the lens ?

Ans. When the convex lens is nearer to the object at L_1, the image is magnified. When the convex lens is nearer to the screen at L_2, the image is diminished.

8. Can you determine the size of the object from the sizes of the above two images ?

Ans. Yes ; If the sizes of the images be I_1 and I_2 and O. the size of the object, then $O^2=I_1.I_2$ or, $O=\sqrt{I_1.I_2}$.

9. Can you find out the power of the concave lens from this experiment ? What is the unit of power ?

Ans. Yes ; Knowing f_1 we can find its power from the formula $P=\frac{1}{f_1/100}$ dioptre. Unit of power is dioptre. It is the power of a lens whose focal length is 1 metre or 100 cm.

10. What is the minimum distance between an object and its real image formed by a convex lens ?

Ans. The minimum distance is four times the focal length of the lens.

3.9. Determination of the power of a convex lens by displacement method :

Apparatus : An. optical bench, a convex lens, stands, paper screen, illuminated wire-gauge, index rod etc.

Theory : For a fixed object and a fixed screen, there are, in general, two positions of a convex lens in between them, for each of which a sharp image of the object is formed by the lens on the screen. If D be the distance between the object and the screen and x the displacement of the lens, then $f=\frac{D^2-x^2}{4D}$

Now, if P dioptre be the power of the lens, then $P=\frac{100}{f}$ where f is expressed in centimetre.

Experimental procedure : (1) Fix a wire-gauge in a stand and put it near one end of the optical bench (Fig 9). Place an electric bulb behind the wire-gauge and wrap it up with white tissue paper. The illuminated wire-gauge will serve the purpose of an object. Fit the paper screen in another stand and place it on the optical bench sufficiently away from the wire-gauge. Fix the convex lens L in a suitable lens holder and place the lens in between the wire-gauge and the paper screen.

(2) Adjust the heights of the wire-gauge, the lens and the paper-screen such that their centres lie on a straight line parallel to the length of the optical bench.

(3) Place the paper screen at such a distance from the lens that a sharp image of the wire-gauge is formed on the screen. In this condition, the distance between the wire-gauge (the object) and the paper screen (the image) is, at least, four times the focal length of the lens. Now shift the paper screen a little towards left so that the above distance is more than $4f$, where f is the focal length of the lens. Note the positions of the object P and the screen Q with the help of their index marks and the bench scale. The difference of these two readings gives the apparent distance (D') between the object and the screen. This distance needs correction for index error.

(4) Without changing the positions of the object and the screen, change the position of the lens (keeping it nearer to the object) until a sharp and magnified image of the cross-wire is formed on the screen. Suppose L_1 is the position of the lens. Read this position of the lens with the help of its index mark. Repeat the observation twice and find the mean value. This gives the first position of the lens.

(5) Now take the lens nearer to the screen. Adjust its position until another sharp image of the cross-wire is formed on the screen. This image will be diminished. Let the position be L_2. Note the position from the bench scale with the help of the index mark. Repeat the observation twice and find the mean value. This gives the second position of the lens.

(6) Find the difference between the readings corresponding to the two positions of the lens. This gives the value of x.

(7) Repeat the operations no. (3), (4), (5) and (6) at least twice by altering the distance between the object P and the screen Q by 3 or 4 cm.

(8) Now index error of the distance between the object and the screen is to be determined. Find the length of the index rod (y_1) by a metre scale. Hold the index rod in a horizontal position by a suitable stand. Place the stand on the optical bench and bring the left pointed end of the index rod in contact with the cross-wire. Now shift the screen Q along the optical bench so that it just touches the right pointed end of the index rod. Find the distance between the object and the screen with the help of their index marks. Let this distance be y_2. In this case, the index error $e=y_1-y_2$.

[If $y_1>y_2$, their difference is to be *added* with D' to get the correct value of D. If, however, $y_1<y_2$, the difference is to be *subtracted* from D'.]

(9) Taking the correct value of D ($D=D'\pm e$) in each observation and the value of x, calculate the value of f with the help of the equation given in the theory. Find the mean value of f and from it calculate the power of the lens.

Measurements : (*a*) *Focal length of the lens* :

No. of Obs.	Position of he object P (a) cm.	Position of the screen Q (b) cm.	Apparent distance $(D'=a\sim b)$ cm.	Correct distance $(D=D'\pm e)$ cm.	Readings for lens				Displacement $(x=L_1\sim L_2)$ cm.	$f=\frac{D^2-x^2}{4D}$ cm.
					1st position	Mean (L_1) cm.	2nd. position	Mean (L_2) cm.		
1.	..	..	..	..	$\left.\begin{matrix}..\\..\\..\end{matrix}\right\}$	..	$\left.\begin{matrix}..\\..\\..\end{matrix}\right\}$	..	..	..
2.	..	..	..	..	$\left.\begin{matrix}..\\..\\..\end{matrix}\right\}$	..	$\left.\begin{matrix}..\\..\\..\end{matrix}\right\}$	..	..	..
3.	..	..	..	..	$\left.\begin{matrix}..\\..\\..\end{matrix}\right\}$	..	$\left.\begin{matrix}..\\..\\..\end{matrix}\right\}$	..	.	..

Mean focal length $f=$ .. cm.

(*b*) *Index error between the object and the screen* :

Length of the index rod (y_1)	Position of the index mark		Distance between P and Q according to index mark ($y_2 = a \sim b$)	Index error ($e = y_1 - y_2$)
	of the object P (a)	of the screen Q (b)		
.. cm.	.. cm.	.. cm.	.. cm.	.. cm.

Calculations : Power $P = \frac{100}{f} = + ..$dioptre

Remarks : (1) There is no need of determining the index error for the displacement of the lens because it is automatically eliminated. (2) The value of D should not be very large. (3) Displacement method is more reliable than $u-v$ method in the matter of determining the focal length of a lens. (4) The power of a convex lens is regarded as positive although its focal length is negative.

Oral questions

1. What do you mean by focal length and power ? What is the unit of power ?

Ans. Consult any text book. The unit of power is dioptre. The power of a lens whose focal length is 100 cm or 1 metre is taken as 1 dioptre.

2. What is the relation between focal length and power of a lens ?

Ans. Power increases with the decrease of focal length and vice versa.

3. Does focal length of a lens depend on the colour of light ?

Ans. Yes ; With the increase of wavelength (*i.e.*, towards red), the focal length increases and with the decrease of wave-length (*i.e.*, towards violet), the focal length decreases.

4. In this experiment, you measured the index error for D but not for x. Why is this difference ?

Ans. To determine x, we take the difference of the readings corresponding to the two positions of the lens. If there be any index error, it will be automatically eliminated while the above difference is taken. Suppose, the readings of the two positions of the lens are x_1 and x_2 and the index error in each position is $+e$. So, the correct readings in the two positions are x_1+e and x_2+e. Hence $x=(x_1+e)-(x_2+e)=x_1-x_2$.

5. The result obtained in this method is more reliable than that obtained from $u-v$ method. Why ?

Ans. In $u-v$ method index correction is required for both u and v while in the displacement method index correction is required only once for D. This is why the result obtained from displacement method is more reliable.

6. How can you get the focal length of a lens in displacement method by drawing graph ?

Ans. $-f=\frac{D-x^2}{4D}$ or, $\frac{x^2}{D}=D+4f$. [The focal length of a convex lens is negative]. It resembles the equation $y=mx+c$. This means $\frac{x^2}{D}-D$ graph is a straight line. The intercept made by the straight line on the X-axis is $-4f$ because when $\frac{x^2}{D}=0$; $D=-4f$.

[The questions on expt 3.8 are also applicable here]

3.10. Determination of the focal length of a concave lens by auxiliary lens method :

Apparatus : A concave lens, a convex lens (whose power is less than the power of the concave lens), an optical bench, an electric lamp, an index rod, a metre scale etc.

Theory : A convex lens L produces a real image P' on a screen of an object P. Now introduce the concave lens L' in between the screen S and the convex lens [Fig 10]. Then the rays converging to P' become less convergent. Let the screen be moved further away to S' to obtain the sharp image Q again. Then for the concave lens, the first image P' behaves as a virtual object and Q is its real image.

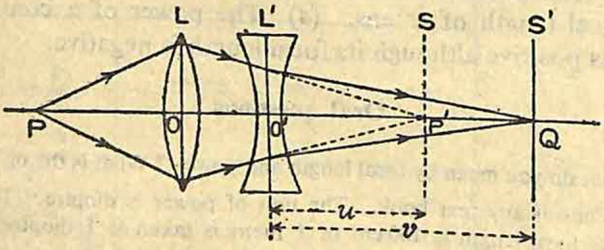

Fig. 10.

In the present case, both $u\,(=O'P')$ and $v\ (=O'Q)$ are negative. Hence from the equation, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$, we get,

$$\frac{1}{f}=-\frac{1}{v}-\left(-\frac{1}{u}\right)=\frac{1}{u}-\frac{1}{v} \text{ or, } f=\frac{u.v}{v-u}. \quad ..\text{(i)}$$

Hence, power of the concave lens $P=-\frac{100}{f\,(\text{in cm})}$ dioptre. ..(ii)

[The power of a concave lens is regarded as negative].

Experimental procedure : (1) Place a wire-gauge P backed by an electric lamp [not shown in the fig. 10(*a*)] on one end of the optical bench. The illuminated wire-gauge will act as an object.

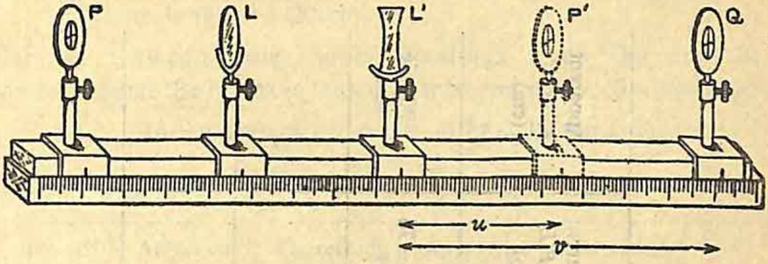

Fig. 10(*a*).

(2) Mount the auxiliary convex lens *L* on a suitable stand and place it on the optical bench at a distance from *P* greater than the focal length of the convex lens. The convex lens will form a real image of the wire-gauge. Receive the image on a white paper-screen *P'* placed at a suitable distance from the convex lens. Adjust the position of the screen *P'* until the image is sharp. The position of the screen is noted against the scale of the optical bench. Repeat the operation twice by slightly displacing the screen. Find the mean value of this position of the paper screen (*P'*).

(3) Now, keeping the position of the convex lens *L* unchanged, shift the paper screen 5 or 6 cm. away to the position *Q*. Note this position of the screen. Place the concave lens *L'* in between the convex lens and the screen and adjust its position until a sharp image of the cross wire is formed on the screen. Displace the concave lens a little and again find its position when a sharp image of the object is formed on the screen. Find the mean value of this position (*L'*). In this case, apparent $u_1 = L'P'$ and apparent $v_1 = L'Q$.

(4) The position of the screen is shifted twice by steps of 5 cm. or 6 cm. and for each position of the screen, the position of the concave lens is adjusted independently thrice to obtain sharp image of the cross-wire on the screen. The mean value of each position of the concave lens is found out and from these apparent values of u and v are found out in each case.

(5) Then the index error (e) between the concave lens and the screen is determined with the help of an index rod in the usual way and the corrected object and image distances *i.e.*, $u = u_1 \pm e$ and $v = v_1 \pm e$ are calculated.

(6) Substituting the values of corrected u and v in the equations (i) and (ii) mentioned in the theory, the focal length and the power of the concave lens are calculated.

Measurements : (*a*) *Measurement of u_1 and v_1* .
Position of the object P (*i.e.*, the wire-gauge)=4 cm.
,, ,, ,, convex lens (L)=24 cm.

No. of obs.	Position of image by convex lens (P') cm.	Mean value (P') cm.	Positions of (cm)		Mean value in cm. (L')	Apparent $u_1=P'L'$ (cm).	Apparent $v_1=QL'$ (cm).
			Image by concave lens (Q)	Concave lens (L')			
1.	84·1 84·0 84·0	84	90	75·9 76 76·1	76	84−76 =8	90−76 =14
2.	,,	,,	96	.. .. ..	..	..	..
3.	,,	,,	102	.. .. ..	..	..	..

(*b*) *Measurement of index error* :

Length of index rod=(i) 20 cm. (ii) 20·1 cm. (iii) 20 cm.
Mean length $y=20$ cm.

Difference between index mark readings when the ends of the index rod touch the concave lens and the screen$=(95{\cdot}6-76)=19{\cdot}6$ cm.

$\therefore$ Index correction $e=20-19{\cdot}6=0{\cdot}4$ cm (+).

(*c*) *Measurement of f* :

No. of obs.	Apparent u_1	Corrected $u=u_1+e$	Apparent v_1	Corrected $v=v_1+e$	f (cm).
1.	8 cm.	8+0·4 =8·4 cm.	14 cm.	14+0.4 =14.4 cm.	20·16
2.	..	..	..	..	..
3.	..	..	..	..	..

$\therefore$ Mean value of $f=..$ cm.
Power $P=..$ dioptre.

Calculations :

$$\text{(i)}\quad f=\frac{u.v}{v-u}=\frac{8{\cdot}4\times14{\cdot}4}{14{\cdot}4-8{\cdot}4}=20{\cdot}16\text{ cm.}$$

$$\text{(ii)}\quad f=\frac{u.v}{v-u}=\ ..\text{ cm.}$$

$$\text{(iii)}\quad f=\frac{u.v}{v-u}=\ ..\text{cm.}$$

$$\text{Power}\ =-\frac{100}{f(\text{mean})}=-\frac{100}{..}=..\text{ dioptre}$$

Remarks : (1) The focal length of the auxiliary convex lens should be less than that of the concave lens but the difference should not be large. (2) Index error is same for the object distance and the image distance.

Oral questions

1. What type of image does a concave lens produce ?

Ans. For real object, the concave lens always produces a virtual image. But if the object is virtual and is situated within the focal length of the lens, the image is real.

2. What type of object and image you are dealing with in this experiment ?

Ans. Object is virtual and the image is real.

3. Is the concave lens always a diverging lens ?

Ans. No ; if the refractive index of the surrounding medium with respect to the lens material is more than 1, the concave lens becomes a converging lens.

4. Why do you use an auxiliary convex lens in this experiment ?

Ans. Concave lens produces a virtual image for a real object and the image cannot be cast on a screen. In order that the concave lens can produce a real image receivable on a screen, an auxiliary convex lens is used.

5. Can convex lens of any focal length serve your purpose ?

Ans. Yes ; but the convex lens should have a focal length less than that of the concave lens.

6. What happens when the convex lens has formed an image beyond the focal length of the concave lens ?

Ans. The concave lens will then produce a virtual image which can not be cast on a screen.

7. Why should you place the convex lens at a distance from the wire-gauge more than its focal length ?

Ans. If the wire-gauge is within the focal length of the convex lens, the lens will produce a virtual image which cannot be received on a screen.

8. Why is the index error same for object and image distances in this experiment ?

Ans. Because both the distances are measured with reference to the concave lens and the screen.

3.11. The Spectrometer :

The spectrometer is an essential instrument for observing the spectrum of a source of light and for determining the refractive index of the material of a prism. The students will have to perform a number of experiments in the laboratory with this instrument. As the instrument is complex consisting of a number of components, the students should get themselves well-acquainted with the different parts of the instrument. Fig. 11 shows the appearance of a spectrometer frequently used in a laboratory. It essentially consists of the following parts :

(*a*) *The prism table* (*P*) for mounting the prism used for producing the spectrum of the light incident on it. (*b*) *The collimator* (*C*) for producing a beam of parallel light. (c) *The telescope* (*T*) for viewing the spectrum. (d) *A circular scale and Verniers* (V_1 and V_2) for the

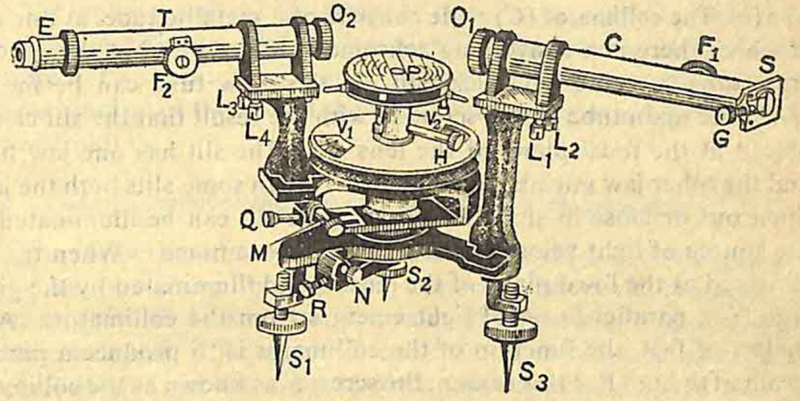

Fig. 11.

measurement of angle of rotation either of the telescope or of the prism table.

Below are given the descriptions of each part in detail.

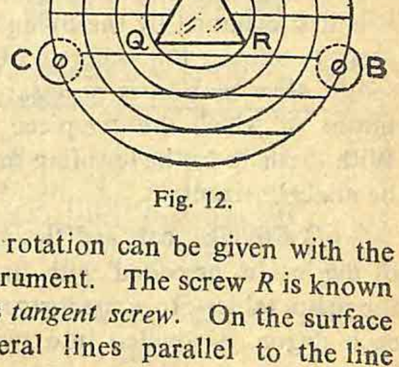

Fig. 12.

(*a*) **The prism table** (*P*) : It is a circular plane table capable of rotating about the vertical axis of the instrument. The table is provided with three levelling screws *A*, *B*, *C* [Fig. 12] by means of which the table can be levelled. The prism *PQR* is placed on this table. The height of the prism table can be adjusted by raising or lowering the rod carrying the table and then clamping it in position by screw *H*. When the prism table is rotated, two verniers V_1 and V_2 move over a circular scale which is kept just below the verniers and measure the rotation of the table. If the fixing screw *R* is clamped, the prism table cannot be rotated by hand but slow rotation can be given with the tangent screw *N* at the base of the instrument. The screw *R* is known as the *fixing screw* and the screw *N* as *tangent screw*. On the surface of the prism table are drawn several lines parallel to the line obtained by joining the levelling screws *B* and *C* of the prism table [Fig. 12]. Some concentric circles are also drawn on the surface of the table, the centre of the circles being situated on the axis of the instrument. With the help of these circles and straight lines, the prism can be placed in its proper position.

(*b*) **The collimator** (*C*) : It consists of a metallic tube, at one end of which there is a converging achromatic lens O_1 and at the other a draw tube carrying a vertical slit S_1; the draw tube can be moved inside the main tube by the screw F_1 with the result that the slit can be placed at the focal plane of the lens O_1. The slit has one jaw fixed and the other jaw movable by the screw *G*. In some slits both the jaws open out or close in simultaneously. The slit can be illuminated by the source of light whose spectrum is to be examined. When the slit is placed at the focal plane of the lens L_1 and illuminated by the given source, a parallel beam of light emerges from the collimator. As a matter of fact, the function of the collimator is to produce a parallel beam of light. For this reason, the screw F_1 is known as the collimator focussing screw. There are two levelling screws (L_1, L_2) below the collimator tube. With the help of these screws, the axis of the collimator tube can be set horizontal and hence perpendicular to the vertical axis of the instrument.

(*c*) **The telescope** (*T*) : It is an astronomical telescope consisting of an objective lens O_2 which is an achromatic doublet of convex and concave lens and a Ramsden type eye-piece *E*, carrying a pair of cross-wires. The telescope can be set so as to admit a parallel pencil of light which forms a sharp image of the slit on the cross-wire. The telescope can be rotated about the vertical axis of the prism table and its rotation can be measured with the help of the verniers V_1 and V_2 (Set at 180° with each other) and the circular scale. Like the prism table, the telescope is also provided with a fixing screw *Q* and a tangent screw *M*. When slow motion of telescope is wanted, it is first clamped by the fixing screw and then moved slowly by the tangent screw. For accurate observation of the rotation of the telescope, slow motion is necessary. There is a focussing screw F_2 by means of which the eye-piece can be focussed on the cross-wires. With the help of the levelling screws L_3 and L_4, the telescope axis can be made horizontal.

(*d*) **Circular scale and the verniers** : A circular scale graduated in degrees is provided with the instrument. The scale is coaxial with the prism table. In some instruments the scale is graduated in $\frac{1}{2}°$ *i.e.*, each degree is divided into two equal parts and in other sensitive instruments, the division is carried to four equal parts *i.e.*, $\frac{1}{4}°$. The telescope tube is rigidly fixed with this circular scale with the result that with the rotation of the telescope, the circular scale rotates below the two verniers V_1 and V_2 fixed with the prism table at angle of 180° between them. When the prism table rotates, the verniers also rotate over the circular scale which remains fixed. In some verniers there are 30 equal divisions while in sensitive instruments, the divisions may be more. Two verniers are given in order to eliminate the eccentric error arising out of the axis of the circular scale not coinciding with the axis of rotation.

3.12. Determination of the vernier constant of a spectrometer :

The vernier constant of a spectrometer should be first determined before it is put to any observation. Suppose, the vernier of a spectrometer contains 30 divisions and the value of the *smallest* division of the circular scale is $\frac{1}{2}°$; If 0-mark of the vernier is made to coincide with any full division of the circular scale, then it will be found that 30 divisions of the vernier have coincided with 29 smallest divisions of the circular scale. The vernier constant, in this case, can be found out in the following way :

30 vernier divisions = 29 smallest divisions of the circular scale

$$\therefore \; 1 \;\;,, \;\;,, \;\; = \frac{29}{30} \;\;,, \;\;,, \;\;,, \;,, \;,, \;,,$$

Hence, vernier constant = 1 division of the circular scale − 1 division of the vernier

$$= 1 - \frac{29}{30} = \frac{1}{30} \text{ of the smallest division of the circular scale}$$

$$= \frac{1}{30} \times \tfrac{1}{2}° = \frac{1}{60} \text{ of } 1° = 1' \text{ (minute)}$$

The smallest angle that this vernier can measure accurately is 1 minute. The following is the method of taking a reading with the above vernier :

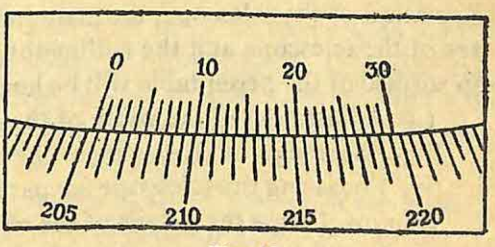

Fig. 13

Suppose, in one case, the vernier reads as shown in fig. 13. The 0-mark of the vernier has crossed over 205° mark of the circular scale but not 205° – 30′ mark. Hence, the circular scale reading is 205° If the vernier divisions are observed carefully, it will be seen that 25th division mark of the vernier has coincided with a certain division of the circular scale. The vernier reading ,in this case, is 25. So, the total reading$=205^\circ+25\times$vernier constant

$$=205^\circ+25\times1'=205^\circ25'$$

Example : The degree division of the circular scale of a spectrometer is divided into four equal parts. 30 divisions of the vernier scale coincide with 29 smallest divisions of the circular scale. What is its vernier constant ?

Ans. 30 divisions of the vernier$=$29 smallest divisions of the circular scale

$\therefore$ 1 ,, ,, ,, $=\frac{29}{30}$,, ,, ,, ,,

So, the vernier constant$=\left(1-\frac{29}{30}\right)$ of the smallest division of the circular scale

$$=\left(\frac{1}{30}\times\frac{1}{4}\right)^\circ$$

$$=\left(\frac{60}{120}\right)' \text{ minute}=30'' \text{ (seconds)}$$

The smallest angle that this venier can measure accurately is 30″ (seconds).

3.13. Adjustments of spectrometer :

Before proceeding to make any observation with a spectrometer, some initial adjustments are essential. These adjustments are time consuming. The students are, therefore, advised to spend one day over these adjustment works. Once they get fully conversant with the adjustment procedures, they will feel no difficulty in handling the instrument later on. The sequence in which these have to be done is given below :

(i) Levelling of the instrument ; by this adjustment (*a*) the axes of rotation of the telescope, the prism table etc will be vertical, (*b*) the axes of the telescope and the collimator will be horizontal and (*c*) the top surface of the prism table will be horizontal.

(ii) Adjustment of the width of the slit and its illumination.

(iii) Focussing the cross-wire of the eye-piece of the telescope.

(iv) Focussing the telescope for parallel rays.

We now discuss the details of the above adjustments :

(i) **Levelling** : If you look at the instrument, you will find three levelling screws at the three legs of the instrument [Fig. 11]. One levelling screw S_3 is just below the collimator tube and the other two

S_1 and S_2 are on the side of the telescope tube. Turn the telescope so that it lies parallel to the line joining the screws S_1 and S_2. Place a spirit level on the telescope parallel to the axis of the tube. If necessary, the spirit level may be fastened with the tube by a rubber band. If the bubble of the spirit level is not at the centre, turn the levelling screws S_1 and S_2 *equally but in the opposite directions* (for example, if one is turned clockwise the other should be turned anti-clockwise) to bring the bubble half way to the centre and half way by turning the levelling screws L_3 and L_4 below the telescope tube equally in the same direction. Now turn the telescope through 180° from this position and see if the bubble is still in the centre or not. If the bubble lies at the centre, the adjustment is complete ; in case it is displaced, it should be brought back to the centre by the previous operation *i.e.*, half way by turning the screws S_1 and S_2 equally but in the opposite directions and half way by turning the screws L_3 and L_4 equally in the same direction. The procedure may have to be gone through three or four times before the bubble remains at the centre for both the positions of the telescope. Now turn the telescope and set it in line with the collimator tube. The bubble of the spirit level might be displaced from its central position. If it is displaced, turn the third levelling screw S_3 at the base until the bubble returns to the central position. With the completion of levelling of the instrument, the axis of rotation of the telescope becomes vertical and the axis of the telescope tube horizontal. The bubble of the spirit level will now remain at the centre in all positions of the telescope.

Levelling of collimator : Now the spirit level is to be placed on the collimator tube with its axis parallel to the axis of the tube. If, in this position of the collimator, the bubble is found to be displaced from its central position, turn the levelling screws L_1 and L_2 provided with the collimator tube, in the same direction to bring the bubble back to its central position. This makes the axis of the collimator tube horizontal.

Levelling of prism table : There are three more levelling screws A, B and C just below the prism table for levelling the table [Fig. 12]. It has been mentioned earlier that several parallel lines are drawn on the prism table parallel to the line joining the screws B and C. Placing the spirit level parallel to these lines, the bubble is to be brought to the central position by turning the screws B and C *equally in the opposite directions*. Now place the spirit level with its axis perpendicular to the line BC. If the bubble is disturbed, bring it to the central position by turning the screw A alone. This makes the axis of rotation of the prism table vertical.

(ii) **Adjustment of the slit and its illumination** : Place a Bunsen burner 10 – 12 cm away from the slit. Prepare some common salt solution. Wrap some asbestos in an iron rod and moistening the asbestos in salt solution, hold it in the non-luminous part of the bunsen flame. The flame will be coloured with golden yellow colour—the characteristic colour of sodium. Place a perforated screen [screen may be made by fixing a piece of black tin plate in a wooden frame and making a narrow long slit on the tin plate] between the flame and the slit. If the height of the slit in the screen be equal to the slit of the collimator, then light passing through the screen slit will illuminate the collimator slit. [If sodium vapour lamp is available, Bunsen burner, salt solution etc. are not needed]. Looking through the collimator tube, see whether, illuminated slit (golden yellow colour) is visible or not. If necessary, slightly alter the position of the burner and make the image of the slit brightest. Now, bringing the telescope in line with the collimator, look through the telescope and try to see the image of the slit. Perhaps the image will be blurred. If so, turn the focussing screw F_2 of the telescope (if necessary, also the focussing screw F_1 of the collimator) and make the image sharp and bright. In this position, the edges of the slit will appear very sharp. If the slit be not vertical, turn it and make it vertical. The width of the slit should be very small (1 or 2 mm). If it is wide, make it narrow by turning the screw G. Sometimes, it may so happen that inspite of levelling the instrument with the levelling screws, the image of the slit seen through the telescope does not occupy the central portion of the field of view of the telescope —it is either raised a little or lowered. In such cases, the image should be made to occupy the central portion of the field of view by turning the levelling screws L_1 and L_2 provided with the collimator tube.

(iii) **Focussing the cross-wires of the eye-piece** : Turn the telescope towards the illuminated slit of the commutator. The cross-wires of the eye-piece ought to be clearly visible in the background of bright and illuminated field of view. If they are blurred, focussing is necessary. To do this, the draw-tube carrying the focussing lens of the eye-piece should be pushed in or out till the cross-wires are very distinct. After this, the focussing lens should not be disturbed. Turning the eye-piece in its own plane, adjust its position such that the vertical wire passes through the centre of the slit. [Some prefer to have the intersection of the cross-wires at the centre of the slit.]

(iv) **Focussing the telescope for parallel rays : Schuster's method** : Place a prism ABC on the prism table with its centre coinciding with the centre of the prism table. The refracting edges of the prism should be vertical. Turn the prism table so that the rays of light coming from the collimator may be incident on one refracting face (say, *AB* in fig. 14) of the prism at an angle of about 45°. The rays of light will emerge from the prism through the other refracting face *AC* and will be deviated towards the base *BC* of the prism. Looking through the prism from the side *AC* as shown in the fig. 14, try to see the image of the collimator slit. Moving the eye a little towards right and left, a bright golden-yellow image will be visible. Keeping the eye in that position, if the prism table is rotated slightly this way or that way, the image will also be seen to move likewise. Now, slowly turn the prism table so that the image moves towards the side of lesser deviation. Follow the movement of the image with naked eye. Soon a position of the prism table will be available where the image stops moving momentarily and then turns back in the opposite direction although the prism table is rotating in the same previous direction. The position of the prism table where the image of the collimator just starts retracing its path is called the *minimum deviation position*.

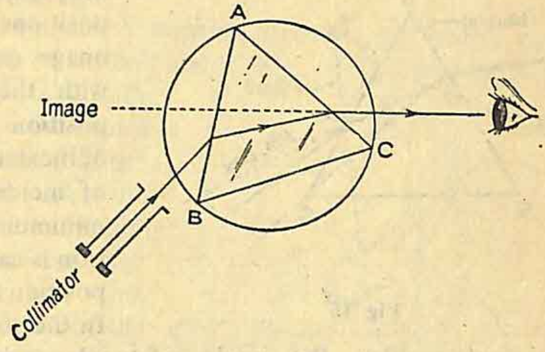

Fig. 14

Having ascertained the position of minimum deviation of the prism with naked eyes, remove the eyes a little behind keeping the image in view and then bring the telescope between the eye and the prism. Now, looking through the telescope, the image of the slit will be clearly visible. Keeping the eye in the same position, turn the prism table slightly, if necessary, so that the image is just on the point of turning *i.e.*, just in the minimum deviation position. Without changing the position of the prism table, turn the telescope slightly towards the side of greater deviation so that the image of the slit may stand at one side of the field of view of the telescope [Fig. 15]. In this position, in whatever

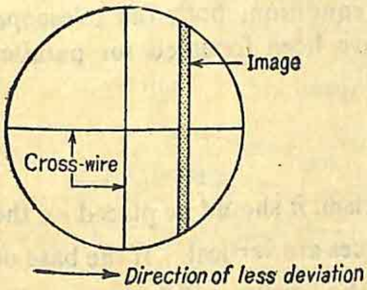

Fig. 15

direction the prism table is rotated, the image will retrace its path after reaching the cross-wires *i.e.*, the prism can be placed in two positions for each of which the image can be made to coincide with the cross-wires. In one position of the prism, the angle of incidence is more than the angle of incidence corresponding to the minimum deviation and the position is called *slant position*. In this position the image is very narrow. In the other position, known as *normal position*, the angle of incidence is less than the angle of incidence corresponding to the minimum deviation and the image is broader [Fig. 16].

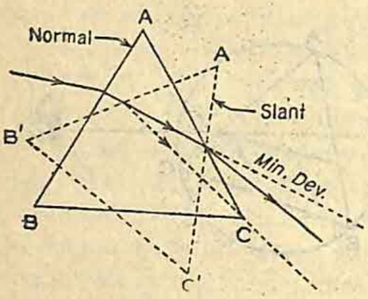

Fig. 16

Now turn the prism table so that it is placed in the slant position and the narrow image of the slit coincides with the cross-wires. The image may appear blurred. Make the image distinct by turning the focussing screw F_2 of the *telescope*. Now turn the prism table to the normal position. The image also moves to the minimum deviation position and then turns back. When it turns back, it becomes broader but blurred. The image is to be made sharp and distinct by turning the focussing screw F_1 of the *collimator*. In general, the above process of focussing the telescope and the collimator alternately will have to be repeated till the image remains in good focus for both the positions of the prism. This adjustment is completed usually in three to four alternate focussings. In this condition, both the telescope and the collimator may be said to have been focussed for parallel rays.*

3.14. Optical levelling of the prism :

In performing experiment with a prism, it should be placed on the prism-table such that its refracting surfaces are vertical. If the base of the prism is perpendicular to the sides, the levelling of the prism table by the levelling screws *A*, *B* and *C* [Fig. 12] ensures that the refracting faces of the prism are vertical. If, there exists some discrepancy between the base and the faces, optical levelling is necessary. It is to be done in the following way :

---

* *Mnemonic for Schuster's method* : Focussing process can be easily remembered by the mnemonic *BC* meaning *B*road image to be focussed by the Collimator.

Swing the telescope in such a position that its axis makes an angle of 90° with the axis of the collimator (Fig. 17). Now place the prism *PQR* on the prism table with its centre coinciding with the centre *O* of the table (or, the apex *P* of the prism coinciding with the centre of the table whichever is convenient) and one of its refracting surfaces (say *PQ*) perpendicular to the line joining the screws *B* and *C* of the prism table. Next turn the prism table slowly till the rays of light coming from the collimator are reflected by the surface *PQ* into the telescope [Fig. 17]. Looking through the telescope, an image of the slit will be visible. If the image is situated symmetrically with respect to the top and bottom of the field of view of the telescope, the levelling is alright and no optical levelling is necessary. If, on the other hand, the image is found to be raised or lowered, bring it to the central position of the field of view by turning the screws *B* and *C equally in the opposite directions*. Now, rotate the prism table, till the rays of light are reflected into the telescope by the other refracting surface *PR* of the prism. If the image is still unsymmetrical, bring it to the central position by turning the third levelling screw *A*. Repeat the process of alternate levellings a few times. Finally, the image will be found to remain symmetrical whatever surface of the prism is used to reflect the rays of light. This ensures that the refracting surfaces of the prism are vertical.

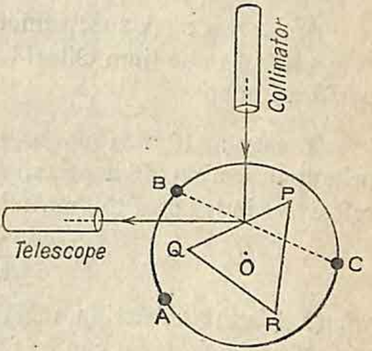

Fig. 17

3.15. Precautions in using a prism : In using a prism, care should be taken, first of all, to see whether the refracting surfaces [*PQ* and *PR* in fig. 17] are clean or not. If the surfaces are smeared with oil or greasy substances, they are to be cleaned by cotton moistened with alcohol. In handling a prism, care should be taken not to touch the refracting surfaces with hand. It

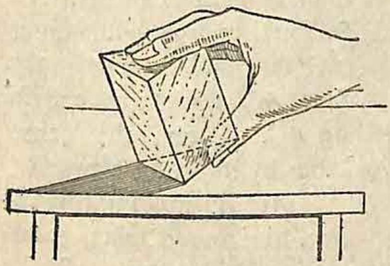

Fig. 18

should be held with fingers pressing the top and the bottom of the prism as shown in fig. 18.

3.16. Adjustment of a spectrometer for parallel rays by Schuster's method and to determine refractive index of the material of a prism by minimum deviation method :

Apparatus : A spectrometer, prism, spirit level, burner, asbestos ring soaked in sodium chloride solution, a wooden or tin screen with perforation etc.

Theory : If a monochromatic ray of light passes through the principal section of a prism with minimum deviation δ_m, then the refractive index μ of the material of the prism is given by,

$$\mu = \frac{\sin \frac{1}{2}(\delta_m + A)}{\sin \frac{1}{2}A}$$

where A is the refracting angle of the prism. So, knowing A and δ_m, the refractive index μ can be found out.

Experimental procedure : (1) Examine the graduations on the circular scale of the spectrometer and find the vernier constant as explained on page 155.

(2) Complete the adjustment operations (i), (ii), (iii) and (iv) mentioned in art 3.12. The instrument will then be focussed for parallel rays and be ready for subsequent observations.

Determination of the refracting angle (A) of the prism :

(3) Place the prism on the prism table with its apex *A* coinciding with the centre of the prism table and the base *BC* perpendicular to the parallel rays coming from the collimator [Fig. 19]. Allow the light from the collimator to fall on the prism in a manner such that half of it falls on the face *AB* and the other half on *AC*. Clamp the prism table by its fixing screw *R*.

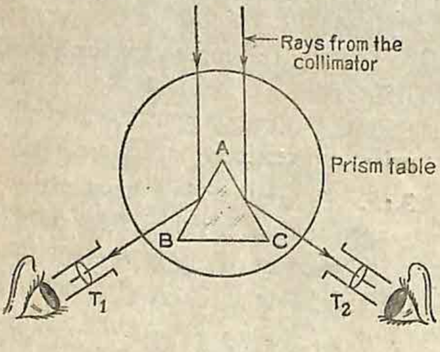

Fig. 19

(4) By moving the eye in a horizontal plane containing the axis of the collimator, look for the reflected image of the slit from the face *AB*. A slight movement of the eye will enable you to see the bright

image of the slit. Swing the telescope to this direction (T_1) to get the image of the slit on the cross-wires. Turn the tangent screw (N) of the telescope to give it a slow motion so that the vertical cross-wire may just pass through the centre of the image [Fig. 19(a)]. Take the readings of the verniers V_1 and V_2 against the circular scale. Check that the difference between these two readings is very nearly 180°. Repeat the operation twice and get the mean reading.

(5) Turn the telescope to see the reflected image from the face AC and make a similar adjustment in the position T_2 [Fig. 19]. Take the vernier readings.

(6) Find the difference between the two readings of the same vernier. Take the mean of the two results obtained from the two verniers. Half of it gives the angle of the prism (A.).

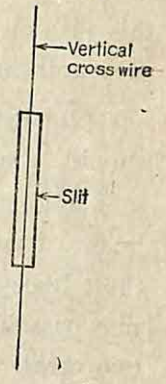

Fig. 19(*a*)

Determination of the angle of minimum deviation (δ_m) :

(7) For this purpose, place the prism on the prism table with its centre coinciding with the centre of the table and light from the collimator falling on a refracting surface (AB in fig. 20) of the prism. Rays of light will emerge through the face AC being deviated towards the base BC of the prism [Fig. 20].

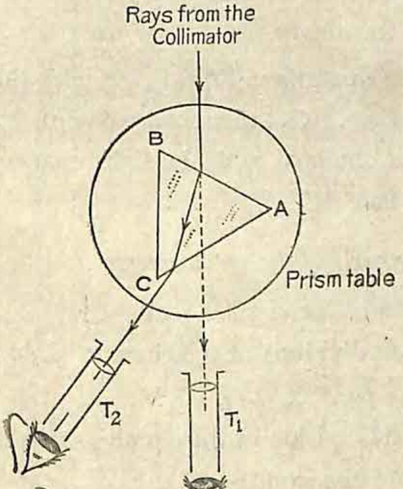

Fig. 20

(8) Moving the eye in a horizontal plane containing the axis of the collimator, try to look for the refracted image of the slit from the side AC. Having obtained the image, follow the operation (iv) of the art 3.13 and place the prism *almost* in the minimum deviation position with naked eye. Clamping the prism table with its fixing screw, swing the telescope in the position T_2 and look for the image of the slit through the telescope. Give the prism table a slow motion by its tangent screw and set it at the position where *the image just starts to retrace its path.* This is the correct minimum deviation position of the prism.

Now slowly rotate the telescope in suitable direction in order to bring the vertical cross-wire exactly at the centre of the slit image. Read this position of the telescope by the two verniers and the circular scale. Repeat the operations, at least, twice and get the mean reading.

(9) Remove the prism without disturbing the prism table, which should be securely clamped. Swing the telescope so that its axis (T_1) may lie in a line with the collimator axis. In such a case, the direct image of the slit will be visible through the telescope. Slowly rotate the telescope with its tangent screw so that the vertical cross wire may pass through the centre of the image. Having done this, read the two verniers. Repeating the operation, at least, twice, get the mean direct reading. The angle of minimum deviation is the angle between the direct rays (T_1) and the emergent rays (T_2) in the operation (8). To get this, take the difference of readings of the same vernier for the two positions of the telescope. Take the mean of the two results obtained from the two verniers. This gives the value of δ_m.

(10) Calculate the value of μ after substituting the values of A and δ_m in the equation mentioned in the theory.

(11) If time permits, repeat the operations (7), (8), (9) with the light falling on the face AC of the prism. Get the mean value of δ_m by taking the average of the results obtained with the light incident first on the face AB and then on the face AC.

Measurements : (*a*) *Vernier constant of the spectrometer* :

The value of the smallest division of the circular scale= . .

. . Vernier divisions= . . smallest divisions of the circular scale

∴ 1 ,, ,, = . . ,, ,, ,, ,, ,,

So, Vernier constant= . . smallest division of the circular scale

= . . (minute or second)

(*b*) *Determination of the refracting angle of the prism* :

No. of obs.	Vernier	Reading of the 1st image (T_1)			Mean (a)	Reading of the second image (T_2)			Mean (b)	Difference ($a \sim b$) *	Mean difference ($2A$)	Refracting angle (A)
		Circular scale	Vernier	Total		Circular scale	Vernier	Total				
1.		..	..	..		..	..	..			..	..
2.	V₁	..	..	..	..	..	..	..	..	..		
3.		..	..	..		..	..	..				
1.		..	..	..		..	..	..				
2.	V₂	..	..	..	..	..	..	..	..	..		
3.		..	..	..		..	..	..				

[* Sometimes it so happens that while shifting the telescope from the position T_1 to the position T_2 (Fig. 20), 0° mark of the circular scale is crossed over. In that case, the difference $= 360° - (a \sim b)$; For example, the reading for the position T_1 is 270° 25′ and that for the position T_2 is 30° 13′. It is easy to realise that 0° has been crossed over. So, the difference of the readings between the two positions $= 360° - (270° 25' - 30° 13') = 119° 48'$. Students are, therefore, advised to be vigilant over this point.]

(*c*) *Determination of the angle of minimum deviation :*

No. of obs.	Vernier	Reading for min^m dev. position (T_2)				Reading for direct position (T_1)				Angle of min^m dev. ($\delta_m = a \sim b$)	Mean δ_m
		Circular scale	Vernier	Total	Mean (a)	Circular scale	Vernier	Total	Mean (b)		
1.		..	..	..		..	..	..			..
2.	V_1	..	..	..	..	..	..	..	..	..	
3.		..	..	..		..	..	..			
1.		..	..	..		..	..	..			
2.	V_2	..	..	..	..	..	..	..	..	..	
3.		..	..	..	(a')	..	..	..	(b')	($a' \sim b'$)	

Calculations : $\mu=\frac{\sin\frac{1}{2}(\delta_m+A)}{\sin\frac{1}{2}A}=\frac{\sin\frac{1}{2}(..+..)}{\sin\frac{1}{2}..}=..$

Remarks : (1) In focussing the slit, parallax should be avoided between the cross-wires and the slit. (2) The accuracy of the measurement depends on the correct way of placing the prism on the table. (3) In measuring the angle of the prism care should be taken to see whether 0° mark of the circular scale is crossed over. (4) While using the tangent screw, the telescope or the prism table should be clamped before hand by the fixing screws.

Oral questions

1. What is angle of minimum deviation ? What is the condition for the deviation to be minimum ?

Ans. Consult any text book. When the angle of incidence of a ray on any refracting face of the prism equals the angle of emergence from the other face, the deviation becomes minimum.

2. Why do you use sodium light in this experiment ? Can any other light be used ?

Ans. Sodium light is monochromatic and can be easily produced. This is the reason why sodium light is used. As a matter of fact, any monochromatic light can be used.

3. Why two verniers are provided in the instrument ?

Ans. See art 3.11(d). [Page 154].

4. Why are telescope and collimator focussed foi parallel rays ?

Ans. If the incident rays are either diverging or converging but not parallel, the distance of the image from the prism will vary with the change of position of the prism. As a result, if the image is focussed at a given position of the prism, it will be out of focus in another position. But if the collimator and the telescope are focussed for parallel rays, the image will remain in focus in all positions of the prism.

5. Why is the instrument levelled ?

Ans. If the instrument is not levelled, the position of the image will vary with the change of position of the telescope.

6. Why is optical levelling sometimes necessary ?

Ans. When the base of the prism and the refracting surface are not perpendicular to each other, optical levelling is necessary.

7. What is the advantage of tangent screw ?

Ans. Tangent screw enables us to give the prism table or the telescope a slow-motion, which is necessary for fine or accurate adjustment.

8. Which type of eye-piece is provided with the telescope ?

Ans. The telescope is provided Ramsden's type of eye-piece, because cross-wires can be used in this eye-piece.

9. When light falls on the refracting surface of a prism, is all light refracted through it ?

Ans. No ; a small fraction is reflected.

10. Can you use white light in your experiment ?

Ans. No ; white light contains seven wave-lengths, each of which will produce an image of the slit and the whole thing will be very much confused.

3.17. To draw i − δ curve of a prism by a spectrometer and hence to find out the angle of minimum deviation :

Apparatus : As in experiment of art. 3.16.

Theory : In fig 21, a ray *PQ* is incident on the refracting face *AB* of a prism at an angle *i*. A part of the light is reflected along *QM* and the remaining part is refracted into the prism along *QR* which finally emerges along *RS*. In this case, the angle of incidence $i=\angle PQN=\frac{1}{2}\angle PQM$.

$$\therefore\ 2i=\angle PQM=180^\circ-\angle TQM,$$

where *QT* is the direct ray from the collimator. So, knowing the angle between the reflected ray *QM* and the direct ray *PQT*, the angle of incidence *i* can be found out.

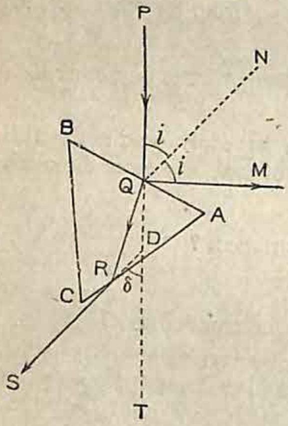

Fig. 21

The deviation of the above incident ray $\delta=\angle SDT$; hence knowing the angle between the emergent ray *RS* and the direct ray *PQDT*, the angle of deviation δ can be found out.

Now, the angle of deviation δ depends on the angle of incidence *i* ; but it is found that at a given angle of incidence, the angle of deviation becomes minimum. So, the angle of minimum deviation can be found out from $i-\delta$ graph.

[N.B. Determining δ_m and *i* from the graph, μ of the material of the prism can be found out by the following formula : The angle of the prism is given by,,

$$A=2i-\delta_m\quad\therefore\ \mu=\frac{\sin A+\delta_m}{2}\Big/\sin\frac{A}{2}\Big]$$

Experimental procedure : (1) Carefully note the graduations of the circular scale of the spectrometer and from it, calculate the vernier constant of the instrument (See art 3.12).

(2) Complete the operations (i), (ii), (iii) and (iv) in connection with the adjustment of the instrument as described in art 3.13. The instrument is now focussed for parallel rays and is ready for subsequent experiment.

(3) Make the screw *H* [Fig. 11] of the prism-table a little loose so that it can be rotated by hand *without rotating the scale or vernier.* In this experiment, rotation of the vernier scale or the circular scale should take place only with the rotation of the telescope.

(4) Illuminate the slit of the collimator with sodium light. Bring the telescope in a line with the collimator so that light from the collimator can directly enter into the telescope. Turn the tangent screw of the telescope and slowly set the telescope in the position *T* [Fig. 21(*a*)] so that the vertical cross-wire may pass through the centre of the image of the slit. Read the circular scale and the two verniers and compute the total reading for this position of the telescope. This gives the direct reading of the telescope. Let it be x.

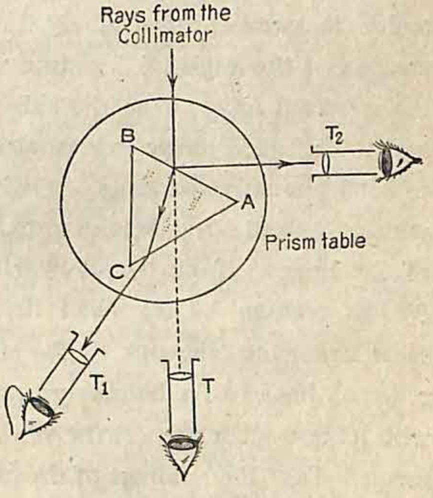

Fig. 21(*a*)

(5) Now place the prism on the prism-table with its centre coinciding with the centre of the prism-table so that light from the collimator may be incident on one of the refracting faces [face *AB* in fig. 21(*a*)] of the prism. The correct setting of the prism can be judged from the concentric circles drawn on the prism-table. Turning the top of the prism-table (without turning the scale) set the prism approximately in the minimum deviation position [say, *ABC* in fig. 21(*a*)] following the procedure described in art 3.13 (iv). Swing the telescope in the position T_1 and look for the *refracted image* of the slit through the telescope. Slowly move the telescope with its tangent

screw until the vertical cross-wire passes through the centre of the image. Take the readings of both the verniers in this position of the telescope. This corresponds to the reading for the approximate minimum deviation position of the prism. Let this reading be x_1. Then, the deviation of this ray $\delta=(x_1\sim x)$.

(6) Looking from the side *AB* of the prism with naked eye, try to see the *reflected image* of the slit. Swing the telescope with hand to the position of the eye and look for the reflected image through the telescope. Slowly adjust the position of the telescope by means of its tangent screw so that the vertical cross-wire may pass through the centre of the image. In this position (T_2) of the telescope, take the readings of the verniers and the circular scale. Let it be x_2. The angle of incidence of the ray $i=\frac{1}{2}[180° -(x_2\sim x)]$.

(7) Now, turn the top of the prism table (without turning the scale) clockwise through nearly 2°. The angle of incidence will increase and the angle of deviation will change. As before try to see the refracted image from the side *AC* of the prism with naked eye and the reflected image from the side *AB*. Swing the telescope first to catch the refracted image. Giving a slow motion to the telescope with its tangent screw, set the vertical cross-wire exactly at the middle of the image. Take the circular scale and vernier scales reading. If this reading be x_3, then the deviation of the ray $\delta=(x_3\sim x)$. Now swing the telescope to the side *AB* of the prism to catch the reflected image. As before, giving a slow motion to the telescope with its tangent screw, set the vertical cross-wire at the middle of the image. Take the readings of the circular scale and both the verniers. If this reading be x_4, the angle of incidence of the aforesaid ray $i=\frac{1}{2}[180° -(x_4\sim x)]$.

(8) In this way, the top of the prism table is to be rotated through steps of about 2° clockwise and anti-clockwise about the minimum deviation position (*i.e.*, the position *ABC*) and the angles of incidence increased and decreased accordingly. In each case, the circular scale reading and the vernier scale readings are to be noted after catching the refracted and reflected images through the telescope.

(9) The above operations give us readings for seven positions—three on the clockwise, three on the anti-clockwise and one approximately on the minimum deviation position. For all these positions, the values of i and δ are to be found out and then a graph between i and δ is to be drawn. The graph will be very much like a parabola [Fig. 21(*b*)]. From the lowest point of the graph, the angle of minimum deviation (δ_m) and corresponding angle of incidence (i) are to be determined.

(10) Applying the formula mentioned in the theory, μ can be found out.

Measurements : (*a*) *Determination of vernier constant* (Data for illustration)

As in Experiment no. 3.16. The vernier constant=1′.

(*b*) *Table for direct image readings* :

Vernier	No. of Obs.	Circular scale	Vernier scale	Total	Mean (x)
V_1	1.	85°30′	2′	85°32′	85°32′
	2.	85°30′	1′	85°31′	
	3.	85°29′	2′	85°32′	
V_2	1.	265°30′	4′	265°34′	265°34′
	2.	265°30′	3′	265°33′	
	3.	265°30′	5′	265°35′	

(c) *Table for angles of incidence and deviation* $(i-\delta)$:

No. of Obs.	Vernier	Readings for refracted ray				Readings for reflected ray				$\delta = (x_1 \sim x)$	Mean δ	$i = \frac{1}{2}[180° - (x_2 \sim x)]$	Mean i
		Circular scale	Vernier scale	Total	Mean (x_1)	Circular scale	Vernier scale	Total	Mean (x_2)				
1	V_1	33°30′ ,, ,,	5′ 4′ 5′	33°35′ 33°34′ 33°35′	33°35′	154°30′ ,, ,,	5′ 4′ 6′	154°35′ 154°34′ 154°36′	154°35′	51°57′	51°58′	55°29′	55°30′
	V_2	213°30′ ,, ,,	5′ 3′ 6′	213°35′ 213°33′ 213°36′	213°35′	334°30′ ,, ,,	4′ 3′ 4′	334°34′ 334°33′ 334°34′	343°34′	51°59′		55°30′	
2.	V_1	29°30′ ,, ,,	20′ 21′ 20′	29°50′ 29°51′ 29°50′	29°50′	124°30′ ,, ,,	10′ 11′ 10′	124°40′ 124°41′ 124°40′	124°40′	55°42′	55°42′	70°26′	70°26′
	V_2	209°30′ ,, ,,	24′ 23′ 22′	209°54′ 209°53′ 209°52′	209°53′	304°30′ ,, ,,	11′ 10′ 12′	304°41′ 304°40′ 304°42′	304°41′	55°41′		70°26′	
3	V_1	28°30′ ,, ,,	22′ 23′ 21′	28°52′ 28°53′ 28°51′	28°52′	176°30′ ,, ,,	17′ 16′ 18′	176°47′ 176°46′ 176°48′	176°47′	56°40′	56°38′	44°22′	44°23′
	V_2	208°30′ ,, ,,	27′ 27′ 28′	208°57′ 208°57′ 208°58′	208°57′	356°30′ ,, ,,	13′ 13′ 13′	356°43′ 356°43′ 356°43′	356°43′	56°37′		44°25′	
4	etc	..	..	..	..	..	..	..	..	..	..	..	..

(*d*) *Table for drawing graph* [Data obtained from table (c)]

Quantity	1	2	3	4	5	6	7
Angle of incidence (i) ⟶	55°30′	70°26′	44°23′	80°30′	41°27′	85°50′	..
Angle of deviation (δ) ⟶	51°58′	55°42′	56°38′	61°48′	61°38′	66°6′	..

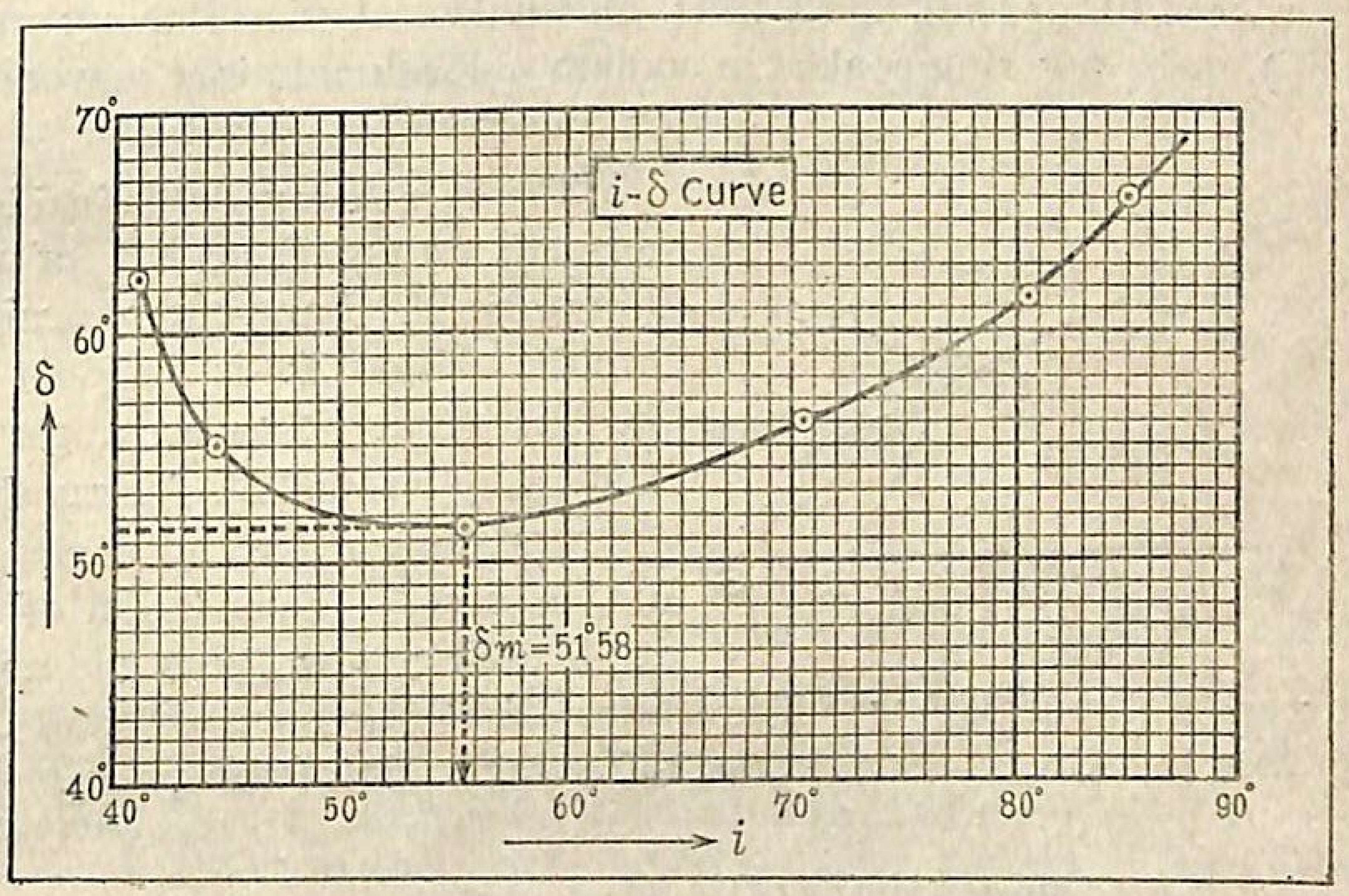

Fig. 21(*b*)

(*e*) From the graph : $\delta_m = 51°58'$ (as an illustration)
$i = 55°30'$

Calculations :

Angle of the prism $A = 2i - \delta_m = 2 \times 55°30' - 51°58'$
$= 49°2'$

$$\therefore \quad \mu = \frac{\sin \frac{1}{2}(\delta_m + A)}{\sin A/2} = \frac{\sin \frac{1}{2}(51°58' + 49°2')}{\sin (49°2'/2)}$$

$$= \frac{\sin 50°30'}{\sin 24°31'} = \frac{0{\cdot}7716}{0{\cdot}4147} = 1{\cdot}86$$

Remarks : (1) Care should be taken so that while rotating the top of the prism table, the positions of the circular scale and the verniers do not change. (2) At no stage should the prism be bodily displaced from its position ; for then the angle of incidence will change and the experiment will have to be repeated from the very begining.

Oral questions

1. How does the angle of deviation depend on the angle of incidence ?

Ans. For a particular angle of incidence, the deviation becomes minimum and for all other angles of incidence, the deviation is greater.

2. What will be the difference between the readings of the verniers at any stage ?

Ans. The difference is 180°.

*3.18. Determination of the refractive index of the material of a thin prism by the method of normal incidence :

Apparatus : A spectrometer, a thin prism (having refracting angle of the order of 10° or less), spirit level, an equilateral prism (having angle of 60°), asbestos ring soaked in sodium chloride solution, a wooden or tin screen with a slit, etc.

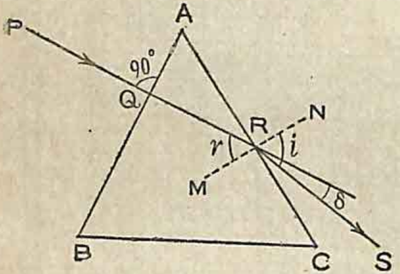

Fig. 22

Theory : A ray PQ is incident normally on the face AB of the thin prism ABC and enters straight into the prism. It is incident on the face AC at R at an angle of incidence $\angle r$ [Fig. 22]. The ray then emerges along RS at an angle $\angle i$. In this case, the deviation of the ray $\delta = i - r$; now $\angle r$ is the angle between PR and MR which are normals to the sides AB and AC respectively. So, $\angle r = \angle A$, the refracting angle of the prism.

$$\therefore \quad \delta = i - A$$

$$\text{Now,} \quad \mu = \frac{\sin i}{\sin r} = \frac{\sin(\delta + A)}{\sin A} = \frac{\delta + A}{A} \quad \text{[Since the angles are small]}$$

$$= 1 + \frac{\delta}{A}$$

Experimental procedure : (1) Look at the graduations of the circular scale of the spectrometer and determine the vernier constant of the instrument.

(2) Perform the operations (i), (ii), (iii) and (iv) in connection with the adjustment of the spectrometer as described in art 3.13. While performing the operation (iv), use the equilateral prism of angle 60°.

(3) Remove the equilateral prism from the prism table. Bring the telescope in line with the collimator so that light from the collimator may enter straight into the telescope. Looking through the telescope, try to see the image of the slit. Move the telescope slowly

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* For North Bengal University only.

by its tangent screw and set the vertical cross-wire just at the middle of the slit image. Note the circular scale reading and the reading of any one vernier (say, the vernier V_1). Suppose the reading is α. Now slowly move the telescope by hand till the vernier V_1 rotates through *exactly* 90° and gives a reading $(\alpha+90°)$ or $(\alpha-90°)$.* In this position, the axis of the telescope will be perpendicular to the axis of the collimator. Clamp tightly the telescope with its fixing screw.

(4) Place the thin prism on the prism-table so that its centre coincides with the centre of the circular prism table. Now slowly rotate the prism table with hand till the light coming from the collimator may, after reflection at the face *AB* of the prism, enter into the telescope *T* [Fig. 22(*a*)]. The image of the slit will be visible through the telescope. Turning the tangent screw of the prism table, bring the vertical cross-wire at the middle of the slit image. Read this position of the prism-table with the help of the circular scale and the vernier. Displace the prism-table a little and perform the above operation twice and get the mean reading (for both the verniers) of the exact position of the prism table. Suppose, the reading for the vernier V_1 is β_1 and that for the vernier V_2 is β_2.

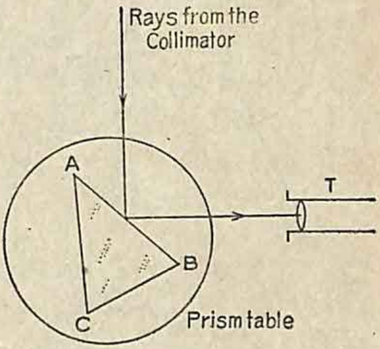

Fig. 22(*a*)

(5) Again turn the prism table by hand and bring the other face *AC* of the prism towards the collimator so that rays of light, coming from the collimator, are reflected by the face *AC* into the telescope. Turning the tangent screw of the prism-table, set the vertical cross-wire at the middle of the slit image and take the circular and vernier scale readings. As before, slightly displace the prism-table and repeat the above operation twice. Find the mean reading of the exact position of the prism-table for each vernier. Let the vernier V_1 now read β_3 and the vernier V_2 read β_4.

In this case, the angle of the prism $A=180°\sim(\beta_1\sim\beta_3)$ or $180°\sim(\beta_2\sim\beta_4)$. Find the mean value of these two readings. This gives the angle *A* of the prism.

(6) Now turn the prism-table from this position (*i.e.*, when the

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* Suppose the telescope is given an anticlockwise rotation. If due to this rotation, the scale reading diminishes gradually, then the vernier V_1 reading should be $(\alpha-90°)$; on the other hand, if the scale reading gradually increases, the vernier V_1 reading should be $(\alpha+90°)$.

vernier V_1 gives a reading β_3) through exactly 45° in the suitable direction so that the present reading of the vernier V_1 is either $(\beta_3+45°)$ or $(\beta_3-45°)$*. In this position of the prism, rays of light from the collimator are incident on one face of the prism normally. [Fig. 22(*b*)]. Clamp the prism-table tightly with its fixing screw.

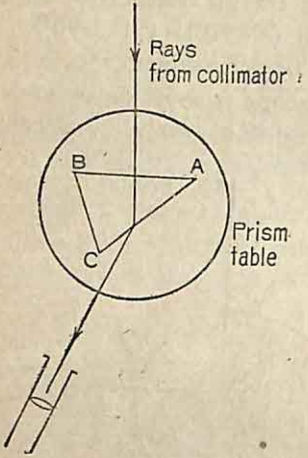

Fig. 22(*b*)

(7) Unfix the telescope and swing it almost in the line with the collimator so that rays of light emerging from the prism may enter into the telescope *T*. Turning the tangent screw of the telescope, bring the vertical cross-wire at the middle of the slit image. Read the circular and vernier scales. Repeat the operation thrice by slightly displacing the telescope. Find the mean readings for both the verniers. Let the mean reading for the vernier V_1 be γ_1 and that for V_2 be γ_2.

(8) Now remove the prism from the prism-table. Bring the telescope exactly in line with the collimator. Try to see the slit image through the telescope. Adjust the position of the telescope by turning its tangent screw so that the vertical cross-wire may lie just at the middle of the slit image. Repeat the operation thrice and get the mean reading. Suppose, the mean reading for the vernier V_1 is γ_3 and that for the vernier V_2 is γ_4. This is direct reading.

Here, the deviation of the ray $\delta=\gamma_1\sim\gamma_2$ or $\gamma_3\sim\gamma_4$. Find the mean value of these two angles. This gives the correct value of the angle of deviation (δ).

(9) Substituting the values of δ and A in the equation mentioned in the theory, calculate the value of μ.

Measurements : (*a*) *Vernier constant of the instrument* :

As in experiment no. 3.16. The vernier constant$=1'$

(*b*) *Determination of the angle* (*A*) *of the prism* :

Keeping the telescope in the direct-reading position (*i.e.*, exactly in the line with the collimator), the reading of the vernier $V_1=32°30'+1'=32°31'$ (α).

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* On turning the prism table in appropriate direction ; if the scale reading gradually decreases, the vernier V_1 should , in that case, read $(\beta_3-45°)$ and if the scale reading gradually increases, the reading should be $(\beta+45°)$.

Setting the telescope at right angles to the line of the collimator, the reading of the vernier $V_1 = \alpha + 90° = 32°31' + 90° = 122°31'$

(The telescope is clamped at this position).

Table for the readings of the prism-table
(Data are given as illustrations)

Vernier	When reflection takes place at the face AB				When reflection takes place at the face AC				$(\beta_1 \sim \beta_3)$	$A = 180° \sim (\beta_1 - \beta_3)$
	Circular scale (S)	Vernier scale (V)	Total ($S + V \times v.c.$)	Mean	Circular scale (S)	Vernier scale (V)	Total ($S + V \times v.c.$)	Mean		
	85°30′	3′	85°33′		260°	18′	260°18′			
V_1	85°30′	2′	85°32′	85°32′	260°	17′	260°17′	260°16′	174°45′	5°15′
	85°30′	2′	85°32′	(β_1)	260°	16′	260°16′	(β_3)		
	..	..	..		..	..	..			
V_2	..	..	..	..	..	..	..	..	$(\beta_2 \sim \beta_4)$	
	..	..	..	(β_2)	..	..	..	(β_4)	..	

$\therefore$ mean value of $A = \dfrac{\cdot\cdot + \cdot\cdot}{2} = 5°15'$

(*c*) *Determination of the angle of deviation* (δ) :

The reading of the vernier V_1 of the prism table when light is reflected by the face AC=260°17 (β_3). The reading of the position of the prism table when it is rotated in the appropriate direction through 45°=260°17′ −45°=215°17′ : (β_3−45°) as the scale reading is decreasing.

(The prism is now set with one if its surfaces at right angles to the axis of the collimator. The prism table is clamped in this position.)

Table for the telescope readings

Vernier	Readings for the refracted ray				Readings for the direct ray				$\delta=\gamma_1\sim\gamma_2$
	Circular scale (S)	Vernier scale (V)	Total ($S+V$)	Mean	Circular scale (S)	Vernier scale (V)	Total ($S+V$)	Mean	
V_1	305°	18′	305°18′		302°30′	17′	302°47		2°30′
	305°	17′	305°17′	305°17′	302°30′	18′	302°48′	302°47′	
	305°	16′	305°16′	(γ_1)	302°30′	16′	302°46′	(γ_3)	
V_2	..	..	..		..	..	..		
	..	..	..	..	..	..	..	..	..
	..	..	..	(γ_2)	..	..	..	(γ_4)	($\gamma_2\sim\gamma_4$,

Mean angle of deviation $\delta=\frac{..+..}{2}$=2°30′

Calculations : $\mu=1+\frac{\delta}{A}=1+\frac{2°30'}{5°51'}=1+\frac{150'}{315'}=1{\cdot}47$

Remarks : (1) In every setting of the prism table and the telescope, parallax between the images of the cross-wire and the slit should be avoided (2) See experiment no. 3.16.

Oral questions

As in experiments no 3.16 and 3.17.

3.19. Determination of the radius of curvature of a lens by Newton's rings method :

Apparatus : Newton's rings arrangement, travelling microscope, short focus convex lens, Bunsen burner, asbestos ring etc.

Theory : Suppose a monochromatic wave-train AB of wave-length λ is incident normally on an air-film entrapped between a plano-convex lens L_1 and a plane glass plate PD [Fig. 23]. A part of the incident ray travels along BC after reflection at B. The other part enters into the air-film and gets reflected at the glass plate and finally emerges from the lens along DEF. These two reflected beams, on being superposed, interfere with each other and produce alternate dark and bright rings. The point of contact between the curved surface of the lens and the glass plate is the centre of the ring system.

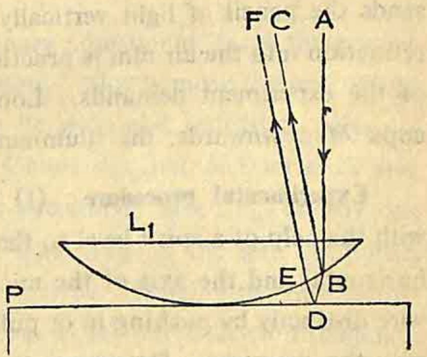

Fig. 23

If D_n be the diameter of the nth *dark* or *bright* ring of the system of rings described above and D_{n+m} that of the $(n+m)$th dark or bright ring, then it may be proved that $D^2_{n+m}-D^2_n=4m\lambda R$ where $R=$ radius of curvature of the curved surface of the lens

$$\therefore\ R=\frac{D^2_{n+m}-D^2_n}{4.m\lambda}$$

Experimental arrangement : Newton's ring apparatus consists of circular metallic frame in which a plano-convex lens L_1 is confined on a plane glass plate P by several screws [Fig. 23(*a*)]. The curved surface of the lens has a large radius of curvature. Uniform pressure can be applied on the circumference of the lens by the screws and the centre of the system of rings can be made to coincide with the centre of the circular frame. If an asbestos ring, soaked in sodium chloride solution, is held in the flame of a burner, a bright source of monochromatic light (S) is produced. A short focus lens L is held at a distance equal to its own focal length from the source of light, so that a beam of parallel rays is obtained. The parallel rays then fall on a glass plate G inclined at 45° to the vertical. The glass plate G sends the pencil of light vertically downward and thus the angle of refraction into the air film is practically zero. This is what the theory of the experiment demands. Looking through a travelling microscope M downwards, the illuminated ring system will be visible.

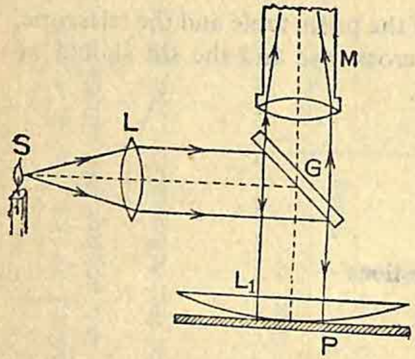

Fig. 23(*a*)

Experimental procedure : (1) Level the travelling microscope M with the help of a spirit level so that the scale on the platform becomes horizontal and the axis of the microscope vertical. Focus the cross-wire distinctly by pushing in or pulling out the focussing lens provided with the eye-piece. Determine the vernier constant of the horizontal scale of the microscope.

(2) Take out the lens and the glass plate from the metallic frame of the Newton's ring arrangement and carefully clean their surfaces with some cotton soaked in alcohol. Place them in their positions in the frame and tighten the screws. If the frame is now held in the sun rays small coloured rings will be visible. If the rings are found to be displaced from the centre of the frame, bring them at their

proper position by adjusting the screws. The centre of the ring system should coincide with the centre of the frame. Then arrange the apparatus as shown in fig. 23(*a*). When bright sodium light falls on the lens and the glass plate, a few dark and bright rings are seen when viewed from above with naked eye.

(3) Having obtained the bright rings, set the microscope properly so that its axis is directed towards the centre of the ring system. Now slowly move the microscope left and right and see whether the intersection point of the two cross-wires passes through the centre of the ring system and one of the cross-wires (the one which is perpendicular to the direction of motion of the microscope) may intersect the rings tangentially. If necessary, turn the Newton's ring frame a little clockwise or anticlockwise and ensure that the above adjustment is effected. Adjust the position of the lens *L* with respect to the source of light so that maximum number of rings are clearly visible through the microscope.

(4) Having made this preliminary adjustment, focus the microscope on to the centre of the ring system. Slowly move the microscope towards left. First few rings may be wide and indistinct. Do not take them in your observation. Count the first distinct dark ring as 1 and then go on counting the successive dark rings one by one till you reach the furthest distinct dark ring on the left. Suppose, the serial number of the furthest ring is 28. Moving the microscope slowly with the help of its tangent screw, set the cross-wire tangental to this ring. Read the horizontal scale and its vernier. Now slowly move the microscope towards *right* so that the cross-wire may be tangential to the 27th dark ring. Again read the main scale and the vernier. In this way, moving the microscope slowly towards right, readings for 4 or 5 successive dark rings (*i.e.*, upto 24th or 25th dark ring) are to be taken.

(5) Then disregarding some of the intermediate dark rings, make the cross-wire tangential again to the 8th dark ring (say) and

take the scale reading. Moving the microscope slowly towards right, go on taking readings for the successive four or five dark rings (*i.e.*, upto the 4th or 5th dark ring).

(6) Now, unfix the microscope and move it towards right slowly with hand. Crossing the centre, the microscope will now go towards the right hand side of the rings. Disregarding first few broad and indistinct rings, start counting the number from the first distinct dark ring as 1 (corresponding to the ring no. 1 on the left) and reach the furthest distinct dark ring no. 28. Clamping the microscope with its fixing screw, give it a slow motion with the help of its tangent screw until the cross-wire is tangential to the 28th dark sing on the right hand side. Take the horizontal scale and vernier reading. Moving towards the *left*, take the readings of the successive rings upto 24 or 25th ring.

(7) As before, disregarding the few intermediate rings, start taking reading with 8th ring and finish it when you reach the 4th or 5th dark ring.

(8) While tabulating the data, place the left hand and right hand readings of a particular ring side by side in a serial order and from this, find the diameter of different rings.

(9) Time permitting, repeat the whole process once again from the furthest left end to the furthest right end of the ring systems.

Measurements : (*a*) *Determination of the vernier constant of the microscope.*

The value of the smallest division of the main scale =..cm.

... vernier divisions=....main scale divisions

∴ 1 ,, ,, = ... ,, ,, ,,

=... cm.

∴ Vernier constant=... cm.

(*b*) Wave length of sodium light $(\lambda)=5896\times10^{-8}$ cm. (given) (*c*) *Determination of the diameters of the rings* (the serial numbers given are for illustration)

Serial no. of the rings (n)	Readings of microscope positions						Diameter $D = a \sim b$	Mean diameter (D)
	Left side readings			Right side readings				
	Linear scale	Vernier scale	Total reading (a)	Linear scale	Vernier scale	Total reading (b)		
No 28	.. ..	.. ..	.. ..	.. ..	... ..	.. ..	.. .. }	..
No 27 etc.	etc.			c.			etc	
No 24	..			..		..	.. .. }	..
No 8 etc.	..	..		..		..	.. .. }	..
No 4	..	..	..			..	.. .. }	..

(*d*) *Determination of the radius of curvature* [Data obtained from table (c)]

Serial no. of the rings (n)	Diameter (D)	D^2	$(D^2_{m+n}-D^2_n)$*	Mean $D^2_{m+n}-D^2_n$	m	$R=\frac{D^2_{m+n}-D_n^2}{4m.\lambda}$ (cm)
No 28	..	.. (D_{m+n})	..			
No 27	..	..	..			
No 26	..	..	..			
No 25	..	..	..			
No 24	..	..	..	..	20	
No 8	..	.. (D_n)	..			
No 7	..	..	..			
No 6	..	..	..			
No 5	..	..	..			
No 4	..	..	..			

* See Notes on page 185.

[*Note* : (*) The difference between the square of the diameter of the 28th ring and that of 8th ring gives the value of $(D^2_{m+n} - D_n^2)$ for 20 rings. Similarly, the above difference between 27th ring and the 7th ring also gives the value of $(D^2_{m+n} - D_n^2)$ for 20 rings. In this way, we may get the value of $(D^2_{m+n} - D_n^2)$ for 20 rings in the case of 26th and 6th rings and so on].

Calculations : $R = \dfrac{D^2_{m+n} - D^2_n}{4m\lambda} = \ldots$ cm.

Calculation of radius of curvature from graph :

If the number of rings (n) is plotted along OX axis and the square of the diameters (D^2) along OY axis, the graph will be a straight line AB as shown in the fig. 23(b). Take two far away points P_1 and P_2 on the straight line and draw perpendiculars P_1Q_1 and P_2Q_2 on the OX axis. If OQ_1 represents nth ring, then P_1Q_1 will represent D_n^2. Similarly, if OQ_2 represents($n+m$)th ring, then $P_2Q_2 = D^2_{n+m}$. So $P_1M = m$ and $P_2M = D^2_{n+m} - D_n^2$

$$\therefore \quad R = \frac{D^2_{n+m} - D_n^2}{4.m.\lambda} = \frac{P_2M}{4\lambda(P_1M)}$$

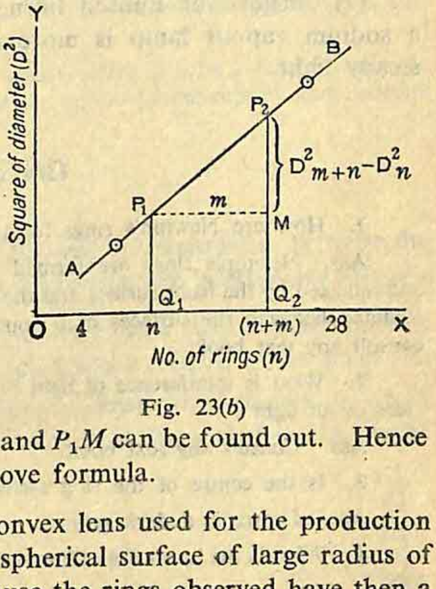

Fig. 23(b)

From the graph, the values of P_2M and P_1M can be found out. Hence R can be calculated from the above formula.

Remarks : (1) The plano-convex lens used for the production of Newton's rings should have a spherical surface of large radius of curvature. This is necessary because the rings observed have then a large diameter and hence the error in the measurement of their diameters will be minimum.

(2) It may happen that the rings seen are not circular but elliptic. This is mainly due to unclean surfaces of the lens and the plate. For this reason, the surfaces are to be cleaned before the commencement of the experiment.

(3) The centre of the ring systems, according to the theory, should be dark. Sometimes, the central spot appears bright. The reason for this is that the thickness at the point of contact, instead of being zero, has a small value.

(4) A few rings near the central spot are rather indistinct and wide. These rings should not be taken into consideration.

(5) Since the slow motion screw of the microscope is used in measurements, it should be initially checked whether its motion covers the diameters of the rings to be measured. Usually the working length of the tangent screw is about 2 cm and it may be, if the above precaution is not taken, that by the time, the measurements are completed, the working length of the screw is exhausted. As a result, the whole set of observations has to be rejected. Such a precaution is, however, not necessary in a microscope having tangent screw for its entire working length of nearly 15 cm.

(6) While moving the microscope right or left, the cross-wire should intersect the rings tangentially.

(7) Instead of Bunsen burner impregneted with common salt, a sodium vapour lamp is more convenient. It gives bright and steady light.

Oral questions

1. How are Newton's rings formed ?

Ans. Newton's rings are formed due to interference between two rays—one reflected by the front surface and the other by the back surface of a film of air confined between the surfaces of a convex lens and a glass plate. For details. consult any text book.

2. What is interference of light ? What conditions are required for interference of light ?

Ans. Consult any text book.

3. Is the centre of the ring system dark or bright ?

Ans. Centre is a dark point.

4. What is the harm if an illuminated slit is used instead of a wide source of light ?

Ans. A part of the ring will be visible.

5. Sometimes you see elliptical rings instead of circular rings. What is the reason ?

Ans. See remark no. 2.

6. What is the harm if white light is used instead of monochromatic light ?

Ans. The rings will be coloured and few in number.

7. Why do you use a convex lens of large radius of curvature in producing Newton's rings ?

Ans. See remark no. 1.

8. On what factors does the diameter of the rings depend ?

Ans. The diameter depends on (*i*) the serial number of the ring (*ii*) the wavelength of the light used (*iii*) the radius of curvature of the curved surface of the lens

9. Can you find out the wave length of light used by this experiment ?

Ans. Yes ; in that case, the radius of curvature of the curved surface of the lens is to be determined by a spherometer and then from the formula $\lambda = \frac{D^2_{m+n} - D_n^2}{4m.R}$, the wave length is to be calculated.

10. Can you find out the refractive index of a liquid by this experiment ?

Ans. Yes ; in that case, a few drops of the liquid are to be poured on the glass plate and over the drops, the lens should be placed. Newton's ring will be formed in the film of liquid. Applying the formula $\mu = \frac{4m\lambda R}{D_{m+n}^2 - D_n^2}$, the refractive index can be found out.

11. The central spot of the ring systems, according to the theory, should be dark. If you get a bright central point, will your measurement for radius of curvature be correct ?

Ans. If the rings are circular, the measurement will be correct because the value of $(D_{n+m}^2 - D_n^2)$ is independent of the nature of the central spot, whether dark or bright.

3.20. Determination of the slit width and the separation between the slits of a double slit by observing the diffraction and interference fringes.

Apparatus : A spectrometer, a double slit, a strip of plane mirror, reading telescope with lamp and scale, sodium light, metre scale etc.

Theory : A parallel beam of monochromatic rays coming from the collimator of a spectrometer, is incident on a double slit placed on the prism table of the spectrometer parallel to the collimator slit. The distance between the slits is b and the width of each slit is a. If the emergent light be received through the telescope of the spectrometer (which is focussed for parallel rays), diffraction fringes will be seen, which will overlap the interference fringes [Fig. 24 side].

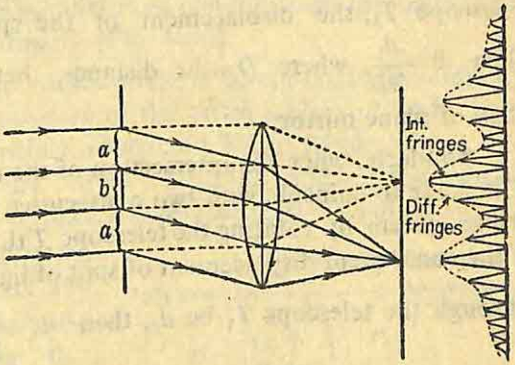

Fig. 24

If θ be the angular width between two consecutive diffraction fringes and ϕ that between two consecutive interference fringes, it can be proved that $a.\theta=\lambda$ or $a=\dfrac{\lambda}{\theta}$ ... (i) where λ=wave length of the incident light.

Also, $(a+b).\phi=\lambda$ or $(a+b)=\dfrac{\lambda}{\phi}$ ... (ii)

Since θ and ϕ are very small angles, a reading telescope with lamp and scale arrangement is used for their measurement. In this arrangement, a strip of plane mirror M is fixed on the arm of the spectrometer telescope (T). The image of a scale formed by the plane mirror is seen through the reading telescope T_1 [Fig 24(*a*)]. When the intersection of the cross-wires of the telescope T is made to coincide with two consecutive dark bands of diffraction fringe system by rotating the telescope through a small angle θ, the spot of reflected light will move a certain distance over the scale. If looking through the reading telescope T_1, the displacement of the spot of light observed is d_1, then $\theta=\dfrac{d_1}{2D}$, where D=the distance between the scale and the strip of plane mirror.

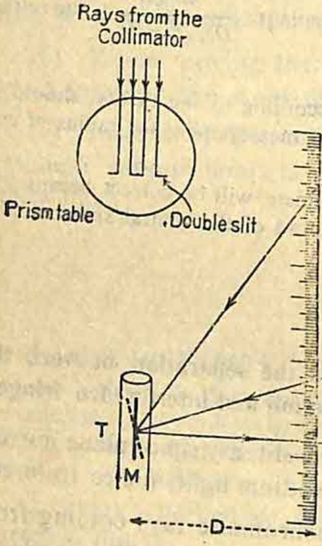

Fig. 24(*a*)

Similarly, when the intersection of the cross-wires of the telescope T is made to coincide with two consecutive dark bands of interference fringe system by rotating the telescope T through a small angle ϕ and if the consequent displacement of spot of light on the scale as observed through the telescope T_1 be d_2, then $\phi_2=\dfrac{d_2}{2D}$.

Hence, $a=\dfrac{2D\lambda}{d_1}$ .. (iii) and $(a+b)=\dfrac{2D.\lambda}{d_2}$ .. (iv).

Knowing D, λ, d_1 and d_2, a and b can be calculated from the above equations.

Experimental procedure : (1) Take a spectrometer and perform all the adjustments, including focussing for parallel rays, as described in art 3.13. Removing the prism from the prism-table, bring the telescope (T) in line with the collimator and allow the rays coming direct from the collimator to enter into the telescope. By adjusting the focussing lens of the telescope eye-piece, focus the cross-wires distinctly and set the vertical cross-wire parallel to the collimator slit.

(2) Attach a small strip of plane mirror (M) on the body of the telescope T with the help of a piece of rubber girder so that the plane mirror is vertical. Place a scale and a telescope (T_1) at some distance (about 2 metres) from the plane mirror. The scale consists of a white glass plate with black graduations. It is so set that its plane is parallel to the plane of the mirror (M). Illuminate the scale with a suitably covered lamp. Focus the cross-wires of the telescope T_1 and adjust the height of the scale such that looking through the telescope T_1, an image of the scale is seen. In this condition, if there be any angular displacement of the plane mirror M, there will be corresponding linear displacement of the scale divisions which can be measured with the help of the cross-wires of the telescope T_1.

Measurement of (*a*) *from diffraction fringes* :

(3) Place the double slit stand on the prism table so that the centre of the stand coincides with the centre of the prism table and the plane of the slit is vertical. Turning the screws below the prism table make the plane of the double slit parallel to the cross-wires of the telescope T. Now, turning the prism table, set the double slit in such a position that the parallel rays coming from the collimator are incident on the slit normally [Fig 24(*a*)].

(4) Arrange the position of the sodium vapour lamp, the width of the collimator slit and the position of the prism table, so that looking through the telescope T, distinct, sharp and bright fringes—both diffraction and interference—are visible. On both sides of the central bright band, equal number of dark fringes (diffraction) will be clearly seen and in the central bright band, equally spaced alternate dark and bright interference fringes will also be visible. [Right side of fig 24].

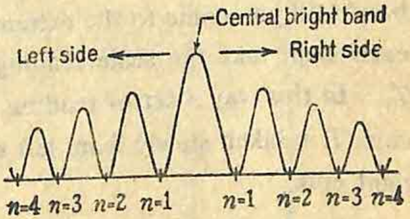

Fig. 24(*b*)

(5) Place the intersection of the cross-wires of the telescope T on the first order *dark* band ($n=1$) on the left side of the central bright diffraction band [Fig 24(*b*).]. Note the scale reading which coin-

cides with the intersection of the cross-wires of the telescope T_1. Now slowly swing the telescope T towards the right of the central bright band and set the intersection of its cross-wires on the first order dark band ($n=1$). Again note the reading on the scale which coincides with the intersection of the cross-wires of the telescope T_1. Difference of these two readings divided by 2 gives the value of d_1 for the first order dark band of the diffraction pattern.

(6) In this way, the cross-wires of the telescope T are to be made coincident with the 2nd, 3rd etc orders (as many as are distinctly visible) of dark bands on both sides of the central bright band and the difference of scale readings available from the reading telescope T_1 are to be found out. Dividing the difference of the scale readings by $2n$ (*i.e.* twice the number of order), the value of d_1 will be obtained for each order.

(7) Calculate the mean value of d_1. Find the distance (D) between the plane mirror M and the scale with the help of a metre scale. Then applying the equation (iii) calculate the value of a.

Determination of $(a+b)$ *from interference fringes* :

(8) Interference fringes are visible within the central bright band of diffraction pattern. The width of the interference fringes is evidently much smaller than that of the diffraction fringes. Set the junction of the cross-wires of the telescope T on the centre of the extreme left bright band. Looking through the telescope T_1 read the scale mark which coincides with the junction of the cross-wires of the telescope T_1. Now slowly turn the telescope T so that the junction of its cross-wires successively coincides with 2nd, 3rd, etc. bright bands till you come to the extreme bright band on the right side. In each step, take the scale reading as observed through the telescope T_1. In this way a set of reading (y_1) will be obtained when the telescope T is taken slowly from left end of the interference fringe to the right end.

(9) Another set of readings (y_2) will have to be taken in the same way. In this case, the telescope T should be taken from right end of

the fringe to the left end and in each step, the reading telescope T_1 should be used to take the scale-reading. (The serial number of fringes will, however, be counted from the left end in both the cases). From these two sets, the mean scale reading for each bright band is to be ascertained.

(10) From the mean readings so obtained, find the difference of the readings between each 5 sets of bands [The difference between the 1st and 6th band, that between 2nd and 7th band and so on.]. Finding the mean value of these 5 sets of band and dividing it by 5, correct value of d_2 will be obtained.

(11) Determine the value of $(a+b)$ from the equation (iv) mentioned in the theory. Then find the value of b from the values of $(a+b)$ and 'a'.

Measurements : The wave length of the monochromatic sodium light used $\lambda=5896\times10^{-8}$ cm. (given).

The distance between the scale and the plane mirror M

$(D)=$(i) ... (ii) ... (iii) ...

Mean distance $(D)=$... cm.

(*a*) *Measurement of diffraction fringes* :

Serial no of dark fringes. (n)	Scale reading when the cross-wires of the telescope T coincide with the dark bands		Diff. of scale readings $x=x_1\sim x_2$ cm	$d_1=\frac{x}{2n}$	Mean d_1
	On the left of central bright band (x_1)	On the right of central bright band (x_2)			
1.	..	..	..	..	
2.	..	..	..	..	
3.	..	..	..	..	..
4.	..	..	..	..	

(*b*) *Measurement of interference fringes* :

Serial no. of bright bands from left	Scale readings when the cross-wires of the telescope T coincides with bright band			Difference of readings of 5 bands (x) cm	Mean x (cm)	$d_2=\frac{x}{5}$ cm.
	From left to right (y_1)	From right to left (y_2)	mean reading $y=(y_1+y_2)/2$			
1.	..	..	..(y_1')	$y_6' \sim y_1'=$..		
2.	..	..	..(y_2')	$y_7' \sim y_2'=$..		.
3.	..	..	..(y_3')	$y_8' \sim y_3'=$..	..	
4.	..	..	..(y_4')			
5.	..	..	..(y_5')	etc		
6.	..	..	..(y_6')			
etc			etc			

Calculations :

(i) $a=\frac{2D.\lambda}{d_1}=$ ... cm, (ii) $a+b=\frac{2.D.\lambda}{d_2}=$ ...cm.

$\therefore \quad b=(a+b)-(a)=$... cm.

Calculation of proportionate error [for Honours students only] :

$$\left(\frac{\delta a}{a}\right)_{max}=\frac{\delta D}{D}+\frac{\delta d_1}{d_1}$$ [λ, being given, is assumed to be correct]

In one observation, D was 250 cm and d_1 was 1·2 cm. Both quantities being measured by a metre scale, $\delta D=0{\cdot}1$cm and $\delta d_1=0{\cdot}1$ cm.

$$\therefore \quad \left(\frac{\delta a}{a}\right)_{max}=\frac{0{\cdot}1}{250}+\frac{0{\cdot}1}{1{\cdot}2}$$

$$={\cdot}000004+{\cdot}08333$$

$$={\cdot}083334$$

It is clear that the greatest error comes in the measurement of width of diffraction fringes. The percentage error $=8{\cdot}33\%$.

Remarks : (1) The source of sodium light should have sufficient intensity, otherwise, fringes are not clearly visible. (2) The distance between the scale and the plane mirror should at least, be 2 metres. More the distance, more is the displacement of the scale image and less the inaccuracy in measuring it. (3) The width of each slit and the distance between the slits may be measured by a microscope and the above results may be compared with the result obtained from microscope measurement. (4) In taking the scale reading with the help of the telescope T_1, parallax between the image of the cross-wires and that of the scale must be avoided. (5) In the case where $a=3$mm and $b=1{\cdot}2$mm, and the distance (*D*) between the scale and the mirror $=250$ cm (nearly), the displacement of the scale image for diffraction fringes is found to be about 1·2cm and that for interference fringes about 3mm ; in such case, 8 or 9 bright interference fringes will be available. (6) Instead of sodium vapour lamp, powerful electric bulb (with straight filament) and red filter may be used. The wave length of red light is $\lambda=6400\times10^{-8}$ cm (nearly). (7) The fringe pattern depends on the ratio of *a* and *b*. The number of fringes increases if *a* is small and *b* is large but in that case, the intensity of the fringes diminishes.

Oral questions

1. What is diffraction ? What is the difference between interference and diffraction ?

Ans. Consult any text book.

2. How does a double slit produce diffraction and interference fringes ?

Ans. While passing through each slit, the beam of light is diffracted and the diffracted beam produces the diffraction fringes. The two diffracted beam coming from the two slits get superposed and hence produce interference fringes.

3. What will happen if the two slits of the double slit arrangement combine ?

Ans. It will cease to remain a double slit and will be converted into a single slit of width 2*a*. The interference fringers will disappear and only the diffraction fringes will be seen.

4. What happens if the width of each slit is increased gradually ?

Ans. The width of diffraction fringes decreases but finally the fringes disappear.

5. What happens when distance between the slits (*b*) is gradually increased ?

Ans. The fringes become gradually narrow and finally they disappear.

6. What happens when the collimator slit is widened ?

Ans. If the collimator slit is widenned, the intensity of the interference fringes increases but the sharpness decreases, and finally the interference fringes disappear at a limiting value of the collimator slit width. The width of the collimator slit should be much narrower compared to the width of each slit of the double slit arrangement.

7. What happens when red light is used instead of sodium light ?

Ans. The wave length of red light is greater than that of the yellow light of sodium. As a result, the angular widths of each two consecutive diffraction fringes as well as interference fringes increases.

8. Why, in the above experiment, readings are taken with the help of reflected light instead of the spectrometer scale ?

Ans. The width of diffraction or interference fringes is usually very small. Error will come in if such small widths are measured by spectrometer scale. Lamp and scale together with reading telescope arrangement is convenient

9. In the case of diffraction fringes, the width of dark bands is measured but in the case of interference fringes, the width of bright bands is measured. Why is this difference ?

Ans. In the diffraction fringes, the dark bands are equally spaced with respect to the central band but the bright bands are not equally spaced. In interference fringes, both the dark and bright bands are equally spaced.

10. Are the interference fringes equally spaced ?

Ans. Yes, the distance between two consecutive dark or bright bands is everywhere the same.

3.21. Determination of the width of a narrow slit by diffraction method.

Apparatus : A spectrometer, a single slit (preferably adjustable), a strip of plane mirror, lamp and scale with reading telescope, a source of monochromatic light (say sodium vapour lamp), metre scale etc.

Theory : A long narrow slit *S* illuminated by monochromatic light sends out rays which are rendered parallel by a convex lens L_1 so that the rays are incident normally on a narrow slit *AB* of width

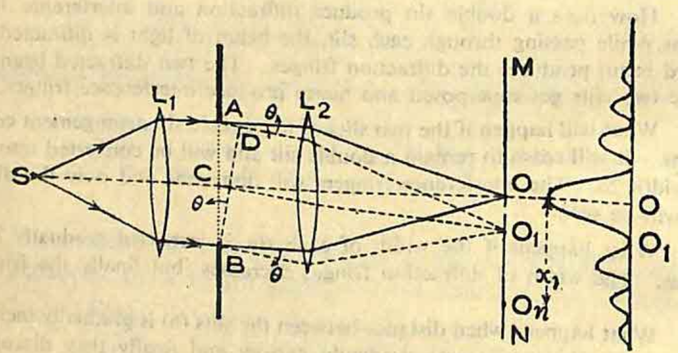

Fig. 25

a. If the light passing through the slit *AB* is received on a screen, diffraction fringes consisting of alternate dark and bright bands [Fig 25] on both sides of the central image *O* of the slit will be found.

If θ_n be the angle of diffraction of the nth *dark band* on the left or right of the central bright band, then it can be shown that $a \sin \theta_n = n\lambda$. Since θ_n is very small, we have $a\,\theta_n = n\lambda$ or $a = \frac{n\lambda}{\theta_n}$. Hence, if θ be the angular width of any two successive dark bands either on the right or on the left of the central bright band, then $a = \frac{\lambda}{\theta}$ ... (i)

Since θ is very small, a reading telescope with lamp and scale arrangement is made for its measurement [Fig 24(*a*)]. As in experiment no 3.20, when the intersection of the cross-wires of the telescope T is made to coincide with two consecutive dark bands by rotating the telescope T through a small angle θ, the spot of reflected light moves over a certain distance over the scale. If looking through the reading telescope T_1, the displacement of the spot of light observed is d, then $\theta = \frac{d}{2D}$, where D = the distance between the scale and the strip of the plane mirror.

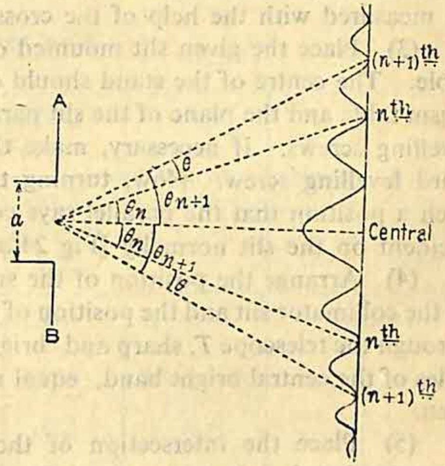

Fig. 25(*a*)

$$\therefore \quad a = \frac{2D.\lambda}{d} \quad .. \quad \text{(ii)}$$

Knowing D, λ and d, the width a of the slit can be found out.

Experimental Procedure : (1) Take a spectrometer and perform all the adjustments including focussing for parallel rays, as described in art 3.13. Removing the prism from the prism-table, bring the telescope T in line with collimator. Illuminate the collimator slit with sodium light (if not used earlier for the adjustment of the spectrometer) and allow the rays coming direct from the collimator to enter into the telescope. By adjusting the focussing lens of the telescope eye-piece, focus the cross-wires distinctly and set the vertical cross-wire parallel to the collimator slit.

(2) Attach a small strip of plane mirror [M, fig 24(a)] on the body of the telescope T with the help of a piece of rubber girder so that its plane is vertical. Place a scale and a telescope T_1 at some distance (about 2 metres) from the plane mirror. The scale consists of a white glass plate with black graduations. It is so set that its plane is parallel to the plane of the mirror M. Illuminate the scale with a suitably covered electric lamp. Focus the cross-wires of the telescope T_1 and adjust the height of the scale such that looking through the telescope T_1 an image of the scale is seen. In this condition if there be any angular displacement of the plane mirror M, there will be corresponding linear displacement of the scale divisions which can be measured with the help of the cross-wires of the telescope T_1.

(3) Place the given slit mounted on suitable stand on the prism table. The centre of the stand should coincide with the centre of the prism table and the plane of the slit parallel to the line joining the two levelling screws. If necessary, make the slit vertical by turning the third levelling screw. Now, turning the prism table set the slit in such a position that the parallel rays coming from the collimator are incident on the slit normally [Fig 24(a)].

(4) Arrange the position of the sodium vapour lamp, the width of the collimator slit and the position of the prism table, so that looking through the telescope T, sharp and bright fringes are visible. On both sides of the central bright band, equal number of dark fringes will be seen.

(5) Place the intersection of the cross-wires of the telescope T on the first dark band on the left side of the fringes. Note the scale reading which coincides with the intersection of the cross-wires of the telescope T_1. In this way, take the scale readings for the successive dark bands on the left hand side till you come to the furthest left hand side but most distinct dark band (say 8th band). Now swing the telescope T to the right hand side of the central bright band. Place the intersection of the cross-wires of the telescope T on the first dark band. Note the scale reading which coincides with the intersection of cross-wires of the telescope T_1. As before, take the scale readings for the successive dark bands on the right hand side till you come to 8th band (say).

(6) Repeat the whole operation, this time starting from the right hand side 8th band and ending on the left hand side 8th band (say). Place the readings obtained for each band side by side in a table and find the mean value of the readings for each band.

(7) The difference of the mean scale readings between 5 dark bands is found out in several instalments from which average value

of 5 dark bands may be obtained. Dividing the average value obtained by 5, width of successive dark bands may be calculated.

(8) Find the distance (D) between the plane mirror M and the scale with the help of a metre scale.

Measurements : The wave length of the light used (Sodium) $=5896\times10^{-8}$ cm.

The distance between the scale and the plane mirror M

=(i)... (ii)... (iii)...

Mean distance (D)=... cm.

(*a*) *Measurement of fringe width* :

Order no of fringes	Scale readings as observed thro' the telescope T_1 (cm.)			Difference for 5 bands (cm.)	Mean diff. for 5 bands (R)	Band width $d=R/5$ (cm.)
	First operation (a)	Second operation (b)	Mean $\left(\frac{a+b}{2}\right)$			
1	..	..	..(x_1)	x_8-x_3..		
2	..	..	..(x_2)			
3	..	..	..(x_3)	$x_7-x_2=$..		
⋮						
8	..	..	..(x_8)	$x_6-x_1=$..		
Central bright band (0)	—	—	—		..	..
1	..	..	..(y_1)	$y_8-y_3=$..		
2	..	..	..(y_2)			
3	..	..	..(y_3)	$y_7-y_2=$..		
⋮						
8	..	..	..(y_8)	$y_6-y_1=$..		

Calculation : $a=\frac{2D.\lambda}{d}=\ldots$ cm.

Remarks : (2) In the diffraction fringe system, the dark bands are equally spaced. Hence the angular width of successive dark bands should be taken. (2) The width of the slit can be found out with a microscope and the result may be compared with that obtained from diffraction experiment . For other remarks, see expt no. 3·20.

Oral questions

1. Can you use any other light source in this experiment ?
Ans. Any monochromatic but bright source will do.
2. Should you keep the collimator slit wide ?
Ans. No ; a very wide slit will make the fringes blurred.
3. Should you keep the given slit wide ?
Ans. No : a wide slit will not produce diffraction fringes.

[*Questions on Expt* 3.20 *are also applicable here*]

3.22. Optical activity.

Polarised light and plane of polarisation : Ordinary light may be regarded as transverse waves due to vibrations of particles of the medium in different directions in a plane perpendicular to the direction of propagation [Fig 26]. This beam of light is also called unpolarised beam of light. When such unpolarised beam of light is passed through a tourmaline crystal, it becomes *polarised i.e.* only those vibrations of light which are parallel to the optic axis of the crystal are transmitted.

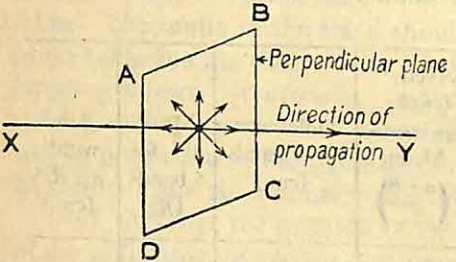

Fig. 26

The plane in which the vibrations of a polarised beam are confined, is called the *plane of vibration*. A plane at right angles to the plane of vibration is called the *plane of polarisation*. In fig. 26(*a*) a polarised beam transmitted through a tourmaline crystal *AB* has been shown. A plane *ABCD* containing the vibrations of the polarised beam and passing through *OG*, the direction of propagation of the beam is the plane of vibration. *EFGH* represents the plane of polarisation.

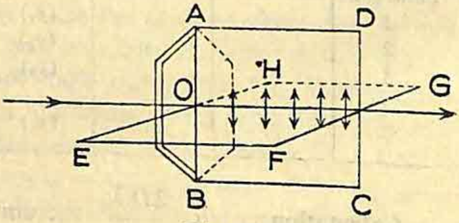

Fig 26(*a*)

Optical activity : If two nicol prisms N_1 and N_2 be kept crossed and a beam of monochromatic light be sent through them, no light emerges from the second nicol N_2. Light, in passing through the nicol N_1 is plane-polarised with vibrations parallel to principal section of N_1 but as the principal section of N_2 is perpendicular to these vibrations, the nicol N_2 obstructs the light. If now, a calcite plate P cut with its optic axis perpendicular to its refracting face, be interposed between N_1 and N_2, no change takes place *i.e.* light remains obstructed by N_2 as before. This shows that the calcite plate cannot bring about any change in the state of polarisation of the beam. If, instead of calcite plate, a similarly cut quartz plate be interposed, some light will come out of the nicol N_2 [Fig 26(*b*)]. The light can be completely extinguished by turning the nicol N_2 a little more from the crossed position. This shows that the beam after coming out of the quartz plate remains plane-polarised but the plane of polarisation has turned through to some extent. The angle through which the analysing nicol N_2 has to be rotated further in order to obtain complete extinction of light, gives the angle of rotation of the plane of polarisation.

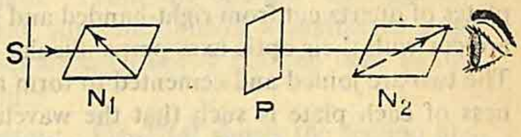

Fig. 26(*b*)

The phenomenon is known as *optical activity*. The substances which bring about optical activity are called optically active substances. Quartz is an optically active substance but quartz when fused loses its optical activity. This shows that optical activity depends on the molecular arrangement of the substance.

Classification : Optically active substances are divided into two classes :

(i) *Dextro-rotatory* : Rotation is said to be dextro or right-handed if, on looking along the path of the beam towards the source, the rotation appears to be clockwise. The substances causing dextro-rotation are known as dextro-rotatory substances. Some quartz crystals, cane sugar etc are the examples.

(ii) *Laevo-rotatory* : Rotation is said to be laevo or left-handed if, on looking along the path of the beam towards the source, the rotation appears to be anti-clockwise. The substances causing laevo-rotation are known as laevo-rotatory substances. Some form of quartz crystals, fruit-sugar etc are the examples.

Half-shade plate : It is a round plate, one half of which is made of quartz and the other half by glass. The quartz plate is cut with its optic axis parallel to the line of contact between the quartz and glass and its thickness is such that it introduces a path-difference of $\lambda/2$ between the *O*- and *E*-rays. The quartz plate, in fact, is a half-wave plate. The thickness of the glass plate is such that it absorbs the same amount of light as the quartz plate does.

Bi-quartz : One of the disadvantages of the half-shade plate is that it cannot be used with any other wave length of light except that for which it is prepared. Bi-quartz is free from this disadvantage, as it can be used even with white light. Quartz crystal may be dextro-rotatory or laevo-rotatory. A bi-quartz consists of two semi-circular plates of quartz cut from right-handed and left-handed samples. They are cut with their optic axes perpendicular to their refracting surfaces. The two are joined and cemented to form a circular plate. The thickness of each plate is such that the wavelength corresponding to the yellow colour is rotated equally in the opposite directions through 90°

If the incident light be *white*, different wave lengths of the incident light will have different rotation. Only the wavelength corresponding to the yellow light will have 90° rotation and will be totally obstructed by the analyser. The emergent light in which yellow light will be missing, will produce a dim grey-violet tint in both the halves, which is called *sensitive tint* or *tint of passage*.

If the position of the analyser is slightly towards right or left, one half will appear bluish and the other reddish with a marked line separating the two. In order to measure the specific rotation, the position of the analyser should be so adjusted that both the halves of the field of view get the tint of passage when some optically active substance is taken in the tube as well as when the tube contains distilled water.

3.23. Calibration of a polarimeter and hence determination of the concentration of sugar solution (Mother solution to be supplied)

Apparatus : Polarimeter with half shade plate or bi-quartz, sodium vapour lamp (for use with half shade plate) or an electric bulb (for use with bi-quartz), pipette, measuring flask with graduations in c.c., beaker etc.

Theory : For a given temperature and a given wavelength of incident light, specific rotation of a substance is defined as the rotation produced by 1 decimeter (*i.e.* 10 cm) of the substance of unit density (for solid) or of the solution of unit concentration (*i.e.* 1 gm. of the optically active substance in 1 c.c. of the solution).

Suppose θ is the total rotation of the plane of polarisation of a plane-polarised beam when it travels a length of l cm (*i.e.* $\frac{l}{10}$ decimeter) of a solution of concentration c (*i.e.* c gm. of the substance present in 100 c.c. of the solution). Then

$$\theta = \frac{s.l.c.}{1000} \quad \ldots \quad \text{(i)}$$

where s is the specific rotation of the solution.

It is clear that a graph drawn between θ and c will be a straight line, passing through the origin. From the straight line graph, s can be found out. Then if the concentration of a sample of sugar solution be not known, the equation (i) above may be used to find out the unknown concentration.

Description of the polarimeter : Fig 27(*a*) shows the section and fig 27(*b*) the actual appearance of the apparatus. In general, it consists of a source of light S, a polariser N_1 and an analyser N_2 provided with a graduated circular scale. The source of light is so placed that it is nearly at the focus of a convex lens L, thus providing a parallel beam of light. The light falls on the polarizing nicol N_1 which produces plane polarised beam. The polarising nicol is immediately followed by either a half shade plate or a biquartz H. The analysing nicol N_2 is in front of a low power telescope E. In between the polarising arrangement and the analysing nicol, is placed a tube T containing the optically active liquid (sugar solution in the present case). The tube is closed on both sides by glass plates which are optically worked and held at the ends of the tube with metal caps. The analysing nicol N_2 and the telescope E are together enclosed in a tube which can rotate about the axis of the instrument. The angle of rotation is measured with the help of the circular scale and the verniers V_1 and V_2.

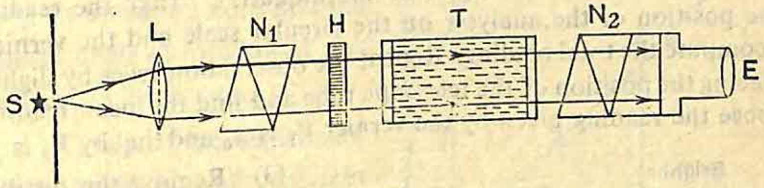

Fig.. 27(*a*)

Experimental procedure : (1) Find the vernier constant of the verniers. Find the distance between the plane glass plates which close the two ends of the polarimeter tube T.

(2) Clean the polarimeter tube with distilled water. Fill the tube with distilled water taking care that there is no air bubble when the end caps have been screwed. Place the polarimeter tube in its proper position. Put the sodium vapour lamp (in the case of half-

shade plate) or a bright electric lamp (in the case of biquartz) behind the slit. Looking through the telescope E, one half of the field of view in general, is found brighter than the other (in the case of half shade plate) as shown in fig. 27(c)] or one half appears red while the other half blue (in the case of bi-quartz). Rotate the tube containing the analyser and the verniers till the two portions of the field of view are equally bright (for the half shade) or aquire the same tint, called the tint of passage (for the biquartz). Take the reading of the position of the analyser on the circular scale and the verniers and compute the total reading. Repeat the observation twice by slightly displacing the position of the telescope tube and find the mean reading. Suppose the reading given by the vernier V_1 is x_0 and that by V_2 is y_0.

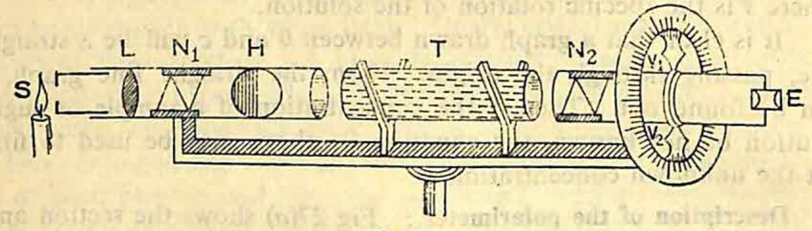

Fig. 27(b)

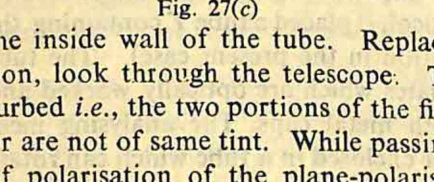

Fig. 27(c)

(3) Remove the distilled water from the tube and rinse the tube with some of the mother solution given to you. Fill it completely with the solution (say $C_1\%$). Take care that no air bubble sticks to the inside wall of the tube. Replacing the tube at its proper position, look through the telescope. The previous setting may be disturbed *i.e.*, the two portions of the field of view are not equally bright or are not of same tint. While passing through the solution, the plane of polarisation of the plane-polarised light will be rotated and the previous setting will be disturbed. Again turn (either clockwise or anti-clockwise) the telescope tube till the two portions of the field of view are equally bright (for half shade) or acquire the grey-tint (for biquartz). Take the readings of the vernier scales against the circular scale. Repeat the operation twice and compute the mean reading. Suppose x_1 is the reading given by the vernier V_1 and y_1 the reading given by the vernier V_2. The rotation of the plane of polarisation, in this case, as given by the vernier V_1 is $\theta_1=(x_1\sim x_0)$ and as given by the vernier V_2 is $\theta_2=(y_1\sim y_0)$. Find the mean of

these two rotations. This gives the angle of rotation $[\theta=\frac{1}{2}(\theta_1+\theta_2)]$ of the plane of polarisation for the solution of concentration C_1%.

(4) Now, a solution of lower concentration C_2% is to be prepared from the mother solution of C_1% concentration. Suppose n c.c. of distilled water is to be mixed with m c.c. of C_1% solution to make the concentration C_2%. Then $n=\frac{C_1-C_2}{C_2}\times m$ c.c. So take m c.c. of mother solution (C_1%) in a pipette and put it in a beaker. Rinse the pipette with distilled water and take n c.c. of distilled water* in it and mix it with the solution of the beaker. Now, fill up the polarimeter tube with some of the newly prepared solution completely and following the operation no. 3 find the mean angle of rotation of the plane of polarisation.

(5) Repeat the experiment with solutions of different strengths (decreasing the strength by steps of 2% from say 16% to 4%). Plot a graph between the angle of rotation along the Y-axis and the concentration along the

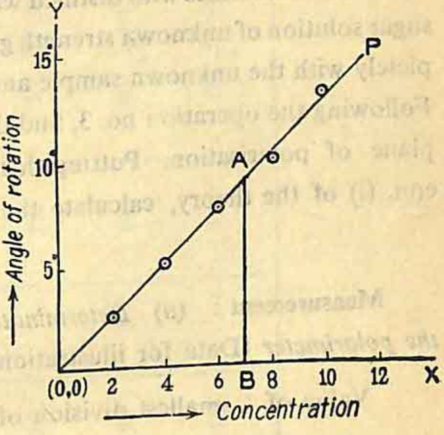

Fig. 27(*d*)

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*C_1% of a solution, by volume, means that there are C_1 gm of sugar in 100 c.c. of the solution. So m c.c. of the solution contains $\frac{mc_1}{100}$ gm of sugar. If n c.c. of distilled water is mixed with m c.c. of the solution, then there will be $\frac{mc_1}{100}$ gm. of sugar in $(m+n)$ c.c. of the solution. Hence, 100 c.c. of the solution contains $\frac{mc_1\times 100}{100(m+n)}=\frac{mc_1}{m+n}$ gm. This is C_2% solution. In other words, $C_2=\frac{mc_1}{m+n}$ $\therefore$ $n=\frac{c_1-c_2}{c_2}\times m$.

Suppose 25 c.c. of 12% solution is to be converted into 10% solution. The required volume of distilled water to be mixed is $=\frac{(12-10)}{10}\times 25=5$ c.c. and the volume of this solution $=25+5=30$ c.c.

X-axis. The graph is a straight line passing through the origin [Fig. 27(*d*)]. Take any point *A* on the straight line and draw a perpendicular *AB* on the *X*-axis. *AB* represents a certain angle of rotation (θ) and *OB* the corresponding strength (c) of the solution. Putting these values in equation (i) of the theory, the specific rotation (s) can be calculated. The straight line graph is called the *calibration curve* of the polarimeter.

(6) Now throw away the solution of the polarimeter tube. Clean the tube several times with distilled water and rinse it with some of the sugar solution of unknown strength given to you. Fill the tube completely with the unknown sample and place it at its proper position. Following the operation no. 3, find the mean angle of rotation of the plane of polarisation. Putting this value of the angle of rotation in eqn. (i) of the theory, calculate the strength of the solution.

Measurement : (*a*) *Determination of the vernier constant of the polarimeter* (Data for illustration) :

Value of 1 smallest division of the circular scale = 1°

10 vernier divisions = 9 smallest divisions of the circular scale

or, 1 ,, ,, $=\frac{9}{10}$,, ,, ,, ,,

$$\therefore \text{Vernier constant} = \left(1-\frac{9}{10}\right) = \frac{1}{10}\text{th. of a degree} = 6 \text{ minutes.}$$

(*b*) Length of the tube (l) = (i) 20 cm. (ii) 20 cm. (iii) 20 cm.
Mean length = 20 cm.

(*c*) Wave length of light used (for half shade plate) = ... A°

(*d*) Temperature at the time of experiment = ... °C.

(*e*) *Vernier reading when the tube is filled with distilled water* : (Data given as illustration)

No of Obs.	First vernier (V_1) readings				Second vernier (V_2) reading (180° apart)			
	Circular scale	Vernier	Total	Mean (x_0)	Circular scale	Vernier	Total	Mean (y_0)
1.	5	$2\times\frac{1}{10}=0{\cdot}2$	5·2	5·2	185	$5\times\frac{1}{10}=0{\cdot}5$	185·5	185·5
2.	5	$1\times\frac{1}{10}=0{\cdot}1$	5·1		185	$4\times\frac{1}{10}=0{\cdot}4$	185·4	
3.	5	$2\times\frac{1}{10}=0{\cdot}2$	5·2		185	$5\times\frac{1}{10}=0{\cdot}5$	185·5	

(*f*) *Preparation of stock solution of* $C_1\%$ (16%) *strength* :

Mass of sugar taken (m gm)	Volume of distilled water added	Volume of solution (V)	Strength $C_1=\frac{m}{V}\times 100\%$
10 gm+5 gm+1 gm =16 gm	.. c.c.	100 c.c.	16%

(g) *Preparation of solutions of different strengths from the stock or mother solution.*

Concentration required	Volume of stock solution taken (m)	Strength of stock solution ($C\%$)	Volume of distilled water to be added $n=[(c_1-c_2)\times m]/c_2$ c.c.
$C_2\%$ (14%)	42 c.c.	16%	6 c.c.
$C_3\%$ (12%)	48 c.c.	14%	8 c.c.
$C_4\%$ (10%)	40 c.c.	12%	8 c.c.
$C_5\%$ (8%)	32 c.c.	10%	8 c.c.
$C_6\%$ (6%)	30 c.c.	8%	10 c.c.

(*h*) *Vernier readings when the tube is filled up with solutions of different known strengths*

Strength of the solution	Readings of vernier V_1 in degree				Readings of vernier V_2 in degree				Angle of rotation from vernier V_1 $(\theta_1=x_1\sim x_0)$ degree	Angle of rotation from vernier V_2 $\theta_2=(y_1\sim y_0)$ degree	Mean rotation $\theta=\frac{1}{2}(\theta_1+\theta_2)$ degree
	Cir. scale	Ver. scale	Total	Mean (x_1)	Cir. scale	Ver. scale	Total	Mean (y_1)			
16%	27	$6\times\frac{1}{10}$ =0·6	27·6		207	$5\times\frac{1}{10}$ =0·5	207·5				
	27	$5\times\frac{1}{10}$ =0·5	27·5	27·5	207	$4\times\frac{1}{10}$ =0·4	207·4	207·5	27·5−5·2 =22·3	207·5−185·5 =22·0	22·15
	27	$4\times\frac{1}{10}$ =0·4	27·4		207	$5\times\frac{1}{10}$ =0·5	207·5				
	24	$2\times\frac{1}{10}$ =·2	24·2		204	$5\times\frac{1}{10}$ =0·5	204·5				
14%	24	$1\times\frac{1}{10}$ =0·1	24·1	24·2	204	$6\times\frac{1}{10}$ =0·6	204·6	204·5	24·2−5·2 =19·00	204·5−185·5 =19·00	.. 19·00
	24	$3\times\frac{1}{10}$ =0·3	24·3		204	$5\times\frac{1}{10}$ =0·5	204·5				
etc	etc	etc				..		..			
6%	..	..	..	..	..	..	..	..	..	..	..

(*i*) *Table for drawing graph* [Data taken from the table (*h*)]

Strength of the solutions (%)	16%	14%	12%	10%	8%	6%
Angle of rotation (θ)	22·15	19·00	..	..	..	..

(*j*) *Vernier readings when the tube is filled up with the unknown solution :*

Readings of vernier V_1				Angle of rotation $(\theta_1 = x \sim x_0)$	Readings of vernier V_2				Angle of rotation $\theta_1 = y \sim y_0$	Mean rotation $\theta = \frac{1}{2}(\theta_1 + \theta_2)$
Cir. scale	Vern. scale	Total	Mean (x)		Cir. scale	Vern. scale	Total	Mean (y)		
..	..	..			..	..	..			
..	..	..	..	..	..	..	..	..	..	..
..	..	..			..	..	..			

Calculations : (*i*) From the graph :

Strength $(OB)=\ldots\ (C)$
Angle of rotation $(AB)=\ldots\ (\theta)$

$$\therefore\quad S=\frac{1000.\theta}{l.C}=\ \ldots\ ^{\circ}$$

(*ii*) From the table (*j*), $\theta=\ldots\ ^{\circ}C$

$$\therefore\quad C=\frac{1000.\theta}{S.l}=\ \ldots\ \%$$

From the graph, $C=\ldots\%$

Proportional and percentage error [For Honours course]

The specific rotation is given by,

$$S=\frac{1000.\theta}{l.C}=\frac{1000.\theta.V}{l.m}$$ [m=mass of sugar and V=volume of the solution]

Maximum proportional error is given by,

$$\left(\frac{\delta S}{S}\right)_{max}=\frac{\delta\theta}{\theta}+\frac{\delta l}{l}+\frac{\delta V}{V}+\frac{\delta m}{m}.$$

In one case, the following data were obtained :

$\theta=22{\cdot}15$ $\delta\theta=\frac{1}{10}$ of a degree$=0{\cdot}1$ [*V.C.* of the scale]
$l=20$ cm. $\delta l=1$ mm$=0{\cdot}1$ cm.
$m=16$ gm. $\delta m=1$ mg$={\cdot}001$ gm.
$V=100$ c.c. $\delta c=0{\cdot}1$ c.c. (minimum measure in the graduated cylinder)

Substituting the values in the above equation,

$$\left(\frac{\delta S}{S}\right)_{max}=\frac{0{\cdot}1}{22{\cdot}15}+\frac{0{\cdot}1}{20}+\frac{0{\cdot}1}{100}+\frac{{\cdot}001}{16}$$
$$=0{\cdot}0045+0{\cdot}005+0{\cdot}001+0{\cdot}00006$$
$$=0{\cdot}01056$$

$$\therefore\quad \%\ \text{error}=\left(\frac{\delta S}{S}\right)_{max}\times 100\%=1{\cdot}0\%$$

Remarks : (1) The angle of rotation of the plane of polarisation depends on the temperature of the solution. So, while measuring the angle of rotation produced by solutions of known and unknown strength, temperature of the solutions should be recorded and mentioned. (2) While filling up the polarimeter tube with solutions, care should be taken so that no air bubbles enter into the tube. (3) If half shade plate is used, the wavelength for which it is effective should be mentioned. (4) The cap of the tube containing the solution should not be tightly screwed to avoid straining of the glass. Strained glass is likely to produce elliptic polarisation which will interfere with the setting of the analysing nicol.

Oral questions

1. What do you mean by optical activity ?

Ans. When a plane polarised light passes through some substances (solid or liquid), its plane of polarisation is found to rotate through certain angle. This phenomenon is called optical activity.

2. What is specific rotation ?

Ans. See theory.

3. Does the specific rotation depend on temperature of the optically active substance ?

Ans. Yes ; for liquids, specific rotation decreases with the increase of temperature but for solids, specific rotation increases with the increase of temperature.

4. Is optical activity for all substances alike ?

Ans. No ; for some substances such as cane sugar, quartz etc, the rotation of the plane of polarisation is clockwise while for some other substances such as fruit sugar etc, the rotation is anticlockwise.

5. Does optical activity depend on the molecular arrangement of the optically active substance ?

Ans. Yes ; Quartz, in its solid state, is optically active but when fused. it ceases to be optically active. This shows that optical activity depends on the molecular arrangement of the optically active substance.

6. What practical application of optical activity do you know ?

Ans. In sugar factory, optical activity is applied in finding the percentage of sugar present in cane juice.

7. What do you mean by polarisation of light ? What is plane of polarisation ?

Ans. Consult any text book.

8. Should you use white light for half-shade plate ?

Ans. Half-shade plate is effective for a particular wave length of light. So instead of white light, the light of given wave length should be used.

9. What do you mean by dextro and leavo-rotation ?

Ans. See page 199.

10. What is the harm if there be air bubbles in the polarimeter tube ?

Ans. Due to reflection and refraction of light at the bubbles, light may go from one half of the field of view to the other and thus disturb the setting of the analyser.

4. MAGNETISM

4.1. Magnetometers :

The instruments ordinarily used for magnetic measurements are called magnetometers. They are employed to (i) compare the magnetic moments of two magnets and (ii) to determine the earth's horizontal intensity.

(a) Deflection magnetometer :

It is provided with a very short magnet $N-S$ pivoted at the centre [Fig. 28] of a circular scale graduated in degrees and divided into four quadrants, each running from 0° to 90°. A long, light aluminium pointer *PP* is fixed to the magnet at its centre and perpendicular to its length so that the tip of the pointer moves over the graduated scale. Any rotation of the magnetic needle can be easily recorded by the corresponding rotation of the pointer on the scale. To avoid parallax, a strip of circular mirror (plane) is fitted beneath the pointer. While taking a reading, the eye is placed in such a way that the image of the pointer is hidden by the pointer itself. The scale and the magnetic needle are enclosed in a wooden box having a glass cover at the top. Two long arms *A* and *B* are attached to the wooden box and they carry scales graduated on them to read distances from the pivot.

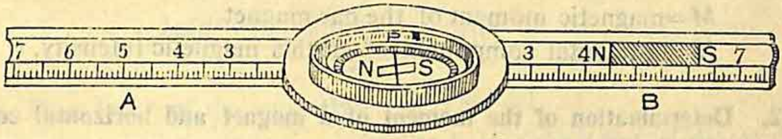

Fig. 28

(b) Vibration or oscillation magnetometer :

It consists of a wooden box *A* fitted with glass windows on the sides [Fig. 29]. There are two slits *M*, *M*, at the top to see the vibrations of the magnet $N-S$ placed on the hanging stirrup *C*. The stirrup

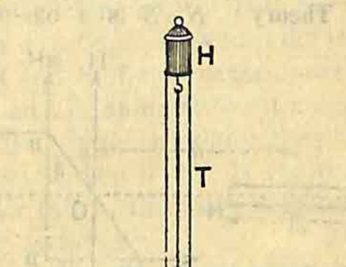

Fig. 29

is suspended by means of an unspun silk thread connected to the torsion head H. The torsion head can be rotated to adjust the position of the magnet. A plane mirror fitted at the base of the box helps to count the number of vibrations of the magnet avoiding error due to parallax. A line PQ is marked on the mirror parallel to the length of the box and is at the middle of the box.

When a magnet is placed horizontally on the stirrup and there is no torsion in the suspension thread, the magnet will face north and south along the magnetic meridian with the line PQ coinciding with the axis of the magnet. If it is now set into oscillation, its time period of oscillation is given by $T=2\pi\sqrt{\frac{I}{MH}}$

where, I= moment of inertia of the bar magnet about the suspension as axis

M=magnetic moment of the bar magnet

H=horizontal component of earth's magnetic intensity.

4.2. Determination of the moment of a magnet and horizontal component of Earth's magnetic field by magnetometers :

Apparatus : Deflection magnetometer, oscillation magnetometer, two bar magnets, (one of them is the experimental bar and the other auxiliary bar), a magnetic compass needle, stop-watch, slide-calipers, balance, weight box etc.

Theory : $N-S$ is a bar-magnet whose magnetic length is $2l$ and magnetic moment is M. It is kept with its axis perpendicular to the magnetic meridian (*i.e.*, along east-west direction). A small compass needle $n-s$ is kept at O at a distance d from the centre of the barmagnet [Fig. 30]. The compass needle under the influence of the horizontal intensity H of the earth's magnetism and the intensity F of the bar-magnet, is at rest, having an inclination θ with the magnetic meridian. Under this condition, it may be proved that

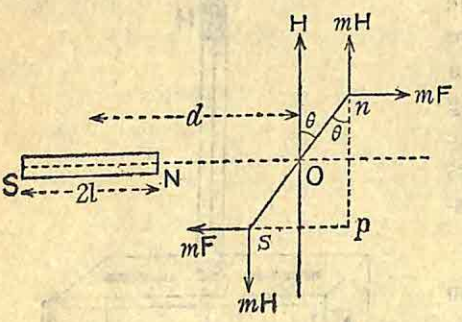

Fig. 30

$$\frac{M}{H}=\frac{(d^2-l^2)^2}{2d}.\tan\theta$$

If l_1 be the geometrical length of the bar magnet, $l=\frac{l_1\times 0{\cdot}85}{2}$ .. (i)

If the bar-magnet be suspended freely in the magnetic meridian with its north pole pointing north and be set into oscillation with small amplitude about a vertical axis, then its time-period is given by,

$$T=2\pi\sqrt{\frac{I}{MH}} \quad \text{or,} \quad MH=\frac{4\pi^2.I}{T^2},$$

where I=moment of inertia of the bar about the vertical axis

$$=\frac{\text{Mass of the bar }(M_1)}{12}\times[(\text{length})^2+(\text{breadth})^2]$$

$$=\frac{M_1}{12}\ (l_1{}^2+b_1{}^2) \quad .. \quad \text{(ii)}$$

$$\text{Hence, } H=\frac{2\pi}{T(d^2-l^2)}\sqrt{\frac{2.d.I}{\tan\theta}} \quad .. \quad \text{(iii)}$$

$$\text{and } M=\frac{2\pi(d^2-l^2)}{T}\sqrt{\frac{I.\tan\theta}{2d}} \quad .. \quad \text{(iv).}$$

Description of the apparatus : See art 4.1.

Experimental procedure : (1) Find the mass (M_1) of the experimental bar-magnet by a balance. Also find its length (l_1) and breadth (b_1) with the help of a pair of slide calipers. Calculate, from the equation (*ii*) of the theory, the moment of inertia (I) of the bar.

(2) Remove all the magnets and magnetic substances far away from the working table and place the deflection magnetometer on the table. The magnetic needle of the magnetometer will lie in the magnetic meridian along the north-south direction. Now turn the instrument in such a way that the arms A and B of the magnetometer are perpendicular to the magnetic meridian (*i.e.*, along east-west direction) or to the length of the magnetic needle. In this condition the pointer attached to the magnetic needle should read 0°-0°. If the pointer does not read 0°–0°, turn the circular case of the magnetometer and bring the pointer in line with 0°–0° divisions.*

(3) Now place the bar-magnet on the arm A (which is on the east side) by the side of the scale with its north pole pointing west-

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* In the following simple way it can be tested whether the arms A and B are perpendicular to the meridian line : Place the bar-magnet on any arm with its axis parallel to the length of the arm and its north pole pointing towards the megnetic needle. Note the deflection of the needle. Keeping the position of the bar-magnet fixed, turn its south pole towards the magnetic needle. Note the deflection. If the two deflections are equal, the arms of the magnetometer are perpendicular to the meridian line.

ward. Adjust the position of the bar-magnet so that the deflection of the pointer is about 45°. Note, from the circular scale, the readings corresponding to both the ends of the pointer.

(4) From the scale attached with the arm of the magnetometer, note the readings corresponding to the two ends of the bar-magnet. Suppose the scale readings are d_1 and d_2. So, the distance of the centre of the bar-magnet from the centre of the magnetic needle $d = \frac{1}{2}(d_1+d_2)$. Keeping the position of the bar-magnet unchanged, turn it upside down and note the readings on the circular scale of the two ends of the pointer.

Keeping the position of the bar-magnet still unchanged, make the north pole of the bar-magnet point towards east (*i.e.* reverse the positions of the polarities of the bar-magnet). Again note the readings of the two ends of the pointer from the circular scale. Now, turn the bar-magnet upside down again and record the readings given by the two ends of the pointer. Note that eight readings are obtained by these operations.

(5) Now place the bar-magnet on the arm *B* (which is on the west side) with its north pole pointing eastward. Make the distance between the centre of the bar-magnet and the centre of the magnetic needle equal to d as before. Following the operations (3) and (4), get another set of eight readings. Find the mean of all these sixteen readings. This gives the correct value of θ.

(6) Removing the deflection magnetometer far away, bring the oscillation magnetometer on the working table. Remove also other magnetic substances or magnets if there be any near by. Place a compass needle near the magnetometer box. Set the box in such a way that the line *PQ* drawn on the mirror at the bottom of the box is parallel to the axis of the compass needle. By turning the torsion head *H* at the top of the box, set the stirrup parallel to the line *PQ*.

(7) Place the bar-magnet on the stirrup with its north pole pointing north. The bar-magnet will lie along the magnetic meridian. Close the glass window of the box and bring the north pole of an auxiliary bar magnet close to the north pole of the suspended magnet (the auxiliary bar magnet will, of course, remain outside the box). Remove the auxiliary bar-magnet as soon as the magnet inside the box starts oscillating. Note, with the help of a stop-watch the time taken for 20 complete oscillations (successive crossings of the line *PQ* in the same direction by a particular corner of the bar-magnet may be seperately regarded as one complete oscillation). Find the time-period (*T*) of oscillation by dividing the total time by the total number of oscillations. Repeat the operation twice and find the mean value of the time-period.

Measurements : (*a*) *Determination of the moment of inertia of the bar-magnet.*

Mass of the bar-magnet (M_1)=. . . gm+. . . gm+. . . gm=. . .gm.

The value of the smallest division of the main scale of the slido calipers =. . . mm

. . . vernier divisions =. . . main scale divisions

=. . . mm or cm.

or, 1 ,, =. . . mm or cm.

∴ Vernier constant =. . . cm.

Quantity to be measured	No. of obs.	Main scale readings (cm)	Vernier reading	Total (cm)	Mean (cm)	Instrumental error	Corrected reading (cm)
Length (l_1)	1.	..	..	..	..	..	..
	2.	..	..	..			
	3.	..	..	..			
Breadth (b_1)	1.	..	..	..	..	..	..
	2.	..	..	..			
	3.	..	..	..			

Moment of inertia of the bar $(I)=\frac{M_1}{12}(l_1^2+b_1^2)=\ldots$ gm-cm^2

(*b*) Half the magnetic length of the bar-magnet $(l)=\frac{1}{2}(l_1\times 0{\cdot}85)=\ldots$cm.

(*c*) *Measurement of d* :

Scale Reading corresponding to one end of the bar (cm)	Scale Reading corresponding to the other end (cm)	$d=\frac{1}{2}(d_1+d_2)$ cm
..(d_1)	..(d_2)	..

(*d*) *Measurement of* θ *from the deflection magnetometer readings* :

Position of the poles of the bar-magnet	Deflection of the pointer (in dgree)				Mean θ (degree)
	Bar-magnet on the arm A		Bar-magnet on the arm B		
	End I	End II	End I	End II	
N-pole pointing west	..	..	..	..	..
Bar upside down with N-pole pointing west	..	..	..	..	
S-pole pointing west	..	..	..	..	
Bar upside down with S-pole pointing west	..	..	..	..	

(*e*) *Measurement of the time-period of oscillation* :

No. of Obs.	Time for 20 complete oscillations (sec)	Time period (sec)	Mean Time period (T) sec
1.	..	..	..
2.	..	..	
3.	..	..	

Calculations : (i) $H=\frac{2\pi}{T(d^2-l^2)}\sqrt{\frac{2.d.I}{\tan\theta}}=\ldots$ Oe.

(ii) $M=\frac{2\pi(d^2-l^2)}{T}\sqrt{\frac{I.\tan\theta}{2d}}=\ldots$ dyne-cm/Oe.

Remarks : (1) If the length of the bar-magnet is 5 cm/6 cm or more than that, slide-calipers can be dispensed with and a metre scale can be used to measure the length. Further, magnetic length of the bar (l) can also be taken to be equal to its geometrical length (l_1). It will simplify matters without incurring much error. (2) The bar-magnet should have sufficient strength ; for in that case the values of θ and d may be large and the error in their measurement will be small, (3) While working with a magnetometer, all magnets and magnetic substances should be removed far away. There should not be any current-bearing wire near by. (4) The value of d should preferably be much larger compared to the length of the bar-magnet (5) While oscillating the bar-magnet in an oscillation magnetometer, the amplitude of oscillation should be small ; it should be within 7°/8°. (6) While keeping the bar-magnet on the stirrup care should be taken so that the suspension wire may pass through the C.G. of the bar. (7) In the measurement of M/H, the deflection $\theta=45°$ gives minimum error. This is why the distance of the bar-magnet from the centre of the magnetic needle is so adjusted that the deflection is nearly 45°. At the same time the value 'd' should, by no means, be small.

Oral questions

1. What is magnetic moment ? Is the magnetic moment of a bar magnet same at all places of the earth ?

Ans. Consult any text book. The magnetic moment of a bar magnet is same at all places of the earth provided its pole strength and length remain unaltered.

2. What is the magnetic length of a bar magnet ? What is its relation with the geometrical length of the bar ?

Ans. The distance between the poles of a bar-magnet is called its magnetic length. Magnetic length $=0{\cdot}85$ times the geometrical length.

3. Can the experiment be done if the axis of the bar-magnet set parallel to the magnetic meridian ?

Ans. No ; in this case, the field due to the bar-magnet will be parallel to the horizontal component of earth's magnetic field and the tangent law will not be applicable.

4. What is the tangent law ?

Ans. If a magnet is freely suspended or pivoted in two mutually perpendicular magnetic fields H and F, then the angle θ made by the axis of the magnet in its equilibrium position with the direction of the field H is given by, $\tan\theta = F/H$ This is known as the tangent law.

5. Can the experiment be done with the bar-magnet kept at the tangent-B position with respect to the magnetic needle of the magnetometer ?

Ans. Yes ; in that case the equation is : $M/H = (d^2+l^2)^{\frac{3}{2}}.\tan\theta$.

6. Why is there a strip of plane mirror in the magnetometer ?

Ans. To avoid parallax error while taking the pointer reading. At the time of reading the pointer, the eye should be so placed that the pointer may just cover its image formed by the plane mirror.

7. Why is the mirror circular in shape ?

Ans. The tip of the pointer moves along the circumference of a circle. The circular shape of the mirror helps us to see the image of the tip in all positions.

8. Why do you take readings of both the ends of the pointer ?

Ans. To avoid eccentric error ; if the centre of the circular scale does not coincide with the centre of the magnetic needle, the reading of the pointer will entail eccentric error.

9. Why do you take readings by turning the bar-magnet upside down ?

Ans. If the geometric axis of the bar-magnet does not coincide with the magnetic axis, some error comes into the reading which can be removed by taking readings with the bar-magnet turned upside down.

10. Why do you take readings by reversing the positions of the poles of the bar-magnet ?

Ans. If the magnetism of the bar be not uniformly distributed throughout the length of the bar, some error is likely to enter in the reading. Error may also come if the axis of the bar-magnet is not exactly perpendicular to the magnetic meridian. These errors are eliminated if readings are taken with the positions of the polarities of the bar-magnet reversed.

11. Why are reading taken with the bar-magnet placed on both the arms of the magnetometer A and B ?

Ans. Error may enter if the 0-mark of the scales attached with the arm is not coincident with the centre of the circular scale. The error is eliminated if readings are taken with the bar-magnet placed on the arms A and B.

12. Why is the deflection of the needle made nearly 45° ?

Ans. See remark no. 7.

13. What is the horizontal component of earth's magnetism ? What is its unit ? Is the value of the quantity same all over the earth ?

Ans Consult any text book. Its unit is Oersted (Oe). Its value is not the same all over the earth.

14. What should be the amplitude of oscillation of the bar-magnet ?

Ans. See remark no. 5.

4.3. Determination of the angle of dip by earth-inductor :

Apparatus : An earth inductor, a ballistic galvanometer, lamp and scale arrangement, a magnetic compass needle, resistance box, flexible twin cord etc.

Theory : The total intensity of the magnetic field of the earth at any place may be resolved into horizontal and vertical components. Hence, at any place there are two kinds of lines of force due to earth's magnetic field : (i) lines of force extending north and south due to horizontal component (H) of the earth's magnetic field and (ii) lines of force extending up and down due to vertical component (V). Keeping the plane of the coil of the earth inductor, at first, perpendicular to the vertical lines of force, if the coil be suddenly turned through 180°, some electric charge flows through the coil and the circuit connected with the coil. It can be shown that the charge q is given by,

$$q=\frac{2n\,A.V}{R}$$

where n=number of turns of the inductor coil ; A=face area of the coil and R=total resistance of the circuit.

If, now, a ballistic galvanometer* be joined in series with the inductor coil, the charge q will also flow through the galvanometer producing a throw θ_1 (in radian) of the galvanometer coil given by,

$$\frac{2nAV}{R}=k.\theta_1(1+\lambda/2)\ldots\text{ (i)}$$

where λ=log decrement of the galvanometer and K=its reduction factor.

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* For description of ballistic galvanometer, see page 233.

The plane of the inductor coil is now held at right angles to the horizontal lines of forces and suddenly rotated through 180°. If the throw of the ballistic galvanometer now be θ_2, then

$$\frac{2n\ AH}{R}=k.\theta_2(1+\lambda/2)\ \ldots\quad \text{(ii)}$$

Dividing (i) by (ii), we get, $\frac{V}{H}=\frac{\theta_1}{\theta_2}=\frac{d_1}{d_1'}$, where d_1=the displacement of the spot of light in the first case and d_1'=the displacement in the second case.

If δ be the angle of dip at the place, then $\tan\ \delta=\frac{V}{H}$.

$$\therefore\ \tan\ \delta=\frac{d_1}{d_1'}.$$

Further, a graph between d_1 and d_1' will be a straight line passing through the origin and the slope of the straight line will be equal to $\tan\ \delta$ from which δ can be found out.

Description of the earth-inductor : It consists of a circular wooden frame on which insulated copper wire is coiled (C). The number of turns of the coil varies from 1000 to 5000. The coil can rotate about a vertical diameter [Fig. 31]. With the help of two suitable stops, the rotation of the coil about the vertical diameter is confined within 180°. The handle H is used to rotate the coil. The frame itself can rotate about a horizontal axis defined by fixed pivots at P and M. A circular scale P provided with the frame denotes whether the plane of of the coil is vertical or horizontal. The two ends of the coil are connected to two slip-rings (S,S) [Fig. 31(a)] which are again connected to two terminals T_1 and T_2 fixed with the rectangular frame. This helps the coil to rotate, yet the two ends of the coil maintain connection with

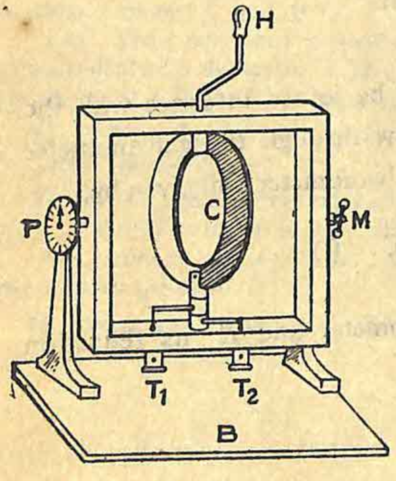

Fig. 31

the terminals T_1 and T_2. When the axis of rotation of the coil is vertical the circular scale P reads 90° ; when the axis of rotation is horizontal, the scale reading is 0°.

Experimental procedure : (1) Connect a ballistic galvanometer G and a resistance box R in series with the terminals T_1 and T_2 of the earth inductor as shown in fig 31(*a*). Keep the earth-inductor sufficiently away from the galvanometer so that the magnet of the galvanometer may not have any influence on the earth-inductor. To ensure it, keep a compass needle near the earth-inductor and see whether the needle freely points to north-south direction. Put a resistance in the resistance box greater than the critical damping resistance (*i.e. C.D.R.*) of the ballistic galvanometer.

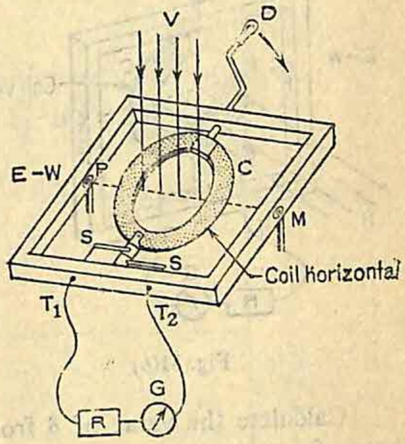

Fig. 31(*a*)

(2) Ascertain the direction of the magnetic meridian with the help of a compass needle and draw two lines by a piece of chalk on the table—one parallel to the meridian line and the other perpendicular to it. Removing the compass needle, place the earth-inductor in its position so that the axis PM of the frame coincides with the line perpendicular to the magnetic meridian. The axis PM will evidently lie along east-west direction. Turning the rectangular frame, make the plane of the coil horizontal [Fig. 31(*a*)]. In this position, the circular scale reading will be 0°.

(3) Adjust the position of 'the scale of the lamp and scale arrangement' so that the spot of light reflected by the mirror of the galvanometer may coincide with the zero mark of the scale. Now give a *quick* rotation to the inductor coil through 180° with the handle D [fig. 31(*a*)]. There will be a sharp deflection of the spot of light. The deflected spot may go out of scale. In such cases, the resistance of the box R should be increased inorder to keep the deflection within scale. When the resistance is adjusted and the spot of light returns to the zero-mark of the scale, again rotate the inductor coil quickly through 180° and note the deflection of the spot of light. The spot of light will come back to the zero-mark after a few oscillations. Then again rotate the coil, in the opposite direction, through 180° and note the deflection. Take the mean of these two deflections (d_1).

(4) Now, turning the rectangular frame, make the plane of the inductor coil vertical. The axis *PM* of the frame will remain along east-west direction as before [Fig. 31(*b*)]. In this position, the circular scale *P* will read 90°. Rotate the coil quickly through 180° by the handle and note the deflection of the spot of light. Similarly rotate the coil through the same angle in the opposite direction and note the deflection. Find the mean of these two deflections (d_1').

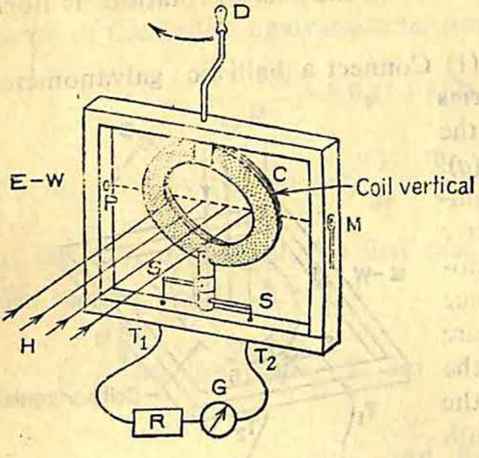

Fig. 31(*b*)

Calculate the value of δ from the values of d_1 and d_1' with the help of a tangent table.

(5) Changing the resistance of the circuit, repeat the whole operation five or six times, and find the mean value of δ from these observations.

δ *from the graph* :

(6) Suppose the mean deflections obtained from five observations with the plane of the coil horizontal are d_1, d_2, d_3. .etc. Similarly let the deflections with the plane of the coil vertical be d_1', d_2', d_3'. . etc. Plotting the former deflections along the *Y*-axis and the latter deflections along the X-axis, a straight line *AB* will be obtained. The straight line will pass through the origin O [Fig. 31(*c*)]. Take any point *C* on the straight line and draw two perpendiculars *CD* and *ED*. Then, $\tan \delta = \frac{d_1}{d'_1} = \frac{CD}{ED}$.

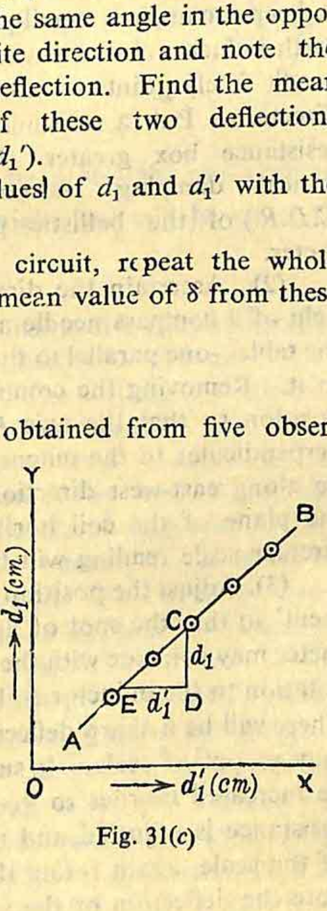

Fig. 31(*c*)

(7) Compare the value of δ obtained earlier with that obtained from the graph.

Measurements :

Locality=. . . .
Critical damping resistance of the galvanometer=... ohms.

No. of obs.	Resistance in the R-box	Defl. when the coil is horizontal			Defl. when the coil is vertical			Ratio $\frac{d_1}{d_1'}$	Mean ratio $\left(\frac{d_1}{d_1'}\right)$
		180° rotation in one way (x_1)	180° rotation in the opp. way (x_2)	Mean= $\frac{1}{2}(x_1+x_2)$	180° rotation in one way (y_1)	180° rotation in opp. way (y_2)	Mean= $\frac{1}{2}(y_1+y_2)$		
1.	..	..	..	..(d_1)	..	..	..(d_1')	..	
2.	..	..	..	..(d_2)	..	..	..(d_2')	..	
3.	..	..	..	..(d_3)	..	..	..(d_3')	..	
4.	..	..	..	..(d_4)	..	..	..(d_4')	..	
5.	..	..	..	..(d_5)	..	..	..(d_5')	..	

$$\therefore \tan \delta = \frac{d_1}{d'_1} = .. \text{ or } \delta = ..^\circ.$$

Table for drawing graph (Data taken from the previous table)

Mean deflections when the coil is horizontal →	. . (d_1)	. . (d_2)	. . (d_3)	. . (d_4)	. . (d_5)	. . (d_6)
Mean deflections when the coil is vertical →	. . (d_1')	. . (d_2')	. . (d_3')	. . (d_4')	. . (d_5')	. . (d_6')

From the graph → $CD = ..$ cm.

$ED = ..$ cm

$$\therefore \quad \tan\delta \; \frac{CD}{ED} = ..$$

or $\delta = ..^\circ$

Remarks : (1) Rotation of the inductor coil should be effected quickly and exactly through 180° (2) The magnet of the ballistic galvanometer should not cause any influence on the inductor coil (3) A flexible twin cord should be used to connect the earth inductor to the ballistic galvanometer. If two separate connecting wires are used, then they may add to the face area of the coil, producing error in the deflection of the galvanometer. (4) A key should be included in the circuit and the key should be closed only when the inductor coil is rotated. If permanent connection is made, a slight change in the position of the earth-inductor coil causes a deflection of the spot of light. It will be difficult, in that case, to bring the spot of light to the zero mark of the scale.

Oral questions

1. What is angle of dip ? Is it same all over the earth ?

Ans. The total intensity of earth's magnetic field at any place can be resolved into two components—one horizontal and the other vertical. The angle made by the horizontal component with the direction of the total intensity is called the angle of dip at that place. It is not same at all places of the earth.

2. What quantity do you measure by an earth-inductor ?

Ans. The angle of dip at a place is measured.

3. Why do you rotate the inductor *quickly* ?

Ans. According to the laws of electromagnetic induction, the induced e.m.f. is proportional to the rate of change of flux linked with the coil $\left(e \propto \frac{d\phi}{dt}\right)$. To get the full value of the induction, the coil is to be rotated very quickly.

4. What is the difference between the ballistic galvanometer and a dead-beat type galvanometer ?

Ans. See page 231 & 233.

5. Should you take a large number or a smaller number of turns in the coil of the inductor ?

Ans. A larger number is preferable. Larger the number, greater is the deflection of the galvanometer.

6. What is the harm if the inductor coil is rotated through 360° instead of 180° ?

Ans. No deflection will take place in the galvanometer. Current will flow through the galvanometer in one direction for 0°--180° rotation while current flows in the opposite direction for 180°--360° rotation. As a result, the galvanometer coil remains in its rest position.

7. No battery is used in the experiment. Still then a current flows through the galvanometer. How is this possible ?

Ans. When the inductor coil is quickly rotated, the number of magnetic flux due to earth's magnetism linked with the coil changes and according to Faraday's law of electromagnetic induction, an e.m.f. is induced in the circuit. This transient e.m.f. causes a flow of current in the circuit.

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5. ELECTRICITY

5.1. Electrical accessories and measuring instruments

(*a*) **Resistance apparatus :** (*i*) **Variable resistance or Rheostat :**

With its help, the resistance of a circuit can be varied at will. It is an essential instrument for current control in a circuit. Fig. 32 shows a rheostat. In this instrument, a wire of high specific resistance (generally nickelin) is wound over a non-conducting cylinder like slate or porcelain cylinder. The turns of the coil are not in contact with each other. There is a metal rod *R* over the coil and along the rod a slider *S* can slide always keeping contact with the coil. There are two screws—one at the end of the rod *R* and the other at the end of the coil. If a circuit be connected between the screws, current will flow in the direction as shown by arrow head. So, if the slider be kept at the extreme left end of the cylinder, the entire resistance of the coil will be included in the circuit. Again when the slider is kept at the extreme right end of the cylinder, the resistance is cut off. Thus, by keeping the slider at various places, resistance can be altered at will.

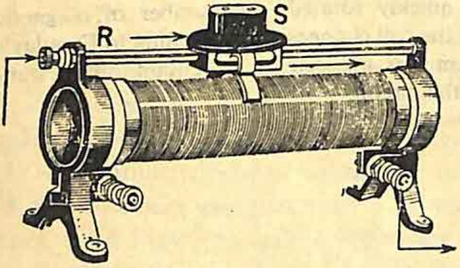


Fig. 32

(*ii*) **Resistance coil and Resistance box :** For various electrical works, resistances of fixed magnitude are necessary. Such block resistances are known as resistance coils [Fig. 33]. These coils are made of manganin or constantant wire and their resistances are either multiples or sub-multiples of 1 ohm. These wires are covered by silk and are wound over a bobin, which is placed within a cylinder made of some non-conducting material. The free ends of the wire are soldered to two screws on the top of the cylinder. Resistance coils of different denominations are available and their magnitudes are engraved on top of the cylinder.

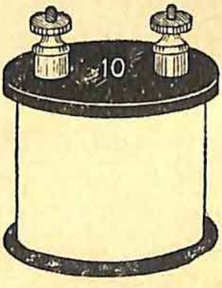


Fig. 33

A resistance box may be regarded as an assemblage of a number of resistance coils of different magnitudes connected in series [Fig. 33(*a*)]. The free ends of each coil are soldered to two adjacent pieces of brass blocks which are kept a little apart. These brass blocks are fixed on an ebonite plate. The magnitude of each resistance coil is engraved by its side on the ebonite plate. If a plug is taken out, the resistance of that coil is included in the circuit. Thus by taking out various plugs, the resistance can be altered.

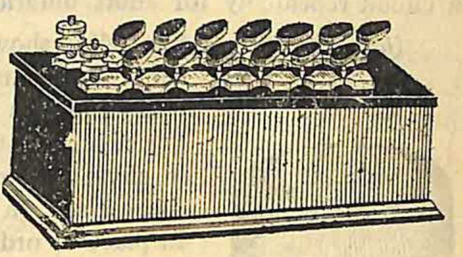


Fig. 33(*a*)

(*b*) **Keys** : Keys of various types are in use for starting or stopping electric current in a circuit. Their descriptions are given below.

(*i*) **Plug Key** : Fig. 34 shows a plug key. *A* and *B* are two thick metallic blocks fixed on an ebonite plate, a little apart from each other. A plug *P* can fit in the space between the blocks. Two screws S_1 and S_2 are fitted with the blocks. If a circuit is connected between the screws and the plug *P* inserted, the circuit becomes complete. When the plug is taken off, the circuit becomes broken. Plug keys are generally used when a continuous current is to be sent.

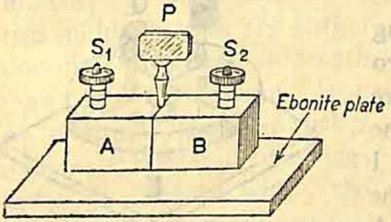


Fig. 34

(*ii*) **Tap key** : Fig. 34(*a*) shows a tap key. Two screws S_1 and S_2 are fitted on an ebonite base. There is a springy brass strip *P* attached to the screw S_2, which is provided with a cap at one end. When the cap is pressed, a pin at its botton may come in contact with a button on the ebonite base. The button is connected with the screw S_1 by a wire at the bottom of the base (shown by dotted lines). When a circuit is connected between the screws S_1 and S_2 and the cap is pressed, the circuit is completed. When the cap is released, the spring

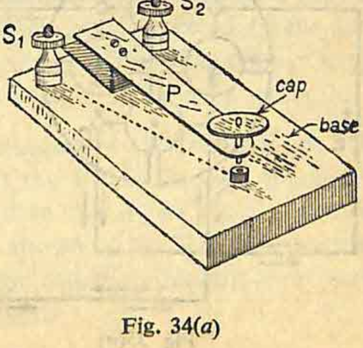


Fig. 34(*a*)

action of the strip will, at once, cause the strip to move up and the circuit is broken. Tap keys are used when current is to be sent in a circuit repeatedly for short duration.

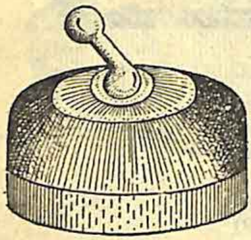


Fig. 34(*b*)

(*iii*) **Switches :** Fig. 34(*b*) shows a switch. Switches are generally used in household electrical circuits regulating fans and lights. With its help, current can be sent or stopped very quickly. Sometimes, in electrical experiment current is drawn from the mains. In such cases, switches are used in place of ordinary plug keys.

(*c*) **Commutators** : Sometimes, the direction of current in a circuit requires reversal. This is done by an apparatu. known as commutators.

(*i*) **Plug commutator** : This type of commutator is frequently used in a laboratory. Fig. 35 shows a plug commutator in which *A, B, C* and *D* are four thick brass blocks fitted on an ebonite base *E*. The blocks have got four gaps among themselves. No. 1 and 2 signify two such gaps and the gaps no 3 and 4 are closed by two plugs *P* and *Q*. The plugs can also be put in the gaps 1 and 2. The circuit in which the direction of current is to be reversed, is connected across the two diagonal screws fitted on to the blocks and the terminals of a cell are joined to other two diagonal screws. Having made such connections, if the plugs *P* and *Q* are put in the gaps 3 and 4, current will flow in the circuit in one direction and if the plugs are now put in the gaps 1 and 2 instead of 3 and 4, the current will flow in the opposite direction.

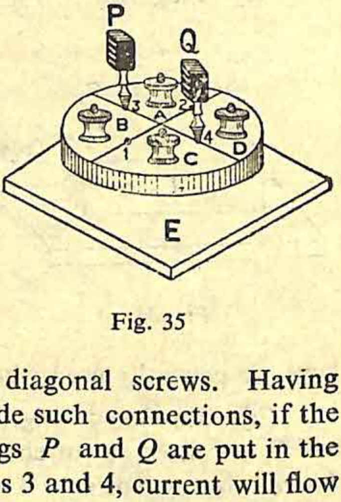


Fig. 35

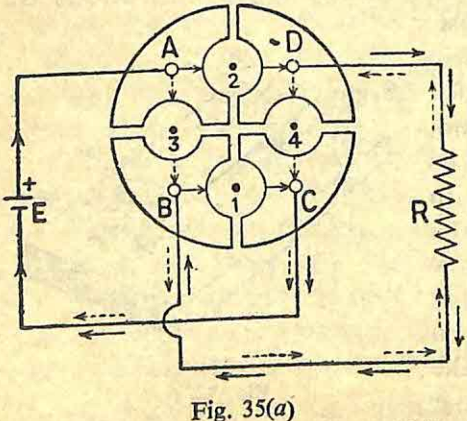


Fig. 35(*a*)

To understand the action, suppose the terminals of a cell *E*, are connected to the screws *A* and *C* and those of a circuit *R* are connected to the screws *D* and *B* [Fig. 35(*a*)]. Plugs are first put in the gaps no.

1 and 2. In this case, current from the cell arrives at A, then flows through the plug no. 2 to D and afterwards through the external resistance R to the point B and finally through the plug no. 1 to the point C and thence back to the cell. This direction of flow has been shown in the fig. 35(a) by continuous arrow head.

Now, the plugs are to be put in the gaps no. 3 and 4. Consequently the current arriving at A, goes to the point B through the plug no. 3 and then through the external resistance R to the point D and finally through the plug no. 4 to the point C wherefrom the current goes back to the cell. This direction has been shown by broken arrow head. Evidently, this direction is reverse of the previous direction.

(*ii*) **Phol's commutator** : Fig. 35(b) shows the sketch of a Phol's commutator. Six holes are bored on an ebonite platform and the holes are filled up with clean mercury. Each mercury hole is connected to the nearest terminal by wire passing below the ebonite platform. A metallic rocker with six legs is placed on the platform in such a way that the legs dip into the mercury of six holes. The two middle legs of the rocker are slightly longer in length than the other four legs. As a result, when the rocker remains at rest, four of its legs come in touch with the mercury, the other two remaining slightly above the mercury. There is a handle H made of ebonite on the top of the rocker. The rocker may be rocked this way or that way by giving suitable push to the handle. The mercury holes which are connected to the terminals A_1 and C_1 are again cross-connected by a short but thick copper wire. Similarly, the mercury holes which are connected to the terminals A_2 and C_2 are also cross-connected by a similar copper wire. The cross-connectors are, however, insulated from each other.

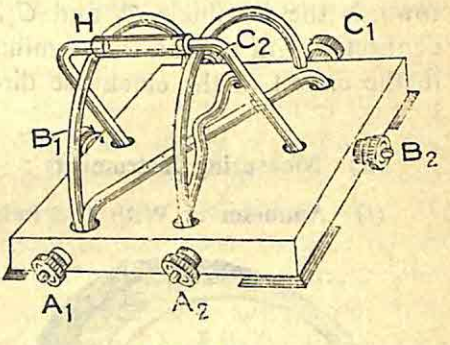
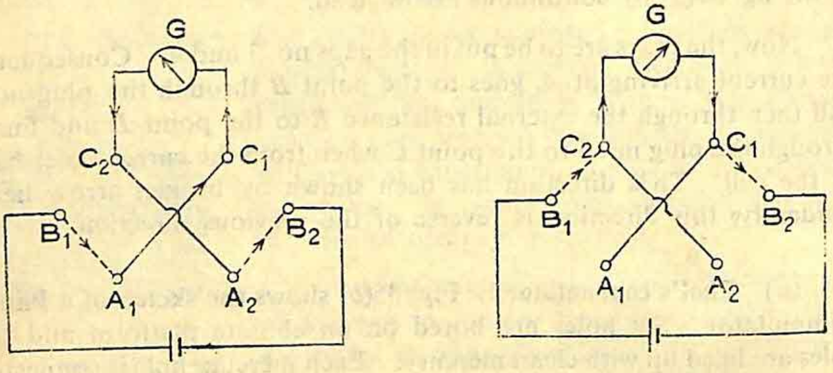


Fig. 35(b)

The two poles of a battery are connected to the terminals B_1 and B_2 of the commutator and the ends of the circuit through which the current needs to be reversed are joined to A_1, A_2 or C_1, C_2. When the rocker is pushed towards the left as shown in fig. 35(b), the current flows through the circuit in one direction but the direction is reversed if the rocker is pushed towards the right.

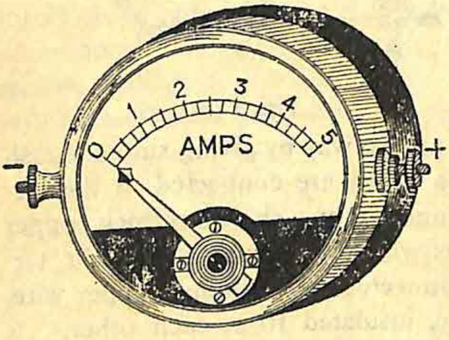
Fig. 36 explains this process of reversal clearly. When the rocker is pushed towards the terminals A_1 and A_2, the terminal B_1 becomes directly connected with A_1 and the terminal B_2 with A_2 and the current flows in a counter clockwise direction (shown by arrow heads) in the circuit [Fig. 36(i)] When the rocker is pushed towards the terminals C_1 and C_2, the terminal B_1 becomes directly connected with C_2 and the terminal B_2 with C_1 and the current flows in the circuit in the clockwise direction [Fig. 36(ii)].



(i) Fig. 36 (ii)

(*d*) **Measuring instruments** :

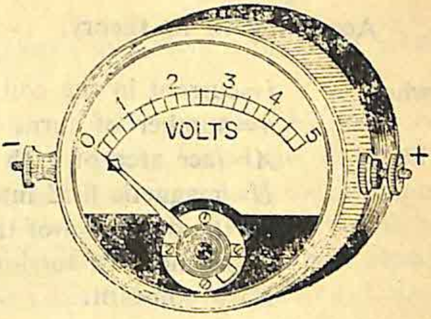
(*i*) **Ammeter** : With the help of this instrument, current in a circuit can be measured directly in amperes. It is a galvanometer of very low resistance [Fig. 37(*i*)]. It is graduated in amperes and subdivisions of amperes and a pointer can move over the scale. Two terminals marked + and – are provided with the instrument. Ammeter is to be connected in the circuit always *in series* and the positive pole of the cell is to be joined with the terminal marked + and the negative pole of the cell to the terminal marked –.



Ammeter

Fig. 37(*i*)

(*ii*) **Voltmeter** : With the help of this instrument, the p.d. between two points in a circuit can be measured directly in volts. It is also a galvanometer but of very high resistance [Fig. 37(*ii*)]. Like an ammeter this instrument is also provided with a scale and a pointer. The scale is graduated in volts and submultiples of volt. The instrument is to be connected *across* two points whose p.d. is wanted. The terminal marked + is to be connected to the high potential point of the circuit.



Voltmeter

Fig. 37(*ii*)

(*iii*) **Suspended coil galvanometer** : In this galvanometer, the coil is suspended freely, and can rotate in a uniform magnetic field produced by a permanent magnet. The instrument is very sensitive and can measure current as low as 10^{-9} or 10^{-10} ampere.

The instrument consists of a rectangular coil *ABCD* of several turns of fine copper wire wound on a light metallic (brass or aluminium) frame. The coil is suspended by a thin phosphor-bronze strip between the poles *N* and *S* of a horse-shoe type permanent magnet [Fig. 38]. The lower end of the coil is joined to a fine spring *S* made of phosphor-bronze which provides a flexible coupling of the coil to the case. The phosphor-bronze suspension is attached to a torsion head *T*. Current enters *via* phosphor-bronze suspension, passes through the coil and leaves *via* the spring *S*. Between the pole-pieces *N* and *S*, lies a cylindrical core of soft iron *E* fixed to the wooden board. The coil can freely turn in the space between the pole-pieces and the soft-iron cylinder without touching either of them. The soft iron cylinder intensifies the magnetic field. Attached to the phosphor-bronze suspension is a small concave mirror *M* which allows small deflections of the coil to be read with a lamp and scale arrangement.

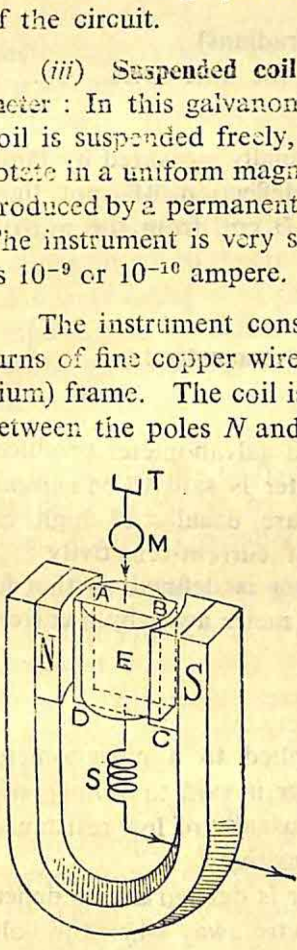


Fig. 38

In very sensitive instruments, the pole-pieces are curved to form parts of a cylinder co-axial with the suspension of the coil. The soft-iron cylindrical piece *E* is also made co-axial with the pole-pieces.

This makes the magnetic field radial to the core E and the pole-pieces over the region in which the coil swings.

According to its theory, $i=\frac{\tau}{nAH}.\theta=k.\theta.$

where i=current in the coil in *e.m.u.*
n=number of turns of the coil
A=face area of each turn of the coil
H=magnetic field intensity
θ=the deflection of the coil (in radians)
τ=moment of torsional couple for unit twist
K=a constant.

The angle of deflection (θ) of the coil is usually measured by lamp and scale arrangement. If d cm. be the deflection of spot light reflected by the mirror M on a scale placed D cm. from the mirror, then $2\theta=\frac{d}{D}$; hence $i=K.\frac{d}{2D}$ *i.e.*, $i \propto d$.

Some important facts about suspended coil galvanometer :

Current sensitivity :

When a feeble current flowing through a galvanometer produces a large deflection of its coil, the galvanometer is said to be *current-sensitive.* Current-sensitive galvanometers are usually of high resistance. The following is the definition of current-sensitivity :

The current-sensitivity of a galvanometer is defined as the deflection in millimeters produced on a scale 1 metre away by a current of 1 micro-ampere (*i.e.* 10^{-6} amp.)

Voltage sensitivity :

When a small potential difference applied to a galvanometer produces a large deflection, the galvanometer is said to *voltage sensitive.* Voltage-sensitive galvanometers are, usually of low resistance. The following is the definition of voltage-sensitivity.

The voltage-sensitivity of a galvanometer is defined as the deflection in millimetres produced on a scale 1 metre away when the voltage applied to the galvanometer is 1 micro-volt (*i.e.*, 10^{-6} volt).

Figure of merit of a galvanometer : The figure of merit of a moving coil galvanometer is defined as the current which produces a deflection of 1 mm. of the spot of light on a scale placed 1 metre away from the galvanometer mirror.

Suppose the scale is placed 1 metre away from the mirror and the spot of light is deflected through S mm. by a current of I ampere. In this case, the figure of merit of the galvanometer $=\frac{I}{S}$ amp/mm. Very sensitive D' Arsonval galvanometer may have a figure of merit of the order of 10^{-11} amp/mm.

(*iv*) **Dead-beat galvanometer** : A galvanometer designed to measure a steady current flowing for a pretty long time, is usually made dead-beat *i.e.*, once deflected, the moving coil of the galvanometer returns to the position of rest very quickly. It does not oscillate for a long time. The coil of such galvanometers is wound on a metallic frame. This increases the damping due to eddy current and electromagnetic effect.

(*v*) **Ballistic galvanometers** : In the case where a certain charge passes momentarily through an instrument due to sudden discharge, a ballistic galvanometer is used to measure it. The time of passage of charge being very short, the coil of the galvanometer receives an impulse and a "throw" is registered. In order that the first throw of the galvanometer coil can measure the momentary charge flowing through it, the ballistic galvanometer has the following characteristics : (*a*) the time-period of oscillation of the moving coil is made high by increasing its moment of inertia (*b*) the damping is made a minimum by winding the coil on a non-conducting frame (*i.e.*, frame of bamboo, ivory or wood).

5.2. Precautions for electrical experiment :

The appliances required for an electrical experiment are mostly sensitive and costly. So, they require careful handling. Besides, some fundamental precautions are always necessary for successful operation of an electrical experiment. The essential precautions are as follows :

(*a*) All connections in the circuit should be tight. Loose connections bring in various troubles. Before building up a circuit by connecting wires, cotton insulations at the two ends of the wire are to be removed and the ends to be rubbed by sand-paper. *Do not rub a screw or a plug by sand-paper.* Bend the end of the wire in the form of an incomplete ring and slide it into the screw terminal which is then tightened. Pull the wire a little to see whether the connection is tight. Remember that the connection should neither be too tight nor too loose.

(*b*) For wrong connections or for heavy passage of current sufficient heat may be produced damaging the appliances used. For this reason, after completing a circuit and before sending current, ask the teacher to check your connections. If the connections are alright then send the current. Allow the current to flow only as long as it is required for taking the necessary readings. *Current should never be allowed to flow unnecessarily.*

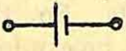
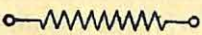
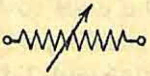
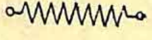
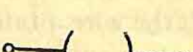
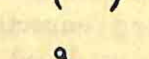
(*c*) While using instruments like resistance boxes and plug keys, care should be taken to insert into or to take out a plug from its socket. While inserting, put it in its gap and then slightly turn it. While taking it out, slightly turn it and then pull it out. Loose plugs create various troubles. After taking out a plug from its socket, do not allow it to be soiled by dust or dirt. Put it in a clean place. Do not mix up plugs of one instrument or of one key with those of other instruments or other key. Plugs of one resistance box, say, may not be proper fit to the sockets of another box.

(*d*) While using a cell—specially a storage cell—always use a resistance in series with it and do not allow the cell to be short-circuited.

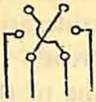
(*e*) Do not connect sensitive apparatus like galvanometers, ammeters etc directly with a cell. Take care about the positive and negative terminals while using voltmeters, ammeters etc.

5.3. Symbols for electric appliances :

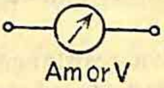
Symbols are used to represent appliances used in an electric circuit in diagrams. It is desirable that students should get familiar with those symbols. The symbols, used frequently, are as follows :

Appliances	**Symbols**
1. Cell	 (longer line represents + ve pole and the short one – ve pole)
2. Resistance	
3. Variable resistance or Rheostat	 or 
4. Plug Key	
5. Tap key or Switch	

6. Commutator



7. Ammeter or Voltmeter



5.4. Determination of the value of the resistance of a wire by a metre-bridge making end-corrections and hence the calculation of specific resistance of the material of the wire.

Apparatus : A metre bridge, table galvanometer, a resistance box (0-500 ohms), a 1-ohm block resistance, a commutator, a Leclanche's cell, a piece of copper or constantan wire, a screw gauge, a metre scale etc.

Theory : (*i*) *For the measurement of end-correction* :

In the metre bridge, the wire *AC* is soldered at its both ends to two thick copper strips *AE* and *CN* [Fig. 39]. However thick the strips may be, they have some resistance. Further, the soldering of the bridge wire at *A* and *C* also introduces some resistance. Moreover, the wire *AC* may not be exactly 1 metre long. For all these reasons, it is supposed that some extra resistances are introduced

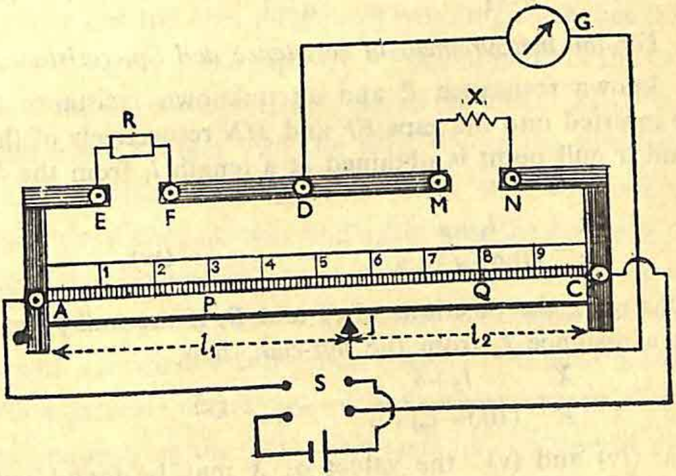


Fig. 39

at the left and right ends of the bridge wire. If these resistances are not taken into consideration, the result becomes inaccurate. This error is called 'end-error'. Usually this end-resistances are expressed in terms of the lengths of the bridge wire. The necessary lengths of bridge wire *i.e.*. the end-corrections may be found out in the following way :

Two known resistances P and Q are inserted into the gaps EF and MN respectively of the metre bridge. If l_1 be the length of the null-point from the *left end* of the bridge wire,

$$\frac{P}{Q}=\frac{l_1+\alpha}{(100-l_1)+\beta} \quad .. \quad \text{(i)}$$

where α=end-correction at the left end and β=the end-correction at the right end.

On interchanging the positions of P and Q, if the null point be found at a length l_2 from the *left end*, then

$$\frac{Q}{P}=\frac{l_2+\alpha}{(100-l_2)+\beta} \quad .. \quad \text{(ii)}$$

Solving the equations (i) and (ii) and taking the ratio $P/Q=a$, we get,

$$\alpha=\frac{l_1-a.l_2}{a-1} \text{ and } \beta=\frac{al_1-l_2}{a-1}-100 \quad .. \quad .. \quad \text{(iii)}$$

(*ii*) *For the measurement of resistance and Sp. resistance* :

If a known resistance R and an unknown resistance (a wire, say) X be inserted into the gaps EF and MN respectively of the metre bridge and a null point is obtained at a length l_1 from the *left end*, then,

$$\frac{R}{X}=\frac{l_1+\alpha}{(100-l_1)+\beta} \quad .. \quad .. \quad \text{(iv)}$$

On interchanging the positions of X and R, if the null-point be obtained at a distance l_2 from the *left-end*, then,

$$\frac{X}{R}=\frac{l_2+\alpha}{(100-l_2)+\beta} \quad .. \quad \text{(v)}$$

From eqn. (iv) and (v), the values of X may be found out. The mean of these two values gives the correct value of X.

Further, if L be the length of the wire whose resistance is X and r the radius of its cross-section, then the specific resistance ρ of its material is given by, $\rho=\frac{\pi r^2 X}{L}$ .. (vi)

Description of metre-bridge : Fig. 39 shows the form of a metre-bridge. It is a practical application of Wheatstone bridge. AC is a uniform, straight wire, one metre long, stretched alongside a metre scale fixed on a wooden board. X is an unknown resistance. The two ends of the bridge wire are soldered to two thick copper strips AE and CN of the shape 'L'. There is another straight piece of copper strip FM fixed on the board. The unknown resistance or the wire is put on the gap MN while a resistance box R on the gap EF. G is a sensitive galvanometer, one end of which is connected to a movable jockey J, which can slide along the wire AC. A cell B is connected to the ends A and C of the wire through a commutator S.

Experimental procedure : *Determination of* α *and* β :

(1) Insert the resistance box R in the gap EF and 1-ohm block resistance in the gap MN. Connect the galvanometer G and the battery B (or the Lechlanche's cell) through a commutator S as shown in fig. 39. Pull all the connecting wires a little to see whether all the connections are tight.

(2) Take out 100-ohm plug ($=P$) from the resistance box. The 1-ohm block ($=Q$) is in the gap MN. Put two plugs in the two opposite holes of the plug commutator. Move the jockey J to one extreme end A and make contact with the wire. Note the direction of deflection of the galvanometer pointer. Now slide the jockey to the other extreme end C of the wire and again note the direction of deflection after making a contact between the jockey and the wire. If the deflections are opposite, the connections are right. Now, making contacts with various points over the length of the wire, find the null-point where galvanometer shows no deflection. Ascertain the length of the null-point from the scale. Reverse the direction of the current by putting the plugs in the other two opposite holes of the commutator and again find the null-point. Find the mean of these two null points (l_1).

(3) The resistance box and the 1-ohm block are interchanged. The ratio now becomes 1/100. Again find the null-points with direct and reverse current. Find the mean of these two null-points (l_2).

(4) Find the values of α and β with the help of the equation (iii) of the theory.

(5) Repeat the whole operation twice with 80 ohm and 50-ohm resistances put in the resistance box. This will give two more sets of values of α and β. The mean values of α and β are then found out from the three sets of values.

Determination of resistance and sp. resistance of a wire :

(6) Find, with the help of a screw-gauge the diameter of the given sample of wire at various places and at right angle directions at each place. From all these observations, find the mean diameter and hence the radius (r) of the cross-section of the wire.

(7) Bend about 1 cm. length of the given wire at its two ends at right angles to the length of the wire. Find the length (L) of the wire between *the two bends.*

(8) Make circuit connections as shown in fig. 39. Two opposite terminals of the commutator (S) are to be connected to the cell and the other two opposite terminals to the points A and C of the metre bridge. Connect the resistance box R in the left-hand gap (EF) and the wire (X) in the right-hand gap (MN). While connecting the wire in the gap, take care that the *bent portion at the two ends are put inside the binding screws.* Whenever the wire is connected to any gap, this practice is to be always adhered to. One end of the galvanometer is connected to the jockey J and the other end to the point D. Put a suitable resistance in the resistance box (say, 2-ohm).

(9) Put two plugs in the two opposite holes of the commutator. Move the jockey J to one extreme end A and make contact with the wire. Note the direction of deflection of the galvanometer pointer. Now slide the jockey to the other extreme end C of the wire and note the direction of deflection after making contact between the jockey and the wire. If the deflections are opposite, the connections are right and the null-point is somewhere at the middle. On the other hand, if the deflections are in the same direction, the connections are defective. In that case, the connections are to be checked carefully after taking out the plugs from the commutator. If necessary, the advice of teachers may be sought.

(10) When right connections are made, move the jockey slowly from one end (say, A) towards the other and find the null-point where the galvanometer shows no deflection when the jockey makes contact with the wire. Find the distance of the null-point from the scale attached to the bridge. Now move the jockey from the end C towards the end A and similarly find the position of the null-point. The mean of these two readings gives the final position of the balance point.

(11) Now reverse the current by putting the plugs in the other two opposite holes of the commutator. Repeat the observation no. 10 and find the balance point. The mean of the readings for the balance points obtained in the operation no. 10 and 11 will give the exact position of the balance. Let l_1 be the length of the wire upto the balance point measured from the left-end of the bridge wire.

(12) Add α to l_1 and β to $(100-l_1)$ and find the value of X according to the equation (iv) of the theory.

(13) Interchange the positions of the resistance box R and the wire X *i.e.*, put the resistance box R in the gap MN and the wire in the gap EF. Keeping the resistance in the box R same as before (say, 2 ohm) and following the procedure mentioned in the sections (10) and (11), find the length (l_2) of the null-point.

(14) Add α to l_2 and β to $(100-l_2)$ and find the value of X according to the equation (v) of the theory.

(15) Now put a different resistance (say, 5 ohms) in the resistance box and repeat all the observations mentioned earlier. Find the value of X for this set of observations. If time permits, another set of readings may be taken with another resistance in the box R (say, 10 ohms). The mean of three values of X obtained from the three sets of observations, gives the correct value of X.

(16) Putting the values of X, L and r in the equation (vi) of the theory, calculate the specific resistance of the material of the wire.

Measurements : (*a*) *Determination of* α *and* β :

No. of obs.	Resistance in the gap EF (ohm)	Resistance in the gap MN (ohm)	Ratio $(=a)$	Length of the null point with			α	β
				Direct current	Reverse current	Mean		
1.	100	1	100	..	..	$..(l_1)$	..	..
	1	100	1/100	..	..	$..(l_2)$		
2.	80	1	80	..	..	$..(l_1)$	..	..
	1	80	1/80	..	..	$..(l_2)$		
3.	50	1	50	..	..	$..(l_1)$	..	..
	1	50	1/50	..	..	$..(l_2)$		

Mean value of $\alpha = \dfrac{..+..+..}{3} = ..\text{cm.}$

,, ,, ,, $\beta = \dfrac{..+..+..}{3} = \;..\;$ cm.

(*b*) *Measurement of length* (*L*) *between the two bends of the wire* :

(i) .. cm (ii) .. cm (iii) .. cm. $\therefore$ Mean $L = ..$cm (*c*) *Measurement of diameter of the wire by screw gauge* :

Value of 1 small division of main scale = . . mm

Screw pitch = . . mm

Total number of divisions in the circular scale = . .

$\therefore$ least count (*l.c.*) = . . mm.

No. of obs.	Linear scale reading (M)	Circular scale reading (N)	Cir. scale reading $\times l.c.$ ($N \times l.c.$)	Total reading ($M + N \times l.c.$)	Mean diameter	Instrumental error	Corrected diameter (d)
1 (i)	. . mm	. .	. . mm	. .		± . . mm	. .
1 (ii)	. . mm	. .	. . mm	. .			
2 (i)	etc.	etc.	etc.	etc.			
2 (ii)							
3 (i)	etc.	etc.	etc.	etc.			
3 (ii)							

[N.B. (i), (ii) signify readings taken at two mutually perpendicular directions]

Radius of the wire $r = \frac{d}{2} =$ . . cm.

(*d*) *Measurement of the resistance* (X) *of the wire* :

No. of obs.	Resistance in the gap EF (ohm)	Resistance in the gap MN (ohm)	Length of the null point with			$(100-l_1)$ or $(100-l_2)$	X (ohm)
			Direct current	Reverse current	Mean		
1.	R	X	..	..	..(l_1)	..	..
	X	R	..	..	..(l_2)	..	..
2.	R_1	X	..	..	..(l_1)	..	..
	X	R_1	..	..	..(l_2)	..	..
3.	R_2	X	..	..	..(l_1)	..	..
	X	R_2	..	..	..(l_2)	..	..

Mean value of $X=$. . . ohm.

Calculations : (i) $\alpha = \frac{l_1 - al_2}{a-1} = \ldots = \ldots = \ldots$

$$\beta = \frac{al_1 - l_2}{a-1} - 100 = \ldots = \ldots = \ldots$$

(ii) $\frac{R}{X} = \frac{l_1 + \alpha}{(100 - l_1) + \beta} \quad \therefore \quad X = \ldots = \ldots = \ldots$

$$\frac{X}{R} = \frac{l_2 + \alpha}{(100 - l_2) + \beta} \quad \therefore \quad X = \ldots = \ldots = \ldots$$

(iii) $\rho = \frac{\pi r^2 . X}{L} = \ldots$ ohm-cm.

Remarks : (1) The exact length of the wire upto the null point may be different from that given by the scale. To eliminate the error, the null point should be ascertained by interchanging the positions of R and X and the mean value of the two readings should be found out. (2) A continuous flow of current through the bridge-wire as well as through the junctions of different metals of the bridge may give rise to heat and thermo-electric current, causing a change in the position of the null points. For this reason, observations should be taken by reversing the direction of the current with the help of the commutator and a tap key should preferably be included in the battery circuit in order to allow the current to flow as long as necessary. (3) If the resistance box and the unknown resistance are made of resistance coils, then, at the time of make and break of the circuit, a current may be induced due to self-induction of the coils, which may cause an error in determining the exact position of the balance point. To overcome the difficulty, the battery circuit should be closed first and then the galvanometer circuit. (4) The bridge is most sensitive when the resistances of the four arms are very nearly equal to each other. For this reason, the metre bridge does not give satisfactory result in the cases of low and high resistances. (5) In order to make the proportional error minimum the null-point should be adjusted to remain between 45 cm to 55 cm. (6) End-corrections are usually small. So, the corrections α and β become negligible when the null-point is obtained near the middle of the bridge wire.

Oral questions

1. What is the principle of Wheatstone bridge ?

Ans. Consult any text book.

2. What is the working principle of a metre bridge ?

Ans. The principle of Wheatstone bridge is the working principle of a metre bridge.

3. What do you mean by end-correction in a metre bridge?

Ans. See theory of the experiment.

4. Why should the balance point in the above experiment be preferably found at the central region of the wire ?

Ans. This will reduce the proportional error to a minimum.

5. Why should you take readings in the above experiment with reverse current ?

Ans. Thermo-electric current may flow due to temperature difference at the junctions of different metals of the bridge. To eliminate errors due to thermo-electric current observations should be taken with reverse current.

6. What is the necessity of interchanging the positions of X and R ?

Ans. See remark no. 3.

7. Is there any material change when α is added to l in measuring the resistance ?

Ans. End corrections are usually small—of the order of 0·1 cm or 0·2 cm. If the length of null point lies between 45 cm. to 55 cm. the error is about 1 or 2 in 500. Errors due to other reasons are greater than this error. So, there is no material change provided α is small.

8. Why is the instrument called 'metre bridge' ?

Ans. Since the bridge-wire is one metre in length, the instrument is called a 'metre bridge'.

9. Comment on the desirability of measuring high and low resistances with a metre bridge.

Ans. See remark no. 4.

10. What is the harm if continuous current is sent for long time through the metre bridge ?

Ans. See remark no. 2.

11. What is specific resistance ? What is its unit ?

Ans. See any text book. Its unit is ohm-cm.

12. Does specific resistance of a wire depend on the radius of its cross-section ?

Ans. No ; specific resistance is a property of the material.

5.5. Measurement of resistance per unit length of the bridge wire by Carey Foster method :

Apparatus : A metre bridge with four gaps (it is alternatively called Carey Foster bridge). Two equal resistances (say 1 Ω and 1 Ω), a fractional resistance box (0·1 Ω to 10 Ω), a table galvanometer, Leclanche's cell, a plug commutator, connecting wire etc.

Circuit connection : The necessary circuit connection has been shown in fig. 40. The fractional resistance box *S* is put on the extreme left gap no 1 and a thick copper plate *R* on the extreme right gap no 4. Two equal resistances *P* and *Q* (say, 1 ohm each) are connected to the gaps no 2 and 3. A leclanche's cell (*E*)is joined to the points *F* and *H* through a commutator *C*. One end of a galvanometer *G* is joined to the point *O* and the other end to a jockey which slides along the bridge wire *AB*.

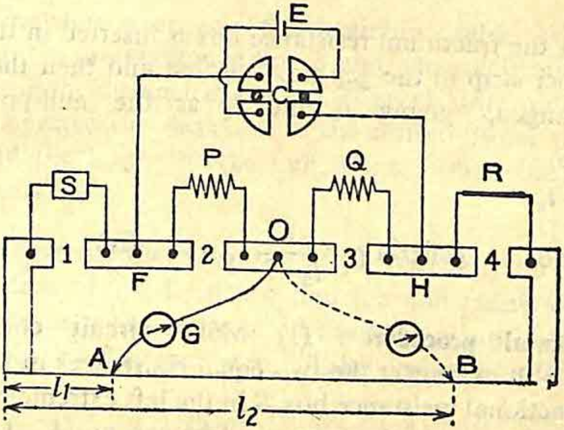


Fig. 40

[*N. B.* If, instead of a table galvanometer, a suspended coil galvanometer is used, a rheostat in series with the Leclanche's cell and a shunt in parallel with the galvanometer should be used as a measure of safety for the galvanometer.]

Theory : In the arrangement shown in fig. 40, if the null point is obtained at a length l_1 measured from the *left end* of the bridge wire, then from Wheatstone bridge principle,

$$\frac{P}{Q}=\frac{S+\alpha+l_1\rho}{R+\beta+(100-l_1)\rho} \quad \ldots \text{(i)}$$

where α and β are the end-corrections at the two ends of the bridge wire and ρ the resistance per unit length.

If the positions of S and R are now interchanged and a null point is obtained at a length l_2 measured from the *left end* of the wire, then,

$$\frac{P}{Q}=\frac{R+\alpha+l_2\rho}{S+\beta+(100-l_2)\rho} \quad \ldots \quad \text{(ii)}$$

Solving these two equations, we get* $\rho=\dfrac{S-R}{l_2-l_1} \quad \ldots \quad \text{(iii)}$

Since a thick copper strip has almost no resistance, $R=0$;

So $\rho=\dfrac{S}{l_2-l_1}$

Again, if the fractional resistance box is inserted in the gap no, 2 and the copper strip in the gap no. 1 at first and then their positions are interchanged, giving l_1 and l_2 as the null-point lengths,

then $\rho=\dfrac{S}{l_1-l_2}$

$$\text{So, in general } \rho=\frac{S}{l_2\sim l_1} \quad \ldots \quad \text{(iv)}$$

Experimental procedure : (1) Make circuit connections as shown in fig. 40. Connect the two equal resistances in the gaps no 2 and 3, the fractional resistance box S in the left extreme gap no. 1 and the thick copper strip in the extreme right gap no. 4. Use thick and short wires instead of usual connecting wires for connecting the resistances in the gaps.

(2) Without taking any plug out of the resistance box S (*i.e.* $S=0$), find the null point. If everything is right, the null point should be very near 50 cm. mark. If the null point is far away from 50 cm. mark, check whether connections are tight and all the plugs in the resistance box are tightly placed in their sockets. If the null point is still away from 50 cm. mark, the resistances P and Q in the gaps no. 2 and 3 are not equal. Replace them by exactly equal resistances.

---

* From eqn. (i) and (ii) we get $\dfrac{S+\alpha+l_1\rho}{R+\beta+(100-l_1)\rho}=\dfrac{R+\alpha+l_2\rho}{S+\beta+(100-l_2)\rho}$

Adding 1 to both sides, $\dfrac{S+R+\alpha+\beta+100.\rho}{R+\beta+(100-l_1)\rho}=\dfrac{R+S+\alpha+\beta+100\rho}{S+\beta+(100-l_2)\rho}$

$\therefore \quad R+\beta+(100-l_1)\rho=S+\beta+(100-l_2)\rho$

or, $(S-R)=\rho(l_2-l_1)$

or, $\rho=\dfrac{S-R}{l_2-l_1}$.

(3) Now take out the lowest resistance (*i.e.* 0·1 ohm) plug from the resistance box *S*. See whether the null point is near the middle of the wire. Slowly increase the resistance in the box *S*. The null point will slowly move towards the left. Find the maximum resistance (say 2·2 ohms) that may be put in the resistance box *S* to get the null-point nearest to (say 3·8 cm) the left end of the wire. This gives an idea of the maximum resistance which can be used in the present circumstances. Null point will not be available for resistance greater than this.

(4) Having thus ascertained the maximum usable resistance, (say 2·2 ohms), put it in the resistance box *S* and accurately find the null-point. Ascertain the distance of the null point (l_1 cm) from the left end of the wire. Reverse the direction of the current by the commutator and again find the length of the null point. Get the average of these two lengths.

(5) Now decrease the resistance in the resistance box *S* by small steps (say by steps of 0·4 ohm) and find the null points at each step for direct as well as reverse currents. Find the mean of the two null point lengths obtained at each step. Take observations for, at least, four such steps.

(6) Now interchange the positions of the fractional resistance box *S* and the copper strip *R* *i.e.* connect the strip in gap no. 1 and the resistance box *S* in gap no. 4.

(7) Now, starting with the same maximum resistance (*i.e.* 2·2 ohms) in the resistance box *S* and decreasing it by the same steps (*i.e.* 0·4 ohm) as before, find the null points at each step for direct and reverse currents. Get the mean of the null points at each step (l_2 cm).

(8) Calculate using equation (iii) of the theory, the value of ρ from the two null point lengths l_1 and l_2 obtained by interchanging the positions of *S* and *R*, for a particular value of the resistance put in the resistance box *S*. Thereafter, find the mean value of ρ from the values obtained from different values of resistance put in the resistance box.

(9) Note that if the bridge wire is of uniform cross-section and the connecting wires used are short and thick, the different values of ρ obtained from different observations will agree and their mean will give the correct value of ρ. If, on the other hand, different values of ρ do not agree, then take the mean of the values of ρ obtained in the cases where *S* has got three larger values out of four of five observations.

Measurements : $P=Q=1$ ohm (as an illustration)

No. of obs.	Resistance in the box S (ohms)	Null point when the copper plate is in						(l_2-l_1) cm	$\rho=\frac{S}{l_2-l_1}$ ohm/cm
		Extreme right gap no. 4			Extreme left gap no. 1				
		Direct current	Reverse current	Mean (l_1) cm	Direct current	Reverse current	Mean (l_2) cm		
1.	2·2	3·8	4	3·9	96·2	96	96·1	92·2	·023
2.	1·8	10·8	10·8	10·8	89·2	89·2	89·2	78·4	·023
3.	..	..	..	..	..	..	..	..	..
4.	..	..	..	..	..	..	..	..	..
5.	..	..	..	..	..	..	..	..	..

Mean value of $\rho=\ldots$ ohm/cm.

Calculations : 1. $\rho=\frac{S}{l_2-l_1}=\frac{2{\cdot}2}{92{\cdot}2}=0{\cdot}023$ ohm/cm.

2. $\rho=\frac{S}{l_2-l_1}=\frac{1{\cdot}8}{78{\cdot}4}=0{\cdot}023$ ohm/cm.

3. etc.

Remarks : (1) In this experiment, the end-corrections of the bridge wire need not be considered ; they are automatically eliminated. (2) If the values of ρ obtained at different positions of the wire be not same, the wire is not of uniform cross-section. Hence, this experiment gives us a method to check the uniformity of the wire (3). At the beginning, it should be seen whether with $S=R=0$, the null point is obtained at a point very near the 50 cm. mark. If the null point is obtained more on the right of 50 cm mark, the resistance P is defective. If, on the other hand, the null-point is more on the left of 50 cm. mark, the resistance Q is defective.

Oral questions

1. Why do you find the null point with direct and reverse currents ?

Ans. When current flows through the bridge wire, heat is produced which changes the resistance of the wire and consequently the position of the null point. If null point is found out with reverse current and the mean of the two readings is taken, the error due to this reason is eliminated.

2. How do the end-corrections in this experiment affect the result ?

Ans. The end-corrections at the two ends of the wire are automatically eliminated and hence they do not affect the final result.

3. Can you ascertain, by this experiment, whether the bridge wire is of uniform cross-section ?

Ans. Yes ; the bridge wire is of uniform cross-section if the values of ρ at various positions of the wire are the same ; In the case when the values of ρ are different, the wire is non-uniform.

4. Measuring the resistance of a wire by a P.O. Box or a metre bridge and then dividing the resistance by its length, ρ can be found out. What is, then, the necessity of a Carey Foster's bridge ?

Ans. P.O. Box method is not as accurate as Carey Foster bridge method. Further, by Carey Foster method, ρ can be found out at various places of the wire.

5. Can the experiment be done with any ratio of P and Q ?

Ans. If $Q \gg P$, then even with $S=R=0$, the null point will be very near the left end of the bridge wire. If now some resistance is put in the box S, then with

$R=0$, the null point will move further left and may go beyond the wire. As a result the experiment fails. For this reason, the ratio of P and Q is kept equal to or very near to 1.

6. Can this experiment be used to determine the difference between two resistances ? Is there any condition for this ?

Ans. In the theory, it is seen that $S-R=\rho(l_2-l_1)$; So knowing ρ, l_1 and l_2, the difference between two resistances S and R can be found out. The limiting condition is that the difference should be less than the total resistance of the bridge wire.

7. If the value of ρ of the bridge wire is 0·023 ohm/cm, what is the total resistance of the wire ?

Ans. The length of the wire=100 cm. So, the total resistance=0·023 × 100=2·3 ohms.

8. What is your idea about the total resistance of the bridge wire ?

Ans. Usually, the total resistance of the bridge wire lies between 2 and 3 ohms.

9. Can the sp. resistance (S) of the material of the wire be found out from the value of resistance per unit length (ρ) ?

Ans. Yes ; Sp. resistance $S=\frac{R\times\alpha}{l}$; but $R=\rho.l$ $\therefore$ $S=\frac{\rho.l.\alpha}{l}=\rho.\alpha=\rho.\frac{\pi d^2}{4}$.

So, finding ρ and then measuring the diameter (d) of the wire by a screw gauge, S can be found out.

10. What is the relation between ρ and diameter (d) of the wire ?

Ans. $\rho \propto \frac{1}{d^2}$ *i.e.* the resistance per unit length varies inversely as the square of the diameter.

11. Will ρ remain same if the material remains the same but the wire becomes thicker or thinner ?

Ans. ρ is smaller for thicker wire and greater for thinner wire.

12. Can you find the resistance of a wire by this experiment ?

Ans. In the theory, it is seen $S=\rho\ (l_2-l_1)$; so knowing ρ, l_1 and l_2, an unknown resistance S can be found out. (See expt no. 5.6)

5.6. Determination of an unknown resistance by Carey Foster's method

Apparatus : A metre bridge with four gaps, two resistances of equal value (say 1 ohm each), a fractional resistance box (0·1Ω to 10Ω), a table galvanometer, a Leclanche's cell, a plug commutator, an unknown resistance (a block resistance or a coil of wire) a thick copper strip etc.

Circuit connection : As in expt no. 5.5.

Theory : In the theory of the previous experiment, it has been seen that if a fractional resistance box S is connected to the extreme left gap no 1 and an unknown resistance R in the extreme right gap no 4 and in this condition a null point is obtained at a distance l_1 cm. from the left end of the wire and then a null point is again obtained at a distance l_2 cm from the left end with the positions of S and R interchanged, then $S-R=\rho(l_2-l_1)$ where ρ=resistance/unit length of the bridge wire.

$$\therefore \quad R=S-\rho(l_2-l_1)$$

Knowing S, ρ, l_1 and l_2, the unknown resistance R can be found out.

Experimental procedure : (1) Following the procedure described in the previous experiment and using the fractional resistance box in one gap and a thick copper strip in the other gap, find the resistance per unit length (ρ) of the bridge wire.

(2) Now connect the fractional resistance box S in the extreme left gap no. 1 and the unknown resistance, instead of the copper strip, in the extreme right gap no. 4. Take out suitable resistance from the resistance box S so that null point is obtained near the central portion of the bridge wire (between 45 cm to 50 cm). [If the null point is obtained far left of the central portion, increase the resistance in the resistance box. The null point will move towards the right. If the null point, on the other hand, is obtained far right of the central portion, decrease the resistance in the resistance box ; the null point will move towards the left]. Note the distance of the null point from the left end of the bridge wire. Reverse the direction of current by the commutator and again find the null point. Calculate the mean distance of the null point (l_1) from these two readings.

(3) Now, increase the resistance of the resistance box by equal steps (say, by steps of 0·2Ω) and at each step find the null point lengths for direct and reverse currents. Find the mean value of the null point lengths in each case. Take, at least, three such observations.

(4) Now interchange the positions of the fractional resistance box S and the unknown resistance R *i.e.* connect the unknown resistance in gap no. 1 and the resistance box in gap no. 4. Take out the resistance plugs from the resistance box in the same order as was done in the previous operation and in each case, find the null points for direct and reverse currents. Find the mean null point length in each case (l_2).

(5) Find the value of R from the two null point lengths (l_1 and l_2) obtained by interchanging the positions of S and R in each case and from these observations ascertain the mean value of R.

Measurements : $P=Q=1$ ohm (say)

(*a*) *Measurement of* ρ :

No. of obs	Resistance in the *S*-box (ohm)	Position of null point when the thick copper strip is						(l_2-l_1) cm.	$\rho=\frac{S}{l_2-l_1}$ ohm/cm.	Mean ρ ohm/cm.
		in the extreme right gap no 4			in the extreme left gap no 1					
		Direct current	Reverse current	Mean (l_1) cm.	Direct current	Reverse current	Mean (l_2) cm.			
1.	..	..	..	..	..	..	..	..	..	
2.	..	..	..	..	..	..	..	..	..	..
3.	..	..	..	..	..	..	..	..	..	
4.	..	..	..	..	..	..	..	..	..	

(*b*) *Measurement of the unknown resistance* R : (Data given as illustrations)

No. of obs	Resistance in the S-box (ohm)	Position of null point when the unknown resistance R is						(l_2-l_1) cm.	$R=S-\rho(l_2-l_1)$ (ohm)	Mean R (ohm)
		in the extreme right gap no 4			in the extreme left gap no 1					
		Direct current	Reverse current	Mean (l_1) cm.	Direct current	Reverse current	Mean (l_2) cm.			
1.	..	45·3	45·1	45·2	54·6	54·8	54·7	(54·7−45·1)	..	
2.	..	31·5	31·3	31.4	67·9	67·7	67·8	(67·8−31·4)	..	..
3.	..	..	..	..	..	..	..	..	..	
4.	..	..	..	..	..	..	..	..	..	

Calculations : $R=S-\rho(l_2-l_1)=..$ ohm.

Remarks : All connections are to be made with thick and short connecting wires instead of usual thin wires. For other remarks see expt. no. 5.6.

Oral questions

See Experiment no. 5.6.

5.7. Measurement of the resistance of a suspended coil galvanometer by half deflection method.

Also calculation of the galvanometer current when there is no resistance directly in series with the galvanometer. Hence determination of the galvanometer current to produce a scale deflection of 1 mm.

Apparatus : A suspended coil galvanometer (D′ Arsonval type), three resistance boxes—one of high range (upto 5000Ω), one of moderate range (upto 500Ω) and one of very low range (between 0·1 to 0·5Ω), a plug commutator, a storage cell, connecting wires etc.

Circuit connection : Fig. 41 shows the necessary circuit connection. A resistance box *R* of high range (upto 5000) is connected in series with the cell *E* (if necessary two storage cells may be connected in series to form a battery) and then to the two opposite terminals (no. 1 and 3) of the commutator *C*. The other two opposite terminals (no. 2 and 4) are joined to the resistance box *S* of low range (*i.e.* between 0·1 to 0·5 ohm). This resistance box *S* is called the shunt box. The galvanometer *G* with the moderate range (*i.e.* upto 500 ohm) resistance box R_1 in series, is connected in parallel with the shunt box *S*.

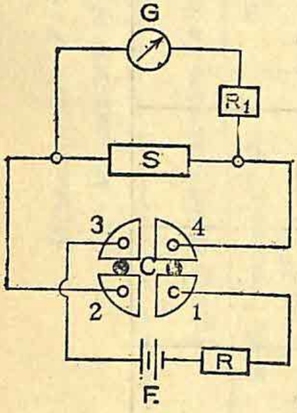


Fig. 41

Theory : (1) *For measurement of resistance :*

From figure 41 it is clear that the resistance $(G+R_1)$ and S are parallel to each other. If r be their equivalent resistance, then

$$\frac{1}{r}=\frac{1}{G+R_1}+\frac{1}{S}=\frac{S+G+R_1}{S(G+R_1)} \quad \therefore \quad r=\frac{S(G+R_1)}{S+G+R_1}$$

If $S \ll G$, the equivalent resistance r is very nearly equal to S. So, whatever may be the value of R_1, as long as $S \ll G$, the equivalent resistance is equal to S. Under this circumstances, the *p.d.* across the resistance S for a given current (main) will be independent of the value of R_1.

If I_g be the current through the galvanometer when $R_1=0$,

$$\text{then } I_G=\frac{V}{G}=k.d. \quad \ldots \text{(i)}$$

where V=terminal *p.d.* of the shunt S ; d=the deflection of spot of light on the scale and k=constant of proportionality.

If, now, the deflection is made exactly half the previous value by taking out the resistance plug in the box R_1 (say R_1 ohm) and the corresponding current through the galvanometer be I'_G,

$$\text{then } I'_G=\frac{V}{G+R_1}=k.\frac{d}{2} \quad \ldots \text{(ii)}$$

From (i) and (ii), we get $\frac{G+R_1}{G}=2$ or $G=R_1$ .. (iii)

(2) *Galvanometer current when* $R_1=0$:

When $R_1=0$, the equivalent resistance of G and $S=\frac{GS}{G+S}$.

$$\therefore \text{ Total circuit resistance}=\frac{GS}{G+S}+R=\frac{GS+R(G+S)}{G+S}$$

$$\text{Main current } I=\frac{E}{\text{Total resistance}}=\frac{E(G+S)}{GS+R(G+S)}$$

$$\therefore \text{ Current through the galvanometer } I_g=\frac{S}{G+S}.\ I$$

$$=\frac{S}{G+S}\times\frac{E(G+S)}{GS+R(G+S)}=\frac{E.S}{S(R+G)+G.R} \quad \ldots \text{(iv)}$$

If d mm. be the deflection produced on the scale by this current, then the current required for 1 mm. deflection

$$=\frac{I_g}{d}=\frac{1}{d}\times\frac{E.S}{S(R+G)+GR} \text{ amp/mm.} \quad \ldots \text{(v)}$$

Experimental procedure : (1) Adjust the galvanometer scale and the spot of light such that the spot of light coincides with the zero mark of the scale. Make circuit connections as shown in fig. 41. Put 0·1 or 0·2 ohm resistance in the shunt box S. Without putting any resistance in the box R_1, put a certain resistance (say 1000 ohm) in the box R which is in series with the cell. Note the deflection of the spot of light. Adjust the resistance R by increasing or decreasing its value so that the spot of light, being deflected, may remain steady somewhere between 10 cm and 16 cm. Note this steady deflection of the spot of light and the resistance put in the box R.

(2) Keeping R unchanged, put gradually increasing resistance in the box R_1 until the *deflection of the spot of light becomes exactly halved*. Note the resistance put in the box R_1. This will be the galvanometer resistance.

(3) Now take out the plugs from the commutator and stop the current flow. See whether the spot of light returns to zero-mark of the scale. If it does not, slightly shift the scale and bring the spot on the zero-mark. Keeping R unchanged, make $R_1=0$ by putting the plug in the box R_1.

(4) Reverse the direction of the current by the commutator. If the plugs of the commutator are inserted in the other two opposite holes than before, the direction of the current will be reversed. Note the deflection of the light stop. The deflection, evidently will be on the other side of the scale.

(5) Now insert gradually increasing resistances from a low value in the box R_1 till the deflection is reduced to half of the previous value. The resistance now required will be exactly equal to or very nearly equal to the resistance obtained in the previous operation. The mean of these two observations gives the resistance of the galvanometer.

(6) Keeping S constant and inserting different resistances in the box R, repeat the whole operation twice or thrice. Similarly, keeping R unchanged and changing the shunt resistance (say, 0·2, 0·3 ohms etc), repeat the operation twice or thrice. Note that in each case, the full scale deflections of the spot of light should be restricted within 10 cm to 16 cm.

(7) Mean of the values of R_1 obtained from the previous six or eight operations is then calculated.

(8) Measure the *e.m.f.* of the cell (or the battery) by a voltmeter and the distance between the mirror of the galvanometer and the scale by a metre scale.

(9) Calculate, with the help of eqn. (iv) of the theory, the current flowing through the galvanometer in each when $R_1=0$. Divide it by the full scale deflection (d) converted into millimetre.

Measurements : *E.M.F.* of the cell (E) = 2 volts [As an illustration]
Distance of the mirror of the galvanometer from the scale = 1 metre

(*a*) *Determination of galvanometer resistance* : (Data given for illustration)

No. of Obs	Resistances (ohm)			Deflections for (cm)		Resistance of Galv. (G)		Mean G (ohm)
	Battery circuit (R)	Shunt (S)	Galv. circuit (R_1)	Direct current	Reverse current	Direct current	Reverse current	
1.	500	0·1	0 130	12·2(d) 6·1($d/2$)	12·4(d) 6·2($d/2$)	130	130	130
2.	800	0·1	0 130	10·2(d) 5·1($d/2$)	10·4(d) 5·2($d/2$)	130	130	
3.	..	..	0 ..	.. ..	.. ..	..	..	
4.	..	..	0 ..	.. ..	.. ..	..	..	
5.	1500	0·2	0 130	14·2(d) 7·1($d/2$)	14·4(d) 7·2($d/2$)	130	130	
6.	.. ..	.. ..	0 ..	.. ..	.. ..	..	..	

(*b*) *Measurement of current for 1 mm deflection* [Data taken from table (*a*)]

No. of obs.	Battery circuit Res. (R)	Shunt resistance (S)	Galv. resistance (G)	Deflection (d) mm	I_g (amp)	I_g/d amp/mm
1.	500 ohm	0.1 ohm	130 ohm	$\frac{1}{2}(12{\cdot}2+12{\cdot}4)$ $=12{\cdot}3$cm $=123$mm	$0{\cdot}3\times10^{-5}$	$0{\cdot}244\times10^{-7}$
2.	..	..	..	..	..	..
3.	..	..	..	..	..	..
	etc		etc		etc	
6.	..	..	..	..	..	..

Calculations :

1. $$I_g=\frac{E.S}{S(R+G)+R.G}=\frac{2\times0{\cdot}1}{0{\cdot}1(500+130)+500\times130}$$
$$=0{\cdot}3\times10^{-5}\text{ amp.}$$

$$\therefore\quad \frac{I_g}{d}=\frac{0{\cdot}3\times10^{-5}}{123}=0{\cdot}244\times10^{-7}\text{ amp/mm.}$$

2. etc.

Remarks : (1) Every time, the full scale deflection should be restricted between 10 cm and 16 cm ; otherwise current through the galvanometer will not be halved at half the scale deflection. (2) At every observation, before the current is reversed it should be checked whether the spot of light coincides with zero mark of the scale (3) The spot of light should preferably have a straight edge. (4)The shunt resistance must be very small compared to the galvanometer resistance. (5) When $S\ll G$, the current through the galvanometer is almost equal to $E.S/R.G$. This simple formula may also be used to calculate I_g.

Oral questions

1. Galvanometer resistance means the resistance of what thing of the galvanometer ?

Ans. It means the resistance of the coil of wire in the galvanometer.

2. Why should you take a small shunt resistance ?

Ans. If the shunt resistance be not very small, compared to the resistance of the galvanometer, the theory will not be applicable (See the theory).

3. What harm is there in the experiment if the galvanometer has a low resistance ?

Ans. If the galvanometer itself has a low resistance, it will be difficult to find a still lower resistance as shunt and the experiment, then, fails.

4. What alternative method can you adopt to find the resistance of a galvanometer of the above type ?

Ans. In such cases, the coil of the galvanometer is to be clamped and its resistance is to be measured by a metre bridge or a P.O. Box by the usual process.

5. Why should you bring the spot of light to the zero mark of the scale before the direction of current is reversed ?

Ans. If the spot of light is displaced then the value of R_1 necessary to reduce the deflection to half value for direct current will be different from the value for reverse current and the measurement will be erroneous.

6. Why do you take deflections with direct and reverse currents ?

Ans. The plane of the mirror, at rest, may not be parallel to the plane of the scale. This will introduce some error in the deflection of the spot of light. The error is eliminated if deflections are taken on the opposite sides of the zero-mark.

7. Why do you restrict the deflection between 10 cm. and 16 cm. ?

Ans. The angular displacement of the mirror is taken proportional to the linear displacement of the spot of light. This proportionality is valid if the linear deflection is small (*i.e.* within 10 cm. to 16 cm. of the scale).

5.8. Measurement of the figure of merit of a suspended coil galvanometer :

Apparatus : A suspended coil galvanometer, a resistance box of low range (0·1 ohm to 5 ohm), a resistance box of high range (10,000 ohm and higher), a plug commutator, a storage cell, a metre scale, connecting wires etc.

Circuit connections : Fig. 42 shows the necessary circuit connections. The high resistance box *R* is connected in series with the storage cell *E* (if necessary a battery consisting of two storage cells in series) which is then joined to the opposite terminals (no. 1 and 3) of a commutator *C*. The other two opposite terminals (no. 2 and 4) of the commutator are connected to the low resistance box *S*, acting as a shunt. The terminals of the galvanometer G are joined to the terminals of the shunt box so they are in parallel combination.

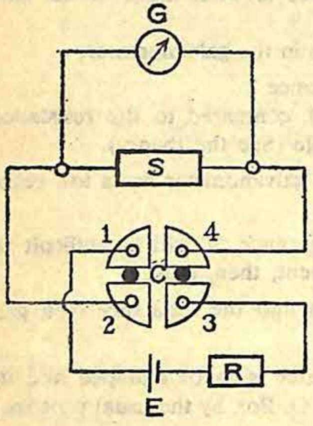


Fig. 42

Theory : Figure of merit of a suspended coil galvanometer is defined as the current which produces a deflection of 1 mm of the spot of light on a scale placed 1 metre away from the galvanometer mirror. According to fig. 42, the resistance of the shunted galvanometer $= \dfrac{S.G}{S+G}$.

If I_g be the current through the galvanometer, then $I_g = \dfrac{S}{S+G} \times$ main current

$$= \frac{S}{S+G} \times \frac{E}{R+\dfrac{SG}{S+G}} = \frac{E.S}{R(S+G)+SG}$$

Let this current produce a deflection of d cm on a scale placed D cm away from the galvanometer mirror. Then the deflection produced on the scale when placed 100 cm (*i.e.* 1 metre) away is given by

$$n = \frac{100 \times d}{D} \text{ cm} = \frac{1000 \times d}{D} \text{ mm.}$$

According to definition, the figure of merit

$$\theta = \frac{I_G}{n} = \frac{E.S}{R(S+G)+S.G} \times \frac{D}{1000 \times d} \text{ amp/mm.}$$

Experimental procedure : (1) Adjust the position of the scale so that the spot of light coincides with the zero-mark of the scale. Make circuit connections as shown in fig. 42. First take out a high resistance plug (say, 10,000 ohm) from the battery circuit resistance box R. Then take out such a resistance plug (say 0·5 ohm or 1 ohm) from the shunt box S so that the spot of light, being deflected, remains steady somewhere between 10 cm and 16 cm of the scale. When the spot of light becomes steady, note the deflection from the scale.

(2) Now stop the current by taking out the plugs from the commutator and see when the spot of light returns to zero-mark. If it does not, slightly shift the scale to bring it to the zero-mark. Put the plugs in the other two opposite holes of the commutator to reverse the direction of the current. Note the deflection of the spot of light.

(3) Determine the mean deflection from these two observations and find the figure of merit with the help of the equation stated in the theory.

(4) Repeat the observations thrice by changing the resistance in the box R but keeping the shunt resistance unchanged and repeat twice more by changing the shunt resistance but keeping the resistance in the box R unchanged. See that the deflection in each case is within 10 cm to 16 cm.

(5) With the help of a voltmeter, find the *e.m.f.* of the storage cell (or the battery) E. Also measure the distance between the galvanometer mirror and the scale by a metre scale. Ascertain the resistance of the galvanometer from the teacher.

(6) Find the mean value of the figure of merit from the results obtained from several observations.

Measurements : The *e.m.f.* (E) of the storage cell (or the battery) =.. volts (measured by a voltmeter)

The distance between the galvanometer mirror and the scale (D) =.. cm (measured by a metre scale)

Galvanometer resistance (G)= .. ohms (supplied)

Measurement of figure of merit :

No. of Obs	Resistance in battery circuit (R ohm)	Shunt resistance (S ohm)	Deflection (d) of the spot of light (cm.)			Fig. of merit (θ) amp/mm	Mean fig. of merit (θ) amp/mm
			Direct current	Reverse current	Mean (d)		
1.	10,000	0·5	10·4	10·4	10·4	..	
2.	10,000	1	12·6	12·6	12·6	..	
3.	20,000	..	..	..	..	..	..
4.	20,000	..	..	..	..	..	
5.	30,000	0·5	..	..	..	..	
6.	30,000	1	..	..	..	..	

Calculations : 1. $\theta = \dfrac{D}{1000 \times d} \times \dfrac{E.S.}{R(S+G)+S.G} = ..$amp/mm.

2. θ =etc.

Remarks : (1) If θ_i be the current sensitivity of the galvanometer, then $\theta_i = \frac{1}{\theta}$. So, current sensitivity of the galvanometer can also be determined by this experiment. Again, if θ_v be the voltage sensitivity of the galvanometer, then $\theta_i = \theta_v \times G$. So, voltage sensitivity can also be determined by this experiment. (2) The scale should be placed at right angles to the reflected ray from the mirror, otherwise the deflections for direct and reverse current will not be same.

Oral questions

1. What do you mean by figure of merit of a galvanometer ? What is its unit ?

Ans. See theory. Its unit is amp/mm or micro.amp/mm.

2. What do you mean by current sensitivity and voltage sensitivity of a galvanometer ? What is their relation ?

Ans. The current sensitivity of a galvanometer is defined as the deflection, in millimetres, produced on a scale 1 metre away by a current of 1 micro-ampere (*i.e.* 10^{-6} ampere).

The voltage sensitivity of a galvanometer is defined as the deflection in millimetres produced on a scale 1 metre away when the voltage applied to the galvanometer is 1 micro-volt (*i.e.* 10^{-6} volt).

Current sensitivity = Galvanometer resistance × voltage sensitivity.

3. What is the harm if a Leclanche's cell is used in this experiment ?

Ans. In a Leclanche's cell, polarisation sets in when continuous current is taken from it. As a result, current diminishes. So, if a Leclanche's cell is used in this experiment, the deflection of the spot of light, after remaining steady for sometime, will begin to diminish.

4. Why is a shunt used in this experiment ?

Ans. Shunt is a safety arrangement for a galvanometer. If a heavy current passes through a galvanometer by any chance, it may get damaged. In a shunted galvanometer, most of the current passes through the shunt and only a small amount passes through the galvanometer. No damage is, therefore, caused to the galvanometer.

5. Why do you restrict the deflection within 10 to 16 cm ?

Ans. We measure the deflection of the coil (θ) in radians. Now, when the coil turns through an angle θ, the reflected ray turns through 2θ. Here, $2\theta = \frac{\text{linear displacement of the spot}}{\text{distance of the scale}}$. If the displacement be not small, $\theta \propto$ displacement relation will not hold good. For this reason, the deflection is restricted within 10 to 16 cm.

6. Will the deflection of the spot of light increase if the distance between the scale and the mirror is increased ?

Ans. If the distance between the scale and the mirror is increased, the deflection is also increased because 2θ remains unchanged.

7. If the distance between the scale and the mirror is increased, will the figure of merit change ?

Ans. No ; whatever may be the distance between the mirror and the scale, deflection will accordingly alter to keep the figure of merit constant.

8. Will the galvanometer current change if the resistance in the box R is increased ?

Ans. With the increase of R, the main current will decrease and hence the galvanometer current will also decrease.

9. What change will occur in the galvanometer current and hence in the deflection if shunt resistance is increased or decreased ?

Ans. If shunt resistance is increased, more current will flow through the galvanometer causing a greater deflection. On the other hand, if the shunt resistance is decreased, less current will flow through the galvanometer causing a smaller deflection.

10. What are the factors on which the figure of merit of a galvanometer depends ?

Ans. The figure of merit of a galvanometer depends on (*i*) the magnetic flux produced by the magnet of the galvanometer (*ii*) the number of turns of the coil (*iii*) the face area of the coil and (*iv*) the torsional rigidity of the suspension wire.

5.9. Determination of the temperature coefficient of resistance of a coil of wire using a metre bridge.

Apparatus : A coil of wire enclosed in a glass tube, a metre bridge, a thermometer, a fractional resistance box, a Leclanche's cell, table galvanometer, a plug commutator, a hypsometer, connecting wires etc.

Description of the coil of wire : A coil of wire (R) whose temperature coefficient of resistance is to be measured (usually copper) is enclosed in a wide glass tube G. Two long copper wires are soldered to the two ends of the coil and the wires are taken out of the tube through two holes in the stopper closing the mouth of the tube. The wires are then soldered to two screw terminals P and Q fitted on the ebonite stopper. Through a hole at the centre of the stopper, a thermometer (T) is inserted into the tube. [Fig 43].

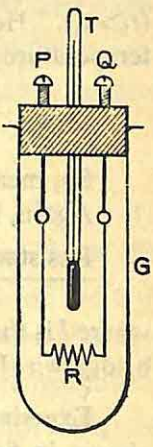


Fig. 43

Circuit connections : Fig 43(a) shows the necessary circuit connections. The terminals P and Q of the heating coil are connected to the gap no. 1 of the metre bridge. The connecting wires should be a bit long because when the heating coil is to be connected to the gap no. 2, it can be done by simply drawing the connecting wires without shifting the tube G. The fractional resistance box S is connected across the right-hand gap no. 2. A Leclanche's cell E is connected to the extreme left and right ends of the bridge wire AB through the commutator C. One end of a table galvanometer is connected to the terminal mid-way between the gaps no. 1 and 2 of the bridge and the other end to the jockey J. A thermometer (T) has been introduced into the tube G through a hole in its stopper. The tube G can be placed inside a hypsometer.

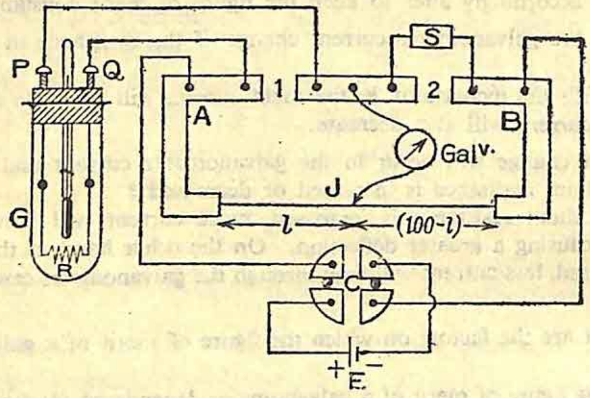


Fig. 43(a)

Theory : Let the resistance of the coil be R_1 at t_1°C and R_2 at t_2°C $(t_2 > t_1)$. Here, $R_1 = R_0\ (1+\alpha t_1)$ and $R_2 = R_0\ (1+\alpha t_2)$ where α is the temperature coefficient of resistance.

$$\therefore \frac{R_2}{R_1} = \frac{1+\alpha t_2}{1+\alpha t_1} \quad \text{or} \quad \alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1} \quad \ldots \quad \ldots \quad \text{(i)}$$

So, measuring R_1 and R_2 by a metre bridge, α can be determined. Again, from the principle of a metre bridge, we know,

$$\frac{\text{Resistance in the left-gap } (R)}{l} = \frac{\text{Resistance in the right gap } (S)}{100-l} \ldots \text{(ii)}$$

where l is the length of the null point measured from the left end of the bridge-wire [Fig 43(a)].

Experimental procedure : (1) Make the circuit connections as shown in fig 43(a). Take a large beaker. Fill it up with water and place the tube G in the water. Keep it fixed by suitable clamp. Note the temperature (t_1°C) of the coil from the thermometer T.

(2) Take out small resistance (say 2 ohms) from the resistance box S and find the null point when the galvanometer shows no deflection. Adjust the resistance of the box S so that the null point remains somewhere between 40 cm and 60 cm. of the brige wire. Note the

null point distance from the scale carefully. Reverse the direction of the current and again find the null point. Ascertain the mean of these two null point lengths.

(3) Change the resistance in the resistance box twice and in each case find the lengths of null points for direct and reverse currents. Take care that the null point always remains within 40 cm and 60 cm. Find the mean of the two readings obtained for each value of the resistance put in the resistance box *S*.

(4) Now interchange the positions of the resistance box *S* and the tube *G* i.e. insert the resistance box *S* in gap no. 1 and the tube *G* in gap no. 2. Take out exactly the same resistances from the resistance box *S* as before and in each case get the null points for direct and reverse current. Find the mean length from the two readings obtained for each value of the resistance put in the box *S*.

(5) Calculate the value of R_1 *separately* from the null point lengths in each of the six cases (three before interchange and three after interchange) with the help of the equation (ii) stated in the theory. From these values, get the mean value of R_1. This gives the correct resistance (R_1) of the coil at t_1°C.

(6) Now introduce the tube *G* into a hypsometer. Take care that the tube does not touch the water in the hypsometer. Heat the water of the hypsometer by a burner. The temperature of the coil will increase when it comes in contact with the steam produced in the hypsometer. When the thermometer *T* shows a steady temperature, repeat the operations (2), (3) and (4) and find the resistance (R_2) of the coil at temperature t_2°C.

(7) Note the steady temperature (t_2°C) shown by the thermometer.

Measurements : (Data given as illustrations)

Temperature	No. of Obs	Resistance (ohm)		Distance of null point (cm)		Mean distance (cm)	Unknown Resistance (ohm)	Mean value of unknown Res. (ohm)
		Left gap	Right gap	Direct current	Reverse current			
t_1°C (room temp)	1.	R (coil)	1·8	45·1	45·3	45·2 (l_1)	..	
	2.	R	2·4	..	..	..	..	
	3.	R	2·8	..	..	..	..	..
	4.	1·8	R (interchanged)	..	..	.. (l_2)	..	(R_1)
	5.	2·4	R	..	..	..	..	
	6.	2·8	R	..	..	..	..	
t_2°C (Steam temp)	1.	R	..	..	..	(l_1')	..	
	2.	R	..	..	..	..	..	
	3.	R	..	..	..	..	..	
	4.	..	R (interchanged)	..	..	(l_2')	..	(R_2)
	5.	..	R	..	..	..	..	
	6.	..	R	..	..	..	..	

Calculations : 1. (*a*) $\dfrac{R_1}{S}=\dfrac{l_1}{100-l_1}=..\quad\therefore\ R_1=..$

After interchange :

$$\text{(b)}\quad \frac{S}{R_1}=\frac{l_2}{100-l_2}\qquad\therefore\ R_1=..$$

$$2.\quad \text{(a)}\quad \frac{R_2}{S}=\frac{l_1'}{100-l_1'}=..\quad\therefore\ R_2=..$$

After interchange :

$$\text{(b)}\quad \frac{S}{R_2}=\frac{l'_2}{100-l_2'}\qquad\therefore\ R_2=..$$

$$3.\quad \alpha=\frac{R_2-R_1}{R_1t_2-R_2t_1}=..\ \text{ohm/°C}$$

Remarks : (1) Heating coil should be kept far away so that the heat of the burner may not warm up the apparatus. The burner should be covered by asbestos board. (2) The error will be minimum if the null point is found near about 50 cm mark. (3) There are some materials whose resistance increases with temperature and some whose resistance decreases with the increase of temperature. (4) If R_0 be the resistance of a wire at 0°C, then actually at t°C, $R_t=R_0$ $(1+\alpha t+\beta.t^2+..)$. If t be small, then only $R_t=(1+\alpha t)$. (5) If temperature coefficient of resistance is found between t_1°C and t_2°C then it should be regarded as the mean coefficient in that specified range of temperature. (6) The connecting wires used for joining the heating coil with the gap of the bridge should be as small as possible.

Oral questions

1. What is temperature coefficient of resistance ?

Ans. The change in resistance per unit resistance of a material for 1°C change in temperature is called the temperature coefficient of resistance of the material.

2. Name two materials whose temperature coefficient of resistance is positive and negative.

Ans. Copper and platinum have positive temperature coefficient while carbon, vulcanised india rubber have negative temperature coefficient *i.e.* resistance of these materials decreases with the increase of temperature.

3. 'Resistance of a material increases with the increase of temperature'— What is the practical use of this property ?

Ans. The practical use of this property is the resistance thermometers. An unknown temperature—specially a high temperature—can be measured by a resistance thermometer.

4. Is there any harm if the connecting wires used to join the heating coil tube with the bridge gap are long ?

Ans. If the connecting wires are long, their resistance will be added with the resistance of the coil. But when the coil is heated, its resistance changes while the connecting wires, which are at room temperature, will have its resistance unchanged. This causes an error in the value of the coefficient.

5. What is the significance of keeping the null-point at the centre of the bridge wire ?

Ans. If there be any error in determining the null point, the value of the unknown resistance found from the null point will be inaccurate. Now, it is found that the error in determining the null point becomes minimum when the null point is near 50 cm mark of the scale *i.e.* at the middle of the wire. In order to determine the unknown resistance with accuracy, the null point should be kept at the centre of the wire.

6. Does the temperature coefficient of resistance of a material same for different range of temperature ?

Ans. No ; temperature coefficient depends on the range of temperature.

7. Is metre bridge method regarded as very accurate in determining the temperature coefficient of resistance of a material ?

Ans. No ; for accurate measurement, Callendar and Griffith bridge should be used.

8. In preparing a standard resistance, manganin wire is usually used. Why ?

Ans. A standard resistance should remain constant at different temperatures *i.e.* it should be made of a material whose temperature coefficient of resistance is negligibly small. Manganin is such a material. It is an alloy of copper, manganese, and nickel.

5.10. Determination of the value of a low resistance by the fall of potential method :

Apparatus : Two low resistances (say 0·01 ohm and 0·03 ohm)—one of which should be a known standard low resistance and the other unknown, a metre bridge, a 4-way plug key ; a storage cell, a rheostat, a plug commutator, a table galvanometer etc.

Description of the standard low resistance : The standard low resistance generally used in laboratories and as illustrated in fig 44 is made of thick manganin strip. The strip, instead of being straight, is made wavy as shown in the adjoining figure. The two ends of the strip are joined to two terminals fitted on an ebonite platform. If a current carrying circuit is connected between the terminals C_1 and C_2, the current will flow through the low resistance. For this reason, C_1 and C_2 are called *current terminals*. But two given points on the strip are connected to two other terminals P_1 and P_2 by thick wires below the platform. The value of the low resistance indicated (for example 0·02) on the upper surface of the platform is the resistance between these two given points *i.e.* between P_1 and P_2. For this reason, these terminals (P_2 and P_2) are called *potential terminals*.

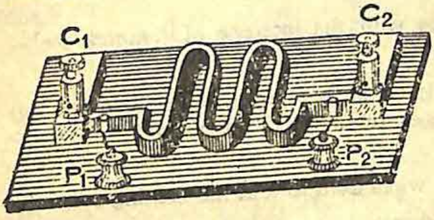


Fig. 44

Circuit connections : The necessary circuit connections have been shown in fig 44(a). Let X_1 and X_2 be the two resistances. The current terminals C_1 and C_2 of the the resistance X_1 are joined to the left gap *MN* of the metre bridge. Similarly, the current terminals C_3 and C_4 are joined to the right gap *RS*. The potential terminals P_1 and P_2 of the first resistance are connected to the screws *A* and *B* respectively of the 4-way plug key while those of the second resistance (P_3 and P_4) are connected to C and D screws respectively. One end of a table galvanometer G is joined to the common terminal E of the 4-way key

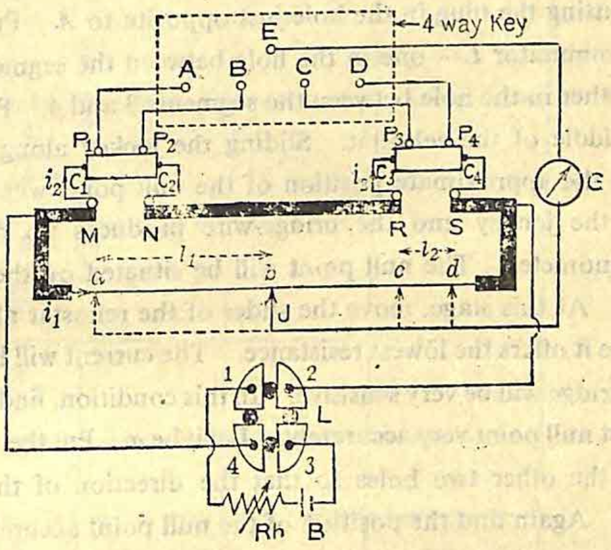


Fig. 44(*a*)

while the other end is in contact with the jockey J. The terminals of a storage cell B are connected to the terminals M and S of the bridge through a rehostat R_h and a plug commutator L.

Theory : Current from the cell arriving at M will be divided into two parts—one flowing through the low resistances and the other through the bridge wire. Suppose i_1 current flows through the bridge wire and i_2 through the low resistances. If the points a, b, c and d on the bridge wire are at the same potentials with the points A, B, C and D respectively, then $i_2X_1 = i_1l_1.\rho$ and $i_2X_2 = i_1l_2.\rho$ where $\rho =$ resistance per unit length of the bridge wire.

$$\therefore \frac{X_1}{X_2} = \frac{l_1}{l_2} \quad \text{or,} \quad X_1 = \frac{l_1}{l_2}.X_2$$

So, measuring l_1 and l_2 and knowing the value of X_2, the value of the low resistance X_1 may be found out.

Experimental procedure : (1) Make circuit connections as shown in the Fig. 44(a).

(2) Connect the galvanometer G with the point A of the 4-way key by putting the plug in the hole just opposite to A. Put two plugs in the commutator L – one in the hole between the segments 1 and 2 and the other in the hole between the segments 3 and 4. Put the slider at the middle of the rehostat. Sliding the jockey along the bridge-wire find the approximate position of the null point where a contact between the jockey and the bridge-wire produces no deflection in the galvanometer. The null point will be situated on the left end of the wire. At this stage, move the slider of the rehostat almost at one end where it offers the lowest resistance. The current will be maximum and the bridge will be very sensitive. In this condition, find the position of the first null point very accurately. Let it be a. Put the commutator plugs in the other two holes so that the direction of the current is reversed. Again find the position of the null point accurately. Mean of these two readings gives the correct position of the first null point a.

(3) In the same way, connect the galvanometer G, one by one, with the points B, C, and D and find the positions of the null points b, c and d respectively for direct and reverse currents. From the positions of the null points, calculate l_1 and l_2 and hence the ratio l_1/l_2.

(4) Now interchange the positions of X_1 and X_2 – *i.e.* put the resistance X_1 in the gap RS and the resistance X_2 in the gap MN. Following the operations no. 2 and 3, find the values of l_1 and l_2 and hence the ratio $\frac{l_2}{l_1}$.

(5) Change the main current twice by changing the position of the rheostat slider. Following the operations no. 2 and 3 find the ratio l_1/l_2 with X_1 in the gap MN and X_2 in the gap RS as well as the ratio l_2/l_1 by interchanging the positions of X_1 and X_2 in each case.

(6) Find the value of X_1 from each observation and then ascertain the mean value.

Measurements : Known value of the low resistance (X_2). = . . ohm.

(*a*) *When X_1 is in the gap MN and X_2 in RS.*

No. of Obs.	Current	Position of null point								l_1 $=(b-a)$ cm.	l_2 $=(d-c)$ cm	$x_1=\frac{l_1}{l_2}x_2$	Mean x_1
		a	Mean *a*	*b*	Mean *b*	*c*	Mean *c*	*d*	Mean *d*				
1.	Direct	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	
	Reverse	. .		. .		. .		. .					
2.	Direct	. .		. .	. .	. .		. .		. .	. .	. .	. .
	Reverse	. .	. .	. .		. .	. .	. .	. .				(*i*)
3.	Direct	. .	. .	. .	. .	. .		. .		. .	. .	. .	
	Reverse	. .		. .		. .	. .	. .	. .				

(*b*) *When X_1 is in the gap RS and X_2 in MN.*

No. of Obs	Current	Position of null point								$l_1 = (b-a)$ cm	$l_2 = (d-c)$ cm	$x_1 = \frac{l_2}{l_1} x_2$	Mean x_1
		a	Mean a	b	Mean b	c	Mean c	d	Mean d				
1.	Direct	..	..	..	..	..	..	..		..	..	..	
	Reverse	..		..		..		..	..				
2.	Direct	..	..	..	..	..	..	..		..	..	..	..
	Reverse	..		..		..		..	..				(*ii*)
3.	Direct	..	..	..	..	..	..	..		..	..	..	
	Reverse	..		..		..		..	..				

$$\text{Mean value of } X_1 = \frac{..(\text{i})+(\text{ii})...}{2} = ..\text{ohm}$$

Remarks : (1) The low resistances are to be connected to the gaps of the bridge by thick and short connecting wires. (2) Observations must be taken with direct and reverse currents in order to eliminate thermo-electric effects. (3) In order to make the bridge sensitive, the resistance in the rheostat should be minimum so that the current flowing in the bridge is high. (4) Current should not be drawn continuously for a long time ; otherwise undesirable effects may be produced due to heating. (5) The method may be used to compare two low resistances. (6) The experiment may be performed with a potentiometer instead of a metre bridge, the theory and the procedure being the same.

Oral questions

1. We generally measure resistance by the wheatstone principle. Why do you adopt a different method in this case?

Ans. By wheatstone bridge principle, we generally measure resistances of middle order value. The bridge becomes sensitive when all the four resistances of the bridge are comparable. For this reason, Wheatstone bridge principle can not be used for the measurement of low or high resistances.

2. Resistances of what value may be regarded as high or low?

Ans. There is, of course, no hard and fast rule ; usually resistances smaller than 1 ohm are regarded as low and resistances greater than 1000 ohm as high. The rest are known as resistances of middle order value.

3. Why do you take direct and reverse currents while finding the null points?

Ans. Observations with reverse current eliminates errors due to thermo-electric effect.

4. Which error is eliminated by taking observations with the positions of the resistances interchanged?

Ans. The bridge wire may not be uniform. Non-uniformity of the wire may bring in some error. This error is eliminated if observations are taken with the resistances interchanged.

5. Should the current flowing through the bridge be high or low?

Ans. The current should preferably be high ; for in that case, the bridge will be sensitive and location of the null point will be accurate.

6. What is the harm if current is drawn continuously for a pretty long time?

Ans. Continuous current produces sufficient heating which may bring in undesirable effects.

7. Under what condition will l_1 be equal to l_2?

Ans. Evidently when x_1 is equal to x_2.

8. In the experimental arrangement, how are the low resistances and the bridge wire connected?

Ans. The low resistances are connected in series but the resistances and the wire are connected in parallel.

9. Why do you use short and thick connecting wires for inserting the low resistances in the bridge gaps?

Ans. Short and thick wires have almost zero resistance. If thin and long wires were used, their resistances may be comparable to the low resistances taken. In that case, the result obtained will be defective.

5.11. Determination of a high resistance by substitution method using a shunt box :

Apparatus : A high resistance box (a decade megohm box containing resistances from 0·1 megohm to 1 megohm), an unknown high resistance, a resistance box of 1 to 500 ohm range, a suspended coil galvanometer of high resistance, a plug commutator, a two-way key, two alkali storage cells, connecting wires etc.

Circuit connections : Two alkali storage cells are joined in series to form a battery E. One end each of the decade megohm box R and the unknown resistance X are connected to one pole (say, negative) of the battery. The other ends of the megohm box and the unknown resistance X are connected to terminals 1 and 2 of the two-way key, the third (or the common) terminal of the two-way key is connected to the commutator C. The positive pole of the battery is also joined to the commutator as shown in Fig. 45. If a plug is put in the hole between no. 1 and 3, the megohm box R will be included in the battery circuit. On the other hand, if the plug is inserted in the hole between no. 2 and 3, the unknown resistance X is included in the battery circuit. So, with the help of the two-way key, the megohm box R and the unknown resistance X may be brought, one by one, in the battery circuit. A galvanometer G, of high resistance with a low resistance box S (1 – 500 ohm) in parallel is connected to the other two screws of the commutator

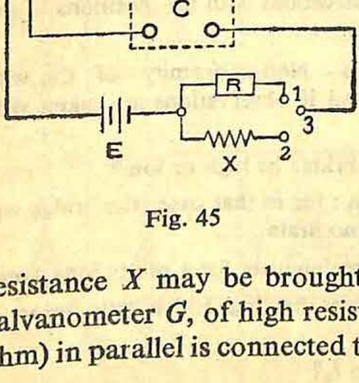


Fig. 45

C. The resistance box S will evidently act as a shunt to the galvanometer.

Theory : If the known resistance R is put in the battery circuit, the total circuit resistance $=R+\dfrac{S_1G}{S_1+G}$

Hence, the main current $I=\dfrac{E}{R+\dfrac{S_1G}{S_1+G}}$. where S_1 = the resistance put in the shunt box.

$\therefore$ The current through galvanometer $I_g=\dfrac{S_1}{S_1+G}\times I$

$$=\frac{S_1}{S_1+G}\times\frac{E}{R+\dfrac{S_1G}{S_1+G}}=\frac{ES_1}{R(S_1+G)+S_1G}.$$

If this current produces a deflection d_1 of the spot of light, then

$$I_g=\frac{E.S_1}{R(S_1+G)+S_1G}=K.d_1 \quad ..\text{(i)} \text{ where } K \text{ is a constant.}$$

If X, the unknown high resistance is included in the battery circuit instead of the known high resistance R and S_2 be the shunt resistance used, then, the galvanometer current I_g' is given by,

$$I_g'=\frac{ES_2}{X.(S_2+G)+S_2G}$$

If this current produces a deflection d_2 of the spot of light, then

$$I_g'=\frac{E.S_2}{X.(S_2+G)+S_2G}=K.d_2 \quad .. \quad .. \quad \text{(ii)}$$

Dividing (i) by (ii) $\dfrac{S_1}{S_2}\ \dfrac{X(S_2+G)+S_2G}{R(S_1+G)+S_1G}=\dfrac{d_1}{d_2}$

As S_1, S_2 and G are small in comparison with R or X, we can neglect S_1G and S_2G. Thus, we get $X=R.\dfrac{d_1}{d_2}.\ \dfrac{S_2(S_1+G)}{S_1(S_2+G)}$ .. (iii)

If X be very high, so that the deflection d_2 remains within the scale without any shunt (*i.e.* $S_2=\infty$), then, $X=R.\ \dfrac{d_1}{d_2}.\ \dfrac{S_1+G}{S_1}$ ... (iv)

Equations (iii) or (iv) may be used to determine the value of an unknown high resistance equation (iv), of course, to be used when X is very large.

Experimental procedure : (1) Make circuit connections as shown in Fig 45. Adjust the position of the scale so that the spot of light coincides with the zero mark of the scale.

(2) Put a plug in the hole between terminals no. 2 and 3 *i.e.* include the unknown high resistance X in the battery circuit and take out a small resistance from the shunt box S. Note the deflection of the spot of light. If the deflection is too much, reduce the shunt resistance and if the deflection is too little, increase the shunt resistance. In this way, adjust the shunt resistance to such a value S_2 that the deflection of the spot of light is within 10 cm and 16 cm range. Note the exact deflection. Reverse the direction of the current by the commutator. Deflection of the spot of light will take place on the other side of the zero-mark. Record the exact deflection and find the mean of these two readings. The mean deflection is d_2.

(3) Now include the megohm box R in the battery circuit by inserting the plug into the hole between the terminals 1 and 3. Put, as before, a small resistance in the shunt box S and the minimum resistance available in the megohm box (*i.e.* 0·1 megohm or 10^5 ohm). Note the deflection. It may be widely different from the previous deflection d_2. Adjust the value of the shunt resistance (say S_1) so that the deflection is *very nearly equal* to the previous deflection. Reverse the direction of the current by the commutator and note the deflection on the opposite side of the zero-mark. Find the mean of these two readings. Let it be d_1. From the values of S_1, S_2, d_1 and d_2 calculate from the eqn. (iii) of the theory, the value of X. [The galvanometer resistance G is to be ascertained from the teacher.]

(4) Having finished the first set of observations in the above way, two more sets of observations are to be taken in the same way. In each of these two observations, new values of shunt resistances are to be used together with increasing values (for example 0·2 and 0·3 megohm) of resistance R so that deflections with X and R are very nearly equal but different from other two cases.

(5) The value of X is calculated for each set of observation. Finally, the mean of three values of X obtained from three sets of observations is ascertained.

Measurements :

Resistance of the galvanometer $(G)=\ldots$ohms (supplied).

No. of Obs.	Resistance in battery circuit	Resistance in shunt box	Deflection in cm. for			X (ohm)	Mean X (ohm)
			Direct current	Reverse current	Mean		
1. (i)	X (unknown)	$..(S_2)$	..	..	$..(d_2)$	..	..
(ii)	R.. (known)	$..(S_1)$	..	..	$..(d_1)$		
2. (i)	X	$..(S_2)$	..	..	$..(d_2)$	..	
(ii)	R	$..(S_1)$	..	..	$..(d_1)$		
3. (i)	X	$..(S_2)$	..	...	$..(d_2)$	..	
(ii)	R	$..(S_1)$	..	..	$..(d_1)$		

Calculations : $X=R.\dfrac{d_1}{d_2}\times\dfrac{S_2(S_1+G)}{S_1(S_2+G)}=\ldots$ohms.

Remarks : (1) In each case, the deflection should be between 10 cm to 16 cm. (2) The constant K (see theory) depends on deflection. For this reason, the deflections d_1 and d_2 for the known and the unknown high resistances should be very nearly equal or exactly equal. (3) By this method, high resistances of the order of $10^4\sim10^5$ ohm may be measured.

Oral questions

1. What is the relation between ohm and megohm?

Ans. 10^6 ohm $=1$ megohm.

2. Why is metre bridge or P.O. Box not used for the measurement of high resistance?

Ans. Metre bridge or P.O. Box becomes very insensitive in the case of a high or a low resistance.

3. Why do you take almost equal deflections in the cases of the unknown and the known high resistances?

Ans. See remark no. 2.

4. How many significant digits should be retained in the final result?

Ans. Not more than three significant digits.

5.12. Determination of mechanical equivalent of heat by Joule's calorimeter :

Apparatus : Joule's calorimeter, lamp board, voltmeter (0 – 20 volts), ammeter (0 – 5 amp), switch, thermometer (1/5°*C*), stop watch turpentine oil, balance, weight box etc.

Description of Joule's calorimeter : *C* is a nickel-coated copper calorimeter [Fig 46]. It has an ebonite lid. Through the holes in the lid, are inserted a thermometer *T*, a stirrer *S* and a coil of wire *R*. The calorimeter is placed in a wider wooden box (not shown in the figure) and the space between the two is packed with wool or cotton. This prevents loss of heat.

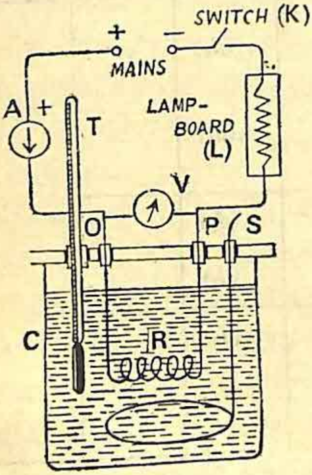


Fig. 46

Circuit connections : At first the positive and negative of 'the mains' are to be ascertained. For this purpose, insert the leads coming from the mains into water kept in a beaker. Take care that *the wires do not touch each other*. It will be seen that along one lead, profuse gas bubbles are coming out. This is the negative of the mains. Connect the negative of the mains to the point *P* of the coil through a lamp-board *L* and a switch *K* as shown in the figure. The end *P* of the coil, therefore, gets negative potential. The lampboard acts as a variable resistance. The positive terminal of the mains is connected to the +*ve* terminal of an ammeter *A*, the other terminal of the ammeter being connected to the point *O* of the coil. The point *O*, therefore, gets positive potential. A voltmeter *V* is connected between the points *P* and *O*. The voltmeter will read the p.d. across the coil of wire.

[**N.B.** If the current is drawn from a battery instead of the mains, then the battery is to be prepared by connecting three or four storage cells in series and in that case a suitable rheostat may be used instead of the lampboard.]

Theory : If a current *i* amp flows through a coil of wire for *t* sec. and if *e* volts be the p.d. across the oil, then the work done *W*

$=e.i.t.$ joules. If H calories of heat are produced due to this, then $W=J.H.$ or $e.i.t.=J.H.$

Now, if the coil of wire be immersed in m_2 gm of oil of sp. heat S_2 kept in a calorimeter of mass m_1 gm and sp. heat S_1, the heat produced will raise the temperature of the oil. If the rise of temperature be $(\theta_2-\theta_1)$°C, then

$$H=(m_1S_1+m_2S_2)\,(\theta_2-\theta_1)\text{ cal}$$

$$\therefore\quad e.i.t.=J.(m_1S_1+m_2S_2)\,(\theta_2-\theta_1)$$

$$\text{or,}\quad J=\frac{e.i.t.}{(m_1S_1+m_2S_2)(\theta_2-\theta_1)}\text{ joules/cal.}$$

Experimental procedure : (1) Clean and dry the calorimeter and the stirrer well and find its mass (m_1 gm) in a balance.

(2) Pour in the calorimeter a quantity of turpentine oil such that the coil may remain fully immersed in it when the lid is placed on the calorimeter. Weigh the calorimeter with oil and the stirrer. From these two weighings, find the mass (m_2 gm) of oil taken.

(3) Place the lid on the calorimeter and see that the coil is completely immersed in the oil. Place the calorimeter inside a wider wooden box and pack the space between them with wool or cotton. Read the temperature (θ_1) of the oil with a thermometer.

(4) Complete the circuit connections, keeping the switch K open Put suitable number of electric bulbs in the lamp-board so that the circuit current may be within 1 to 2 ampere. Close the switch K and at once start the stop watch. Stir the oil gently with the stirrer.

(5) Record the readings of the ammeter, voltmeter and the thermometer at an interval of every one minute. When the rise of temperature is about 5°C, stop the current and the stop-watch. But continue stirring the oil. The temperature will slightly rise and then will become steady. Note the steady maximum temperature (θ_2) attained by the oil. From the stop-watch record the interval (t) during which the current was allowed to flow.

Measurements :

(*a*) *Determination of masses of calorimeter and oil.*

Weights			Specific heats	
Empty cal+Stirrer (m_1)	Calorimeter+oil+stirrer (M)	Oil taken $m_2=(M-m_1)$	Calorimeter (S_1)	Oil (S_2)
..gm+..gm +..mg=..gm	..gm+..gm +..mg=..gm	..gm	..	..

(*b*) *Time—Temperature table* : [Data given as illustration]

Initial temperature of oil $(\theta_1)=32°C$

Time (mnt)	Temperature of oil (°*C*)	Ammeter reading (amp)	Mean ammeter reading (*i* amp)	Voltmeter reading (volts)	Mean voltmeter reading (*e* volts)	*J* (joules/cal)
0	$32°C(\theta_1)$	1·7		5·0		
1	..	"	1·7	"	5·0	
2	..	"		"		
3	..	"		"		
4	36·5°	current		"		
stop-	36·8°	is				
watch	36·8°	stopped				
stopped	36·8° (θ_2)					

Calculation : Interval during which current flows $(t)=..$ sec.

Max^m temperature (θ_2) attained $=..$ °C

Initial temperature (θ_1) of oil $=..$ °C

$$\therefore\ J=\frac{e.i.t.}{(m_1S_1+m_2S_2)(\theta_2-\theta_1)}=..\text{ joules/cal.}$$

Remarks : (1) Before sending current, zero-error of the voltmeter and the ammeter, if any, should be noted. If there be any zero-error, the readings should be corrected accordingly. (2) The rise of temperature should not be more than 5°C/6°C. If this rise of temperature be caused in a short time, by sending heavy current, the radiation loss will be minimum. (3) The oil should be stirred continuously in order to ensure uniformity of temperature throughout its mass. The rise of temperature does not stop as soon as the current is stopped. The rise continues for some time. (4) All precautions should be taken in order to reduce the loss of heat. (5) The thermometer bulb must not touch the heating coil.

Oral questions

1. What is Joule's equivalent? What are its values in the F.P.S. and in the C.G.S. systems?

Ans. Consult any text book. In the C.G.S. system $J=4{\cdot}2$ joules/cal and in the F.P.S. system $J=778$ ft lb/B.th.u.

2. Should you take too much of oil in this experiment?

Ans. The quantity of oil should be such that the coil of wire is completely immersed in it. Too much of oil will delay the rise of temperature. Consequent radiation loss of heat will be high.

3. Should the current strength be high?

Ans. Current strength should be between 1 and 2 amp. so that the rise of temperature may take place quickly.

4. How will you identify the positive and negative of 'the mains'?

Ans. If the mains' leads are dipped into water, profuse quantity of gas bubbles will be produced at the negative.

5. How will you identify the 'live wire' of the mains?

Ans. One end of the test-bulb is to be earth-connected and the other inserted successively into the two holes of the mains. In one hole, the bulb will glow and that is the live wire of the mains.

6. What is the advantage of the switch being connected to the live wire?

Ans. This will eliminate the possibility of a shock.

7. What is the difference between an ammeter and a voltmeter?

Ans. Ammeter is a current measuring instrument ; its resistance is very low. A voltmeter is a p.d. measuring instrument; its resistance is very high.

8. Can you use alternating current instead of direct current in this experiment?

Ans. Yes, but in that case, the ammeter and the voltmeter should be a.c. ammeter and a.c. voltmeter.

9. Can you measure the specific heat of oil by this experiment?

Ans. Yes, in that case, the value of *J* should be supplied.

10. Can you measure the resistance of the coil by this experiment?

Ans. Yes ; by dividing the voltmeter reading (*e*) by the ammeter reading (*i*), we get the resistance of the coil.

11. What is the law of production of heat by current?

Ans. Consult any text book.

5.13. Determination of the reduction factor of a tangent galvanometer, using a copper voltameter :

Apparatus : A copper voltameter, a tangent galvanometer (single or double coil), a battery (one or two storage cells in series) rheostat, Phol's commutator, a stop watch, a spirit level, a balance and a weight box etc.

Description of copper voltameter : It consists of a cylindrical glass vessel of circular (or rectangular) cross-section containing a saturated solution of copper sulphate, slightly acidulated with sulphuric acid. A pair of copper plates (*A*, *A*) connected together form the anode and a detachable copper plate (*K*) of smaller area than the anode plates forms the cathode and it is held symmetrically between the two anode plates as shown in the fig. 47. The anode and cathode plates are connected to two binding screws *P* and *Q* respectively fitted on an ebonite disc which covers the glass vessel.

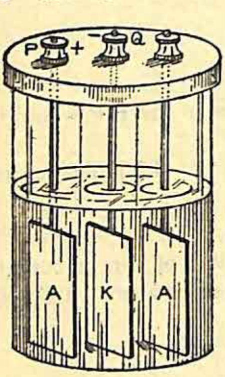


Fig. 47

Circuit arrangement : Fig. 47(*a*) shows the electrical connections necessary for the experiment. A copper voltameter *V* and a rheostat R_h are connected in series with a battery *E* (either a single storage cell or an alkalie cell or two such cells joined in series). The two terminals of the combination are joined to the binding screws B_1 and B_2 of a Phol's commutator. The negative terminal of the battery is, however, connected to the cathode *K* of the voltameter *V*. The binding screws A_1 and A_2 of the commutator are joined to the tangent galvanometer *G*. A freshly prepared solution of copper sulphate is taken in the voltameter.

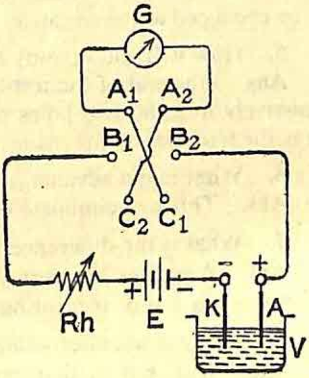


Fig. 47(*a*)

Theory : If *I* amp. of current flowing through a tangent galvanometer produces a deflection θ, then,

$I=10.\ K.\ \tan\theta$ .. (i) where K is the reduction factor of the galvanometer.

Again, if the same current flowing through a copper voltameter connected in series with the galvanometer, for t sec. liberates W gm of copper on the cathode plate, then $W=Z.I.t$

or $I=\frac{W}{Z.t}$ .. (ii) where Z is the $E.C.E.$ of copper. Combining eqns (i) and (ii), we have, $10.K.\tan\theta=\frac{W}{Zt}$

$$\therefore\quad K=\frac{W}{10Z.t.\tan\theta}\ e.m.u.$$

Knowing W, Z, t and θ, the reduction factor K can be found out.

[**N.B.** If K is expressed in amp, the formula is $K=\frac{W}{Z.t.\tan\theta}$ amp]

Experimental procedure : (1) Level the tangent galvanometer with the help of a spirit level. When the galvanometer is properly levelled, the coil of wire will be vertical and the magnetic needle together with the aluminium pointer will move over the horizontal scale freely. Slightly turning the circular wooden frame carrying the coil, about the vertical axis, bring the frame in the same plane with the magnetic needle. In this condition, the coil and the magnetic needle will both lie in the magnetic meridian and the pointer will indicate (0° – 0°) reading. In case the pointer does not read (0° – 0°), the disc containing the needle and the pointer should be slightly rotated to bring the pointer in the (0° – 0°) line. Now, the plane of the coil and the magnetic needle are set exactly in the magnetic meridian.

(2) Make circuit connections as shown in fig 47(a). Take care that the negative terminal of the battery is connected to the binding screw Q of the cathode [Fig 47].

(3) Pour a freshly prepared copper sulphate solution to which is added a little of sulphuric acid, into the copper voltameter so that the anode and cathode plates are fully immersed in the solution. Keeping the rheostat slider in the middle position, pass current through the galvanometer. Note the deflection of the pointer. Adjust the position of the rheostat slider so that the deflection of the galvanometer is exactly 45°. Reverse the direction of the current with the help of the commutator. Note the deflection. It should also be exactly 45°. Remove the rocker from the commutator and stop the current

(4) Take out the cathode plate K from the voltameter. Rub

both the surfaces of the cathode plate by fine-grained emery paper to make it free from oily or greasy substances. Dust the plate with clean cloth and then wash it with distilled water. Dry the plate under electric fan. Do not use direct flame or hot air for drying purpose. It will oxidise the plate.

(5) Weigh the clean and dry plate (W_1) in a balance to the nearest milligram, using a rider. Then place the cathode plate in its proper position in the voltameter.

(6) Now put the rocker on the commutator and at once start the stop watch. Current will flow through the voltameter and the galvanometer and the pointer will be deflected. Record the deflections indicated by both the ends of the pointer. Allow the current to flow for 25 minutes. At the intervals of 5 minutes, reverse the direction of current several times and note the deflections, in each case, indicated by both the ends of the pointer. From all these readings of deflection, find the mean deflection (θ).

(7) When the interval of 25 minutes is over, stop simultaneously the current and the stop watch. Removal of the rocker from the commutator stops the current. Note from the stop watch, the exact time for which current is passed. Now carefully take out the cathode plate from the voltameter and rinse it, first, with tap water and then with distilled water. As before, carefully dry the plate under a whirling electric fan.

(8) Weigh the dry plate in the balance to the nearest milligram using a rider (W_2). Mass of copper deposited will be available by subtracting the previous weight (W_1) of the plate from the present weight (W_2).

Measurements : (*a*) Mass of the cathode plate before current is passed =...gm+...gm+...mg+...mg=...gm (W_1)

Mass of the cathode plate after current is passed

=...gm+...gm+...mg+...mg=...gm (W_2)

$\therefore$ Mass of copper deposited $W=W_2-W_1=$...gm

(*b*) Electro-chemical equivalent of copper (Z)=0·0003293 gm/coulomb (given)

Measurement of deflection (θ) :

Time-interval in mnts	Direction of current	Deflection (Degree)		Mean deflection (degree)
		End I	End II	
0-5	Direct	45°	45°	
5-10	Reverse	45°	46°	..
10-15	Direct	..	..	
15-20	Reverse	..	..	
20-25	Direct	..	..	

Calculations :

$W = \ldots \text{gm}$

$t = 25 \text{ min} = \ldots \text{sec.}$

$Z = 0003293 \text{ gm/coulomb}$

$\theta = \ldots {}^\circ$

$$\therefore \quad K = \frac{W}{10\, Z.t.\tan\theta} = \ldots \text{e.m.u.}$$

Remarks : (1) The galvanometer coil should be vertical and placed in the magnetic meridian. (2) To get a good deposit, the current should be so adjusted that for every 50 sq.cm. area of the cathode plate, it is about 1 amp. Further, the density of the copper sulphate solution should be nearly 1·18 gm/c.c. and for every 100 c.c. of the solution 1 c.c. of sulphuric acid (conc) and 1 c.c. of alcohol should be added. (3) The cathode plate should be absolutely free from oily or greasy substances. After the deposition of copper, the plate must not be dried by the application of direct bunsen flame or any other flame. (4) Cathode and anode plates should be parallel to each other. They must, on no account, touch each other. (5) While reading the deflection of the pointer, parallax should be avoided and while weighing the plate, rider should be used. (6) Readings must be taken with direct and reverse currents.

Oral questions

1. What is reduction factor of a tangent galvanometer ?

Ans. The current in *e.m.u.* that produces a deflection of 45° in the galvanometer is called the reduction factor of the tangent galvanometer.

2. What is the electrochemical equivalent of an element ?

Ans. The mass of ion of the element liberated when 1 coulomb of current passes through any solution of the salt of the element is called the electro chemical equivalent of the element.

3. Why do you keep the deflection restricted to 45° ?

Ans. From the principle of galvanometer it is found that proportional error in the measurement of current is minimum when the deflection is 45°. This is why the deflection is restricted to 45°.

4. Is the direction of current reversed in the galvanometer or in the voltameter in this experiment ?

Ans. The current is reversed through the galvanometer only. Had the current been reversed in the voltameter, the nature of cathode and anode plates would have been reversed alternately and no deposit would have been available.

5. Why do you take the readings of both the ends of the pointer ?

Ans. The pointer may not be pivoted exactly at the centre of the circular scale. In such circumstances, the reading of only one end of the pointer may not be accurate. The consequent error is known as *eccentric error*. To eliminate eccentric error, both the ends of the pointer are read.

6. Which one is better—one coil galvanometer or double coil galvanometer ?

Ans. Double coil galvanometer is better. It is more sensitive and more accurate than one coil galvanometer.

7. Can this experiment be performed keeping the galvanometer surrounded by a magnetic material, say a thin iron sheet ?

Ans. No ; if the tangent galvanometer is surrounded by a magnetic material, lines of force of earth's magnetism will not be able to enter into the material and the galvanometer will not work.

8. When copper is deposited on the cathode plate, is there any loss of weight of the anode plate ? Is the loss of weight of the anode plate equal to the gain of weight of the cathode plate ? If not, why not ?

Ans. Yes, the anode plate loses some weight when copper is deposited on the cathode plate. Theoretically the loss of weight suffered by the anode plate is equal to the gain of weight by cathode plate but in practice the loss of weight is found to be greater. The reason is that with the loss of copper from the anode plate some of the impurities attached with the plate get loose and fall down on the bottom of the vessel. This makes the loss of weight of the anode plate greater.

9. What is the difference between a voltmeter and a voltameter ?

Ans. The vessel in which electrolysis of an electrolyte is carried out with the help of two electrodes is known as a voltameter. The instrument by which the potential difference between two points of a circuit is measured in volts is known as a voltmeter.

10. What do you mean by an electrolyte ? Is mercury an electrolyte ?

Ans. Any salt solution which undergoes ionisation when solution is prepared is called an electrolyte ; for example sodium chloride solution. Mercury, though a liquid, is not an electrolyte.

5.14. Potentiometer and its pricinple :

Description : Fig 48 shows a type of 10-wire potentiometer frequently used in a laboratory. With this instrument, potential difference may be measured. It consists of ten wires of uniform cross-section, each one metre long and connected in series. The wires run parallel to each other and rest on a milk-white glass plate mounted on a wooden platform. One end of the wire is soldered at *A* and the other at *B*. The wires are so selected that their temperature coefficient of resistance is low. Usually manganin or constantant wires are used. A triangular jockey, made of brass, can slide along the wires from left to right or from right to left and one of its legs *L* is always in contact with a brass-strip *R*−*R*. A galvanometer is connected to the terminal *R* fixed at the end of the brass-strip. This brings the jockey in contact with the galvanometer. If the central key *T* be pressed, the central leg touches the wire. By moving the central key forward or backward, the central leg may be brought in contact with any wire.

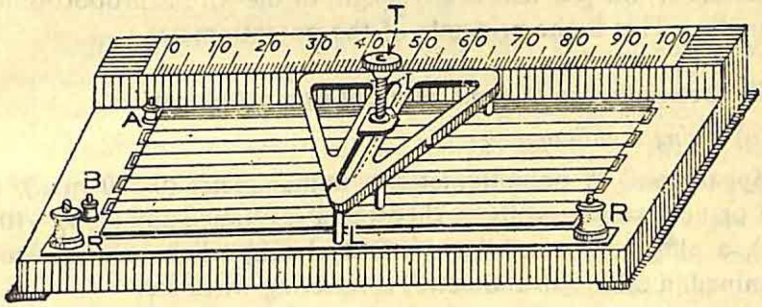


Fig. 48

Principle : Suppose *AB* is the potentiometer wire [Fig 48(a)]. A current is allowed to flow through the wire by means of a fixed e.m.f. battery *E*. Let the resistance per unit length of the wire be σ and the current passing through the wire be I amp.

The terminal p.d. of the wire $=1000\,\sigma\times l$ [$AB=1000$ cm.]

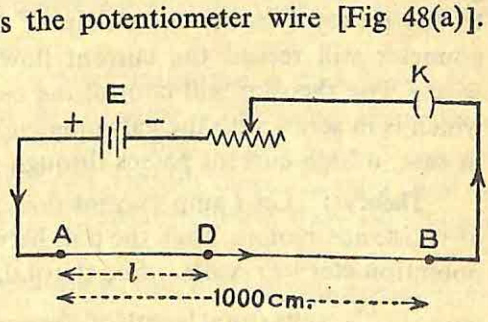


Fig. 48(*a*)

The p.d. of the wire $AD=l\times\sigma\times I$

$$\frac{\text{The terminal p.d. of the wire } AB}{\text{P.D. of the portion } AD}=\frac{1000\,\sigma\times I}{l\times\sigma\times I}=\frac{1000}{l}.$$

Hence, the p.d. of the portion $AD=\frac{l}{1000}\times$terminal p.d. of the wire AB.

Hence, it is seen that when a steady current flows through the potentiometer, the p.d. across any length of the wire is proportional to the length. This is the principle of the potentiometer.

5.15. Determination of e.m.f. of a cell by a potentiometer :

(*a*) *Using a milliammeter* :

Apparatus : A potentiometer, a milliammeter (0-500 m.a.), two alkali or acid storage cells, a rheostat, a resistance box (1000 – 10000 ohms), a plug key, a cell (say, a Daniel cell) whose e.m.f. is to be determined, a table galvanometer, connecting wires etc.

Circuit connections : Fig 49 shows the necessary circuit connections. Two storage cells (either acid or alkali) E are connected in series to form a battery. The positive terminal of the battery is connec ted to the binding screw A of the potentiometer and the negative terminal to the binding screw B, through a milliammeter Am, a rheostat Rh and a plug key K. The positive terminal of the cell E_1 (a Daniel cell) whose e.m.f. is to be found out is connected directly to the binding screw A while its negative terminal is joined to a resistance box S. One end of a table galvanometer G is connected to the resistance box S and the other end to the jockey (via the brass-strip of the potentiometer). The milliammeter will record the current flowing through the potentiometer wire. The rheostat will control the current and the high resistance S which is in series with the galvanometer, will protect the galvanometer, in case, a high current passes through it.

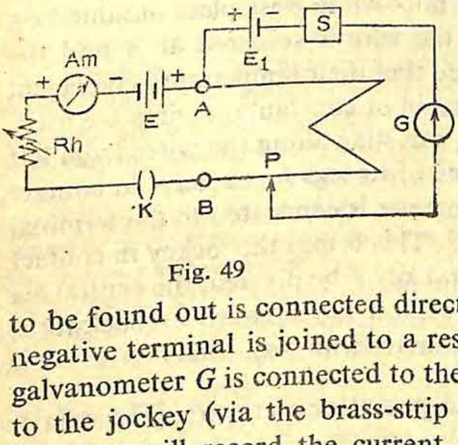


Fig. 49

Theory : Let i amp current flow through the potentiometer wire of resistance r ohm. Then the p.d. between the points A and B of the potentiometer$=i.r$ volts. So, the p.d. per cm. of the potentiometer wire$=\frac{i.r}{1000}$ volts (total length of the potentiometer wire being 1000 m)

When the cell E_1 is balanced against the p.d. between the ends of the potentiometer wire, let a balance point be found at a length l cm. of the wire. Now, the p.d. across the length l is equal to $\frac{i.r.l}{1000}$ volts.

Hence, the e.m.f. of the cell $E_1 = \frac{i.r.l}{1000}$ volts

Experimental procedure : (1) Measure the resistance (r) of the potentiometer wire by a P.O. Box in the usual way. [If, however, the resistance is supplied, this operation need not be performed].

(2) Make the circuit connections as shown in the fig 49. Take care that the positive terminals of the battery E and the cell E_1 are connected to the point A of the potentiometer. Also see that the milliammeter is connected properly in the potentiometer circuit.

(3) Put the slider of the rheostat in the minimum resistance position so that the maximum possible current that the milliammeter can stand is allowed to flow. Put the plug in the plug key K and take out a high resistance (of the order of 10,000 ohms) from the box S. Put the jockey in contact with the beginning of the potentiometer wire at A and note the galvanometer deflection. Next, put the jockey in contact with the end of the wire at B. If the deflections of the galvanometer are opposite, the connections are right. If the deflections are not opposite, the connections are defective. In such cases, check the connections again. Take the help of the teachers, if necessary, to detect the fault in the connections.

(4) Having made proper connections, increase the rheostat resistance (*i.e.* decrease the potentiometer current) till the null point is obtained at the tenth wire. The null point should be accurately found out by decreasing the resistance in the box S to a minimum value. This null point is noted thrice and the mean is determined. Note the milliammeter reading.

(5) Now decrease the rheostat resistance a little. Putting a high resistance (10,000 ohms) in the box S, find out approximate position of the null point at the nineth wire. Then make S zero and accurately find the null point thrice. From these readings, find the mean position of the null point. Note the current from the milliammeter.

(6) Repeat the above operation so that null point is now obtained at the eighth wire.

Measurements : (a) Resistance of potentiometer wire $(r) = \ldots$ ohms.

(Give a table for P.O. Box measurement)

(b) *Determination of null point* : (Data are for illustration)

No of Obs	Current from milliammeter (*m.a*)	Current in amp. (*i*)	Null point length			Total length (*l*)	$E_1 = \frac{i.r.l}{1000}$ volt
			Number of wire	Scale reading	Mean scale reading (*cm*)		
1	70	$\frac{70}{1000} = 0{\cdot}07$	10 th	48 50 48	48·6	948·6 cm	..
2	80	$\frac{80}{1000} = 0{\cdot}08$	9 th	25 24 23	24	824 cm	..
3	90	$\frac{90}{1000} = 0{\cdot}09$	..	..	..	..	..

Mean e.m.f. = . . . volts.

Calculations : 1. $E_1 = \frac{i.r.l}{1000} = \ldots$ volt.

2. $E_1 = \ldots$

3. $E_1 = \ldots$

Remarks : (1) The deflection of the galvanometer may not be opposite when the jockey is made to touch the first and the last wire due to two possible faults in the circuit arrangement *viz*. (i) the positive terminals of the cells E_1 and E, are not connected to the same point A of the potentiometer or (ii) the e.m.f. of the cell E is less than that of the cell E_1. So, before proceeding with the experiment, care should be taken against these two points. (2) In each observation, rough balance point should first be found out with a very high resistance in the box S and then the galvanometer is to made very sensitive by making $S=0$ and then accurate balance point should be found out.

(*b*) *Without using a milliammeter* :

Apparatus : All apparatus required in the previous experiment except the rheostat. In place of rheostat a resistance box (0-500 ohm) and a voltmeter are needed.

Circuit connections : Same as in the previous experiment except that the rheostat is replaced by a resistance box R (0-500 ohm).

Theory : Suppose, E=e.m.f. of the battery in the potentiometer circuit.

r =resistance of the potentiometer wire.

R=resistance put in the box in the potentiometer circuit.

Then the current I flowing through the potentiometer is given by $I=\frac{E}{R+r}$.

Hence, p.d. across the points A and B of the potentiometer

$$=\frac{E.r}{R+r}.$$

So, p.d. per cm. of the potentiometer wire $=\frac{E}{(R+r)}\cdot\frac{r}{1000}$ volt/cm.

If l cm. be the length of the null point when the cell E_1 is balanced, the e.m.f. of the cell $E_1=\frac{E}{(R+r)}\times\frac{r.l}{1000}$ volts.

Experimental procedure : (1) Measure the resistance (r) of the potentiometer wire by a P.O. Box in the usual way and the e.m.f. (E) of the battery in the potentiometer circuit by a voltmeter.

(2) Make circuit connections as shown in fig 49, replacing the rheostat by a resistance box, say R. Take care that the positive terminals of the battery E and the cell E_1 are connected to the binding screw A of the potentiometer.

(3) Put a high resistance (about 10,000 ohms) in the box S and insert the plug in the key K. Without putting any resistance in the box R, bring the jockey in contact with the beginning of the potentiometer wire at A and note the galvanometer deflection. Next put the jockey in contact with the end of the wire at B. If the deflections of the galvanometer are opposite, the connections are right. If the deflections are, however, not opposite, the connections are defective. Check the circuit in such cases and set it right.

(4) Keeping the resistance S unchanged, adjust the resistance in the box R, so that an approximate null point is obtained at the last wire. Now make the galvanometer sensitive by making the resistance S=0 and find the null point accurately. Repeat it thrice and get the mean reading.

(5) Now decrease the resistance in the resistance box R, by such an amount that the null point shifts to ninth wire. First, find the approximate position of the null point by putting a very high resi-

stance in the box S and then accurately by making S zero. Repeat the operation thrice and find the mean position of the null point.

(6) Repeat the above operation once again with a suitable resistance in the box R so that the null point is obtained at the eighth-wire.

(7) Again find the e.m.f. of the battery E by a voltmeter.

Measurements : (a) Resistance of the potentiometer wire (r)=... ohms.

(A table for P.O. Box measurement should be given)

(b) *E.M.F. of the battery in the potentiometer circuit* :

Before the commencement of the experiment =...volts.
After the completion „ „ „ =...volts.
Average e.m.f. E=...volts.

(c) *Determination of null point* :

No of Obs	Resistance in the R-box (R ohm)	Null point length			Total length (l cm)	$E_1 = \frac{E}{(R+r)} \times \frac{r.l.}{1000}$ volt
		Number of wire	Scale reading	Mean scale reading		
1.	..	10 th	.. .. ..	..	..	..
2.	..	9 th	.. .. ..	..	..	..
3.	..	8 th	.. .. ..	..	..	..

Mean e.m.f. E_1=...volts.

Calculations : 1. $E_1 = \frac{E}{(R+r)} \times \frac{r.l}{1000}$=...volts.

2. E_1=...

3. E_1=...

(c) *With the help of a standard cell* :

Apparatus : A potentiometer, two storage cells (acid or alkali), a cell (say, Leclanche's cell) whose e.m.f. is to be found out, a standard cell, a rheostat, a plug key, a two-way key, a high resistance box (0-10,000 ohm), a table galvanometer etc.

Circuit connections : *AB* is the whole potentiometer wire [Fig 50] to which are connected in series a battery *E* (made of two storage cells joined in series), a rheostat R_h and a plug key *K*. The positive terminal of the battery *E* is joined to the binding screw *A* of the potentiometer. E_1 and E_2 are respectively a standard cell (a Daniel cell, say) and a Leclanche's cell. The positive terminals of both the cells are joined to the screw *A*. The negative terminals of the cell E_1 and E_2 are connected to the galvanometer *G* through a two-way key K_1. The other end of the galvanometer is connected to the jockey through a high resistance box *S*. When the plug is put in the hole no. 1 of the two-way key, the cell E_1 is connected to the galvanometer. On the other hand, if the plug be inserted in the hole no. 2, the cell E_2 is included in the galvanometer circuit. The e.m.f. of the battery *E* should be greater than the e.m.f.'s of the cells E_1 and E_2.

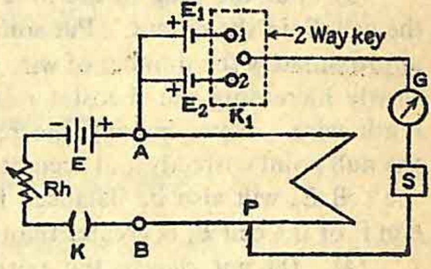


Fig. 50

Theory : If a cell of e.m.f. E_1 is balanced by a length l_1 of the potentiometer wire, then $E_1 \propto l_1$

Similarly, if another cell of e.m.f. E_2 is balanced by a length l_2 of the same potentiometer, then $E_2 \propto l_2$

$$\therefore \quad \frac{E_1}{E_2}=\frac{l_1}{l_2} \quad \text{or} \quad E_2=\frac{l_2}{l_1}. E_1$$

Knowing E_1 and finding l_1 and l_2 experimentally, the value of E_2 may be found out.

Experimental procedure : (1) Making the rheostat resistance zero and the resistance in the box *S* highest (say, 10,000 ohm), put the plug in the key *K*. Insert the plug in hole no. 1 of the two-way key K_1. This brings only the standard cell E_1 (the Daniel cell) in the circuit. Pressing the jockey key, bring the central leg of the jockey in contact with the beginning of the first wire and the end of the last wire. If the deflections in the galvanometer in the two cases are opposite, the connections are right. Bringing the jockey in contact with different wires, find the approximate null point when the galvanometer produces no deflection.

(2) Now, insert the plug in the hole no. 2 of the two-way key K_1. This brings the Leclanche's cell E_2 in the circuit. Keeping the rheostat resistance zero and *S* very high, find the approximate null point as in operation no. 1. The length of approximate balance point for the

cell of higher e.m.f. will be greater than that of the other. Suppose the e.m.f. of the cell E_1 is greater than that of the cell E_2. These operations ensure that the e.m.f. of the battery E in the potentiometer circuit is greater than the e.m.f. of the cell E_1 or E_2. This is essential for the success of the experiment.

(3) Put the plug in the hole no. 1 of the key K_1 again bringing the cell E_1 in the circuit. Put some resistance in the rheostat and find approximately the number of wire on which the balance point lies. By slowly increasing the rheostat resistance, bring the null-point on the tenth wire. Now, making the resistance $S=0$, find the position of the null point correctly and accurately ($l_1$cm). If the cell E_1 is balanced, the cell E_2 will also be balanced for the same current, because the e.m.f. of the cell E_1 is greater than the e.m.f. of the cell E_2.

(4) *Do not change* the position of the slider of the rheostat. Put the plug in the hole no. 2 of the two-way key K_1 bringing the Lechanche's cell E_2 in the circuit. Following the operation (3) find the null point (l_2 cm) first approximately and then accurately.

(5) Decrease the rheostat resistance by moving its slider *i.e.* increase the current through the potentiometer. Push the null point for the cell E_1 to the ninth wire and find also the exact null point for the cell E_2.

(6) In this way, take another set of readings with a slight lower rheostat resistance bringing the null point for the cell E_1 on the 8th wire.

(7) Find the ratio l_2/l_1 in each case. Ascertain the *e.m.f.* of the standard cell (E_1) from the teacher.

Measurements : E.M.F. (E_1) of the standard cell=... volts

No. of Obs	Balance points for						$\frac{l_2}{l_1}$	$E_2=\frac{l_2}{l_1}\times E_1$ volts
	the cell E_1			the cell E_2				
	No of wire	Scale reading	Total reading (l_1)	No of wire	Scale reading	Total reading (l_2)		
1.	10th	..	..	..	..	..	..	..
2.	9th	..	..	..	..	..	..	..
3.	8th	..	..	..	..	..	..	..

Mean value of E_2=.. volts.

Remarks : (1) All positive terminals of the cell E, E_1 and E_2 should be connected to the binding screw A of the potentiometer, (2) The *e.m.f.* of the battery E should be greater than that of the cell E_1 or E_2. (3) Continuous current through the potentiometer wire should be avoided. It produces undue heating causing a continuous fluctuation of null point. The plug of the plug key should be taken off for a while after finding each null point.

Oral questions

1. What is the working principle of a potentiometer ?

Ans. See page 287.

2. Why the cell in the potentiometer circuit be of greater *e.m.f.* than the other cells ?

Ans. If the cell in the potentiometer circuit be not of greater *e.m.f.*, the terminal *p.d.* of the potentiometer wire will be less than the *e.m.f.*'s of the other cells and null point will not be available for the cells.

3. What is the advantage of having the null point on the tenth wire ?

Ans. It will produce minimum percentage of error.

4. Why does the null point change slowly with time ?

Ans. Continuous current produces heating of the wire and a consequent change in the resistance of the wire. This causes a slow fluctuation of null point with time.

5. What is the practical unit of *e.m.f.* ?

Ans. Volt.

6. What is the difference between *e.m.f.* and *p.d.* ?

Ans. Consult any text book.

7. What should be done to bring the null-point on the 10th wire ?

Ans. The rheostat resistance should be increased so that a smaller current flows through the potentiometer. This will reduce the *p.d.* across the potentiometer wire and the null point will be obtained on a lower wire.

8. Should the potentiometer wire be uniform ?

Ans. Yes ; in that case, the fall of *p.d.* along the wire will be uniform.

5.16. Measurement of current by a potentiometer :

Apparatus : A storage (acid or alkali) cell, a Daniel cell, a potentiometer, a high resistance box (1-1000 Ω), a standard low resistance (1 ohm), a table galvanometer, a P.O. box, a resistance coil (20 ohm), two plug keys, voltmeter etc.

Circuit connections. : Connect a resistance coil R' (20 Ω), a standard low resistance r (1 ohm) and a plug key K' in series with a Daniel cell E' (Fig. 51). Current flowing in this circuit is to be found out. In series with the potentiometer AB, connect a storage cell E, a high resistance box R and a plug key K. Then, connect the positive end of the standard low resistance r to the binding screw A of the potentiometer and the negative end to the jockey J through the galvanometer G.

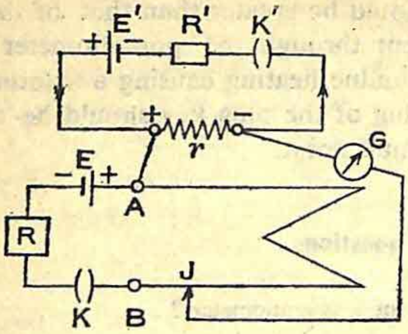


Fig. 51

Theory : Let i be the current flowing through the standard low resistance r. Here, the terminal *p.d.* of the resistance $r = i.r$ [Fig. 51]

If this *p.d.* is balanced against a length l cm. of the potentiometer, then, $ir = l.e$ where e is the *p.d.* per cm. of the potentiometer wire.

Now, $e = \frac{E}{R+R_1} \times \frac{R_1}{1000}$ where E=*e.m.f.* of the battery E in the potentiometer circuit, R_1=resistance of the potentiometer wire and R= resistance put in the box in the potentiometer circuit.

$$\therefore \quad i = \frac{E}{R+R_1} \times \frac{R_1}{1000} \times \frac{l}{r}$$

Experimental procedure : (1) Measure the resistance (R_1) of the potentiometer wire by a P.O.Box in the usual way and the *e.m.f.* (E) of the storage cell in the potentiometer circuit by a voltmeter.

(2) Make the electrical connections as shown in fig. 51. See whether the positive end of the standard low resistance (r) is connected to the binding screw A of the potentiometer. Put some resistance (say, 20 ohms) in the resistance box R. Put the plugs in the plug keys K and K'. Note the deflection of the galvanometer after making contact between the jockey and the beginning of the potentiometer wire. Now bring the jockey in contact with the end of the wire. If the deflection of the galvanometer is opposite to the previous deflection, the connections are right. If not, the connections are defective or faulty. Remove the fault, if necessary, with the help of your teacher.

(3) Having made the correct connections, adjust the resistance of the box R so that the null point is obtained somewhere at the 10th wire. Find the exact position of the null point twice—once by moving the jockey from left to right along the 10th wire and then by moving it from right to left. Find the mean of these two readings.

(4) Decrease the resistance in the resistance box R so that the null point shifts to 9th wire. As before, find the position of the exact null point twice—once by moving the jockey from left to right and then in the opposite direction. Get the mean of these two readings. Repeat the operation again with suitable resistance in the box so that the null points is obtained at the 8th wire.

(5) After the experiment, again find the *e.m.f.* of the battery E with a voltmeter and find the mean of the initial and final values of E. Calculate the value of the current in each observation and the mean therefrom.

Measurements : (*a*) *Resistance of the potentiometer wire*

$R_1=..$ ohms

[Give a table for P.O. Box measurement]

(*b*) *E.M.F. of the storage cell* :

Value before experiment $=..$ volts

,, after ,, ,,

$\therefore$ Mean $E=..$ volts.

(*c*) *Measurement of balance point* :

Standard low resistance used $(r)=..$ ohm.

No. of Obs.	Resistance in R-box (R ohm)	Balance point			Total length (l. cm)	Current (i) amp
		No. of wire	Scale reading	Mean scale reading		
1.	..	10th	.. ..	..	..	..
2.	..	9th	.. ..	..	..	..
3.	..	8th	.. ..	..	..	..

Mean current $=..$ amp.

Calculations : 1. $i=\frac{E}{R+R_1}\times\frac{R_1}{1000}\times\frac{l}{r}=\ldots$ amp.

2. $i=..$ amp.

3. $i=..$ amp.

Remarks : (1) The positive end of the standard low resistance should be connected to the positive end of the potentiometer wire. (2) Continuous current should not be allowed to flow through the potentiometer. It may cause undesirable heating and fluctuation of null point. After each observation, current should be cut off for some time by taking off the plug from the plug key. (3) If a suspended coil galvanometer is used instead of a table galvanometer, a high resistance should be connected in series with it.

Alternative method : With the help of a milli-ammeter :

Apparatus : One or two alkali storage cells, a Daniel cell, a potentiometer, a rheostat (20 ohm), a standard low resistance (1 ohm), a table galvanometer, a P.O. Box, a resistance coil (20 ohm), two plug keys, a milliammeter (0-100 m.a.) etc.

Circuit connections : Fig. 52 shows the necessary circuit connections. A battery of two alkali storage cells in series (or one cell, as the case may be), a rheostat R_h, a milliammeter (*MA*) and a plug key *K* are connected in series between the binding screws *A* and *B* of the potentiometer. With the Daniel cell *E'*, a resistance coil *R'* (20 ohms), a standard low resistance *r* (1 ohm) and a plug key K_1 are connected in series. The positive end of the low resistance *r* is joined to the binding screw *A* of the potentiometer and the negative end to the jockey *J* through a galvanometer *G*.

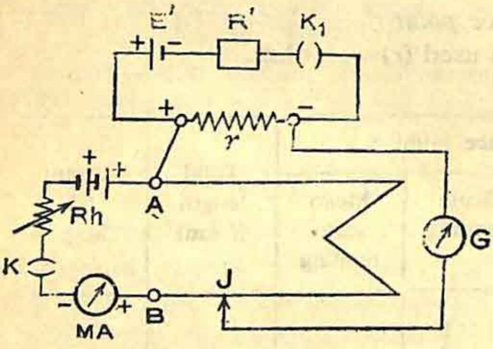


Fig. 52

Theory : Let *i* be the current flowing through the low resistance *r*. Here, the *p.d.* at the ends of the resistance $r=i.r$ [Fig. 52]. If this *p.d.* is balanced by a length *l* of the potentiometer, then, $ir=l.e$ where *e* is the *p.d.* per cm. of the potentiometer wire.

But $e=\frac{I}{1000}\times\frac{R}{1000}$ where I=current recorded by the milliammeter in milliampere and R=resistance of the potentiometer wire.

$$\therefore\quad i=\frac{l}{r}.\ e=\frac{l}{r}\times\frac{I.R}{10^6}=\frac{I.R.l}{r}\times10^{-6}\text{ amp.}$$

Experimental procedure : (1) Measure the resistance of the potentiometer wire by a P.O. Box. See whether there is any zero-error in the milli-ammeter. If the pointer reads zero when no current flows through the milli-ammeter, the instrument has no zero-error.

(2) Make connections as shown in fig. 52. Take care that the positive end of the low resistance r is joined to the positive end (A) of the potentiometer. Keeping the slider of the rheostat R at the middle position ; put the plugs in the keys K and K_1. Then bring the jocky in contact with some point of the wire very near A and note the deflection of the galvanometer. Next make contact with some point of the wire very near B. If the deflection of the galvanometer now be opposite to the previous deflection, the connections are alright. If not, the connections have defect. Check the connections and set them right. If necessary, the help of the teachers may be sought.

(3) Having made the right connections, adjust the position of the slider of the rheostat so that the null point is obtained somewhere at the 10th wire. Find out accurately the position of the null point twice—first by moving the jockey slowly from left to right along the tenth wire and then from right to left. Find the mean of these two readings. Also record the current (I) shown by the milliammeter.

(4) Moving the slider a little, increase the current through the potentiometer so that the null point is shifted to the 9th wire. As before find the position of the null point twice and the mean therefrom. Also note the current from the milliammeter. Repeat the operation once again with the null point obtained at the 8th wire.

(5) The values of current obtained from three observations are to be separately calculated and the mean value is to be found out from them.

Measurements : (*a*) *Resistance of potentiometer wire.*

$R = ..$ ohms

[Give the table for P.O. Box measurements] (*b*) *Measurement of current* :

The standard low resistance (r) used= .. ohm

No. of Obs.	Milliammeter reading (I *m.a*)	Position of the balance point				$i=\frac{I.Rl}{r}$ $\times 10^{-6}$ (amp)	Mean i (amp)
		No. of the wire	Scale reading	Total length	Mean length		
1.	..	10th	.. .. }	.. .. }	..	..	..
2.	..	9th	.. .. }	.. .. }	..	..	
3.	..	8th	.. .. }	.. .. }	..	..	

Calculations :

1. $i=\frac{I.R.l}{r}\times 10^{-6}=..$ amp.
2. $i=..$amp.
3. $i=..$amp.

Oral questions

1. What did you actually measure in this experiment ? Current or potential difference ?

Ans. In this experiment, the *p.d.* across the standard low resistance was measured by the potentiometer and then dividing the *p.d.* by the low resitance, current flowing through the circuit was calculated.

2. Why are you advised to keep the null point at the 10th wire ?

Ans. It keeps the percentage error minimum.

3. Why is it not desirable to draw continuous current through the potentiometer ?

Ans. See remark no. 3, page 295.

4. Which one would you prefer—a table galvanometer or a suspended coil galvanometer ?

Ans. Suspended coil galvanometer. It is more sensitive.

5. Inspite of the positive end of the standard low resistance being connected to the positive end of the potentiometer, the deflection in the galvanometer is found to be always in the same direction. What may be the reason of it ?

Ans. If the *p.d.* between the points *A* and *B* of the potentiometer wire be less than the terminal *p.d.* of the standard low resistance, the deflection will be always in the same direction. To avoid it, the *e.m.f.* of the cell *E* should be more than that of the cell *E'*.

6. Should the potentiometer wire be uniform ?

Ans. Yes ; if the wire be not uniform, the fall of potential along the wire will not be equal.

7. What is the basic principle of a potentiometer ?

Ans. See page 287.

* 5.17. Measurement of the thermo-e.m.f. with a potentiometer and to draw E—T curve :

Apparatus : A potentiometer, a storage cell (either an acid or an alkali cell), a resistance box (0-10,000Ω), a plug key, a large beaker, funnel, a thermometer, a copper-constantan thermo-couple, connecting wire, a voltmeter etc.

Description of the thermo-couple : A thermo-couple is prepared by soldering two wires of different materials, end to end. Here, two pieces of copper wire and a piece of constantan wire have been used to prepare the thermo-couple. One end of a copper wire is soldered to one end of the constantan wire while the other end of the constantan wire is similarly soldered to one end of the second copper wire, leaving one end of each copper wire free. If the two junctions of the thermo-couple so prepared be kept at a temperature difference of 100°C, a potential difference of about 5000 micro-volt will be developed.

Circuit connections : One junction of the thermo-couple is inserted into powdered ice kept in a funnel. Its temperature is 0°C (cold). The other junction is dipped into a test tube containing turpentine. The test tube is placed in a large beaker containing water. When the beaker is heated by a burner, the junction of the thermo-couple immersed in the water is heated (hot). One of the free ends of the copper wire of the couple is connected to the binding screw *A* of the poten-

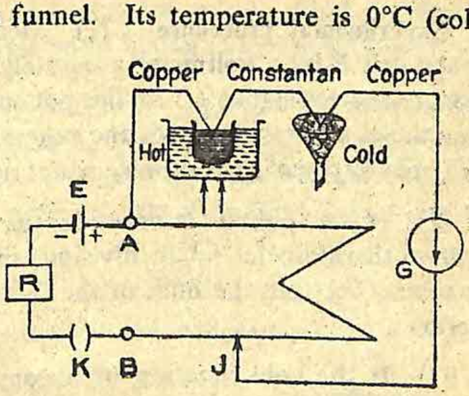


Fig. 53

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* For North Bengal University only

tiometer and the other free end to the jockey J through a suspended coil galvanometer G. A storage cell E, a resistance box R and a plug key K are joined in series with the potentiometer wire AB.

Theory : When one junction of a thermo-couple is kept at 0°C and the other junction at a higher temperature, a thermo-*e.m.f.* is developed in the couple. If this thermo-*e.m.f.* (E) is balanced against the *p.d.* across a length l of the potentiometer wire, then $E = e \times l$, where $e = p.d.$ per cm. of the potentiometer wire. Now, suppose, the *e.m.f.* of the cell E in the potentiometer wire $= E_1$, the resistance put in the box $= R$ and the potentiometer resistance $= r$, then,

$$\text{the } p.d. \text{ between } A \text{ and } B = \frac{E_1 . r}{R + r}.$$

As the length of the wire between A and B is 1000 cm, the *p.d.* per cm. of the wire is given by

$$e = \frac{E_1 . r}{(R + r) \times 1000} \text{ volts} \quad .. \text{ (i)}$$

$$\therefore \quad E = \frac{E_1 . r . l}{1000(R + r)} \text{ volts} \quad .. \text{ (ii)}$$

If the temperature of the hot junction of the thermo-couple be changed, the thermo-e.m.f. is also changed. Measuring thermo-e.m.f's at various temperatures (t) of the hot junction, a graph between E and t may be drawn.

Experimental procedure : (1) Measure the e.m.f. (E_1) of the storage cell E by a voltmeter accurately upto two places of decimal. Measure the resistance (r) of the potentiometer wire by a P.O. Box. (Sometimes, the resistance of the potentiometer is supplied. In such cases, measurement of resistance is not necessary).

(2) Make circuit connections as shown in fig. 53. Insert a sensitive thermometer ($\frac{1}{10}$th division) into the turpentine kept in the test tube. See that the bulb of the thermometer is very near the hot junction.

(3) If the cold junction of a copper-constantan thermo-couple be at 0°C and the hot junction at 100°C, a few thousand microvolt of *p.d.* is developed. To measure it, a *p.d.* of 5000 micro-volt between the points A and B of the potentiometer or 5 micro-volt per centimeter of the wire need be produced. For this purpose, a suitable resistance

is to be put in box *R* so that the *p.d.* per cm. of the wire *i.e.* *e* becomes $5\mu V$. This value of *R* can be calculated from eqn. (1) of the theory*.

(4) Now, it is to be checked whether the positive end of the thermo-couple is joined to the binding screw *A* of the potentiometer. To do this, making $R=0$, allow the jockey to touch the beginning of the wire and note the galvanometer deflection. Now bring the jockey in contact with the end of the wire. If the deflection is opposite to the first, the positive end of the couple has been properly joined to the screw *A*. If the deflection be not opposite, then the connection of the thermo-couple is to be reversed *i.e.* the terminal connected to the galvanometer be now joined to the binding screw *A* and vice versa. Then put resistance in the box *R* as calculated in operation (3) so that a *p.d.* of 5 micro-volt per cm. (*e*) of the wire is produced.

(5) Keep the hot junction of the thermo-couple at room temperature and note the temperature (t_1°C) from the thermometer. Find the null point on the potentiometer wire and read the length of the null point ($l_1 cm$). [Probably, the null point will be obtained at the 1st or 2nd. wire]. Calculate, the thermo-e.m.f. developed at that temperature by multiplying e ($=5\times10^{-6}$ volt) with l_1.

(6) Heat the water in the beaker by a burner and raise its temperature by 10°C. Stir the water gently and by controlling the heat, keep the temperature of the water constant for a few minutes. Note the constant temperature (t_2). While the temperature of water is kept constant, find the balance point and its length (l_2). (Perhaps the null point will be obtained at the 3rd. wire). Find the thermo-e.m.f. corresponding to the null-point length l_2 by the product el_2.

(7) In this way raise the temperature of water by steps of 10°C until the temperature becomes 80°C. At every step, the temperature should be kept constant for a few minutes by controlling the heat and the balance point should be found out when the temperature becomes constant.

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* Suppose the *e.m.f.* of the cell $E=2$ volt (E_1) and the resistance of the potentiometer wire $=20$ ohms. (r). If $e=5\mu V$/cm, then $5\times10^{-6}=\dfrac{2\times20}{(R+20)\times1000}$

$$\therefore\ R=7980 \text{ ohm.}$$

So, if resistance of 7980 ohm is put in the box *R*, the *p.d.* per cm. of the potentiometer wire will be $5\mu V$.

(8) Draw a graph between the temperatures of the hot junction of the couple and the thermo-e.m.f. developed. Temperature is to be plotted along the *X*-axis and the thermo-e.m.f. along the *Y*-axis. The graph, in general, is a parabola but within a short range of temperature as in the present case (0°*C* – 80°*C*), the graph will be a straight line (straight portion of the parabola) as shown in fig. 53 (*a*).

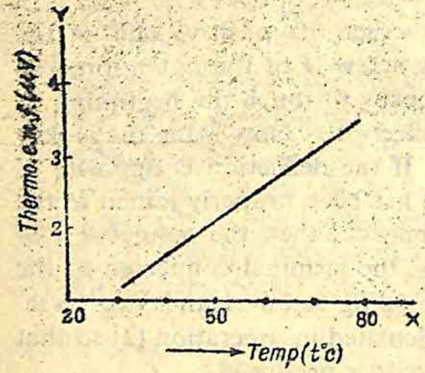

Fig. 53(*a*)

Measurements : (*a*) *E.M.F.* of the storage cell (*E*)=. . . volt (*E_1*)
Resistance of the potentiometer wire (*r*)=. . . ohm

[Table for P.O. Box measurement may be given here]

For calibration of the potentiometer wire (*i.e.* for a *p.d.* of 5×10^{-6} μV per cm), the resistance put in the box $R=$. . . ohms (R)

(*b*) **E – t table** :

Temp of cold junction (°C)	Temp. of hot junction (°C)	Position of null point			*e* (volts)	*E*=*e.l.* (volts)
		No. of the wire	Scale reading	Total length (cm)		
0°	t_1° (room temp)	1st wire	..	..(l_1)	5×10^{-6}	$5\times10^{-6}\times l_1$
,,	(t_1+10)°	2nd wire	..	..(l_2)	,,	$5\times10^{-6}\times l_2$
0°	(t_1+20)°	4th wire	..	..(l_3)	,,	$5\times10^{-6}\times l_3$
etc	etc		etc	etc		
0°	80°	..	..	..	..	..

Remarks : (1) During experiment, the cold junction of the couple should remain always at 0°*C*. The heat of water in the beaker may be conducted to the cold junction through the wire which, by melting ice round the junction, may create an air pocket there. If it happens, the temperature of the cold junction may not be 0°*C*. To prevent it, the mass of ice should be poked from time to time with an iron wire and fresh powdered ice need be added into the funnel. (2) Both the junctions will remain well dipped in ice and turpentine respectively. (3) The difference between the temperatures of cold and hot junctions should not be high because in that case, the graph may not be a straight line. (4) The burner should be sufficiently far away from the cold junction. (5) A shunt should be used with the galvanometer.

Oral questions

1. What are the thermo-electric effects ? What is a thermo-couple ?

Ans. Consult any text book.

2. If a graph is drawn between the temperature of hot junction and the thermo *e.m.f.* developed, the cold junction being always kept at 0°C, will it be a straight line ?

Ans. For small range of temperature, it will be a straight line but for higher temperatures the curve is a parabola.

3. What are the neutral temperature and temperature of inversion of a thermo-couple ?

Ans. Consult any text book.

4. What is the relation between the thermo-*e.m.f.* and the temperature difference between the two junctions of the couple ?

Ans. If the cold junction be at 0°C and the hot junction at t°C, then for moderate range of temperature, the thermo-*e.m.f.* E is given by $E=at+bt^2$ where a and b are two constants which depend on the nature of the thermo-couple.

5. What is the most important practical application of a thermo-couple ?

Ans. The most important application is the measurement of temperature. Very high temperature can be measured by a thermo-couple.

6. How did you calibrate the potentiometer in this experiment ?

Ans. The potentiometer is so calibrated that *p.d.* per cm. of the wire is 5 micro-volt because for a difference of temperature of 100°C, the *p.d.* developed in the given couple is about 5000 micro-volt.

6. SOUND

*6.1. Determination of the frequency of a tuning fork by a sonometer

Apparatus : A sonometer, a tuning fork, a padded hammer, some slotted weights, metre scale etc.

Description of sonometer : Fig 54 shows a sonometer. It consists of a wire stretched on a hollow, rectangular wooden box. One end of the wire is fixed to a peg and the other end hangs vertically supporting a hook. The string passes over two fixed bridges A and B and a pulley P. When weights (W) are put on the hook, the wire becomes taut. Between the two fixed wooden bridges A and B, there is a movable bridge M. If the portion of the wire between the bridges A and M be plucked at the middle, it will go on vibrating transversely and will emit a sound whose frequency depends on the length, the tension and the mass per unit length of the wire. Changing position of the movable bridge M, the length of the vibrating portion of the string may be altered.

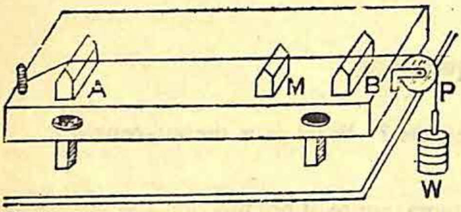

Fig. 54

Theory : The fundamental frequency of transverse vibration of a stretched string is given by $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$.

where l=length of the string vibrating ; T=tension of the string and m=mass/unit length of the wire.

If m_1 be the mass used for stretching the string, then $T=m_1g$

$$\therefore \quad n=\frac{1}{2l}\sqrt{\frac{m_1g}{m}}=\sqrt{\frac{g}{4m}\times\frac{m_1}{l^2}}$$

If a tuning fork vibrates in unison with the above string, the frequency of the fork may be obtained from the above equation.

Experimental procedure : (1) First, the mass per unit length of the sonometer wire is to be determined. For this purpose, take a piece of sample wire and find its length (L) by a metre scale. Then find the mass (ω) of the sample wire by a balance to the nearest milligram. From this, $m=\frac{\omega}{L}$ gm/cm can be found out.

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(2) If the weight of the hook is not known (usually it is supplied), then it is to be detached from the wire and weighed in a balance ($m_2$gm). Then it is to be attached to the wire again.

(3) Put a suitable weight W kg (say 1 kg) on the hook. It will produce a total tension of $(1000W+m_2)$gm. This has been referred to as m_1 in the theory. Due to this tension, the wire will remain taut. Put the movable bridge M [Fig 54] as far away from the fixed bridge A as possible. Take a small paper rider in the form of V and place it at the middle of the portion AM of the wire.

(4) Strike one of the prongs of the tuning fork with a padded hammer. Press the stem of the vibrating fork on the sonometer board in front of the paper-rider. The vibrations of the fork will be transmitted to the wire which will be set into transverse vibrations. Now slowly move the bridge M towards the bridge A until, due to the vibrations of the wire, the paper-rider is violently thrown out. Care should be taken that at each stage of adjustment, the paper-rider is placed at the middle of the portion AM. Measure this length of the wire from A to M by a metre scale.

[There is an easy way to find the resonant length of the wire approximately.

Keep the movable bridge M at a distance from the fixed bridge A. Vibrate the tuning fork and press the lower portion (*i.e.* the portion near the handle) of any of the vibrating prongs lightly against the wire near the fixed bridge A. Now move the fork slowly towards the bridge M. At a point, loud sound will be heard which shows that the vibrating wire is in unison with the fork. Now place the movable bridge M at the point where loud sound was heard. Find the resonant length of the wire accurately with the help of the paper-rider as described in section 4.]

(5) Repeat the above operation, at least, three times and find the mean length l. From this, get the value of m_1/l^2.

(6) Change the weights on the hook twice (say, by taking 2 kg and 3 kg) and repeat the observations no 1 to no 5. Each time, find the value of m_1/l^2 and from all these observations, get the mean value of m_1/l^2.

(7) Calculate the value of n with the help of the formula given in the theory.

Measurements :

(*a*) *Measurement of mass per unit length of the wire* :

No. of Obs	Length of the sample wire (L cm)	Mean length (L cm)	Mass of the sample wire (ω gm)	$m=\omega/L$ (gm/cm)
1.	..		..mg+..mg+	
2.	..	..	..mg	..
3.	..		=..gm	

(*b*) *Measurement of resonant length of the sonometer wire* :

Weight of the hook=. . .gm (m_2)
Value of g=. . . cm/sec²

of No. Obs	Wt. placed on the hook (W kg)	Total tension $m_1=(1000\ W+m_2)$ gm	Length of the resonant wire (l cm)	Mean length (l cm)	m_1/l^2	Mean m_1/l^2
1.	1 Kg	..	.. .. ..	..	..	
2.	2 Kg	..	.. .. ..	..	..	..
3.	3 Kg	..	.. .. ..	..	..	

Frequency of the fork n=. . . Hz

Calculation : $n=\sqrt{\frac{g}{4m}\times\frac{m_1}{l^2}}$=. . .Hz.

Remarks : (1) In adjusting the resonant length AM, care should be taken so that the paper-rider is always situated at the middle of the vibrating portion of the wire. (2) Weight of the hook should be taken into account in finding the tension of the wire. (3) Resonant length of the wire may also be found out by noting the beats.

Oral questions

1. What are the laws of transverse vibration of string?

Ans. Consult any text book.

2. Why is the wire stretched on a hollow wooden box?

Ans. The box and the air within it are thrown into forced vibration by the tuning fork which intensifies the vibration of the wire.

3. Why is the paper-rider thrown out violently?

Ans. When the wire vibrates in unison with the tuning fork, the amplitude becomes very big, due to which the paper-rider is thrown out.

4. Will the resonant length of the wire increase, if the tension is increased?

Ans. Yes ; the length is directly proportional to the square root of the tension.

5. If a thick wire is taken instead of a thin one, will the frequency increase?

Ans. Since the frequency is inversely proportional to the diameter, the frequency will decrease if a thick wire is taken.

6. Should the sonometer wire be thick or thin?

Ans. It should preferably be thin.

7. A fork is in unison with a wire. Will the unison be maintained if the length of the wire is doubled or halved, other factors remaining unchanged?

Ans. Yes ; If n be the frequency of the fork, then for double length, the wire will have a frequency $n/2$ and for half length $2n$.

8. Can you measure the density of the material of a wire by the above experiment?

Ans. Yes ; (See expt. no. 6.2).

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*6.2. Determination of the density of the material of a wire by sonometer method :

Apparatus : A sonometer, a piece of wire (*i.e.* the sonometer wire), a tuning fork of known frequency, a metre scale, a padded hammer, some slotted weights, a screw-gauge, a common balance, a weight box etc.

Description of the sonometer : As in expt. no. 6.1.

Theory : If the sonometer wire be the wire under test, then the frequency n of its transverse vibration is given by $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$ or $m=\frac{T}{4l^2n^2}$, where l=length of the wire vibrating, T=tension of the string and m=mass/unit length of the wire. If m_1 be the mass used for stretching the wire, then $T=m_1g$

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$$\therefore \quad m=\frac{m_1g}{4l^2n^2} \quad \dots \text{(i)}$$

Now, if ρ and d be the density and diameter of the wire, then $m=\frac{1}{4}\pi d^2.\rho$. Substituting in eqn. (i), we get $\frac{1}{4}\pi d^2\rho=\frac{m_1g}{4l^2.n^2}$

$$\text{or } \rho=\frac{g}{\pi n^2d^2}\left(\frac{m_1}{l^2}\right) \quad \dots \text{(ii)}$$

Finding the value of m_1/l^2 from the experiment and measuring the diameter d of the wire, the density ρ may be found out.

Experimental procedure : (1) Find the diameter (d) of the *sample wire* at four different places by a screw gauge in the usual method and at each place measurements are taken at right angle directions. From these observations, get the mean value of the diameter which is then corrected for the instrumental error of the screw-gauge, if any.

(2) Following the operations no. 2, 3, 4 and 5 of the previous experiment find the value of m_1/l^2 *i.e.* the ratio of the mass used for producing tension and square of the resonant length of the wire.

(3) Repeat the observations using two different weights on the hook (say, by using 2kg and 3kg). Each time find the value of m_1/l^2 and from all these observations, get the mean value of m_1/l^2.

(4) Substituting the values of d and m_1/l^2 in the eqn. (ii) of the theory, calculate the density ρ.

Measurements : (*a*) *Measurement of the diameter of the wire by a screw-gauge.*

The value of the smallest division of the linear scale of the screw-gauge=. . . mm ; Screw-pitch=. . . mm. No of divisions on the circular scale=. . .

$\therefore$ least count=. . . mm.

[Here give the table for screw-gange measurement]

$\therefore$ Diameter (d)=. . . cm.

(*b*) *Frequency of the fork* (n)=. . . Hz (supplied) (*c*) *Measurement of resonant length* :

Wt. of the hook=. . . ($m_2$gm)

No. of Obs	Wt. placed on the hook (W Kg)	Total tension= $m_1=(1000W+m_2)$ *gm*	Length of the resonant wire (cm)	Mean length (l cm)	$\frac{m_1}{l^2}$	Mean m_1/l^2
1.	1 Kg	..	.. .. ..	..	..	
2.	2 Kg	..	.. .. ..	..	..	..
3.	3 Kg	..	.. .. ..	..	..	

$\therefore$ Density of the material of the wire $\rho=\ldots$ gm/c.c.

Calculations : $\rho=\frac{g}{\pi n^2 d^2}\left(\frac{m_1}{l^2}\right)=\ldots\ \ \ldots$ gm/c.c.

Remarks : (1) In the expression for ρ, the powers of d and l are 2. Hence they should be determined very accurately.

Oral questions

1. How does density of the material of a wire affect its frequency?

Ans. $n \propto \frac{1}{\sqrt{\rho}}$ other factors remaining constant. Hence, greater the density, less is the frequency and vice versa.

2. Why is a fork set into vibration by a padded hammer?

Ans. It helps the fork to emit its fundamental. Also the hammer being padded, there is no chance of the prong of the fork getting damaged.

[For other questions, see the experiment No. 6.1].

6.3. To draw $(n-l)$ curve with the help of a sonometer and hence to find the frequency of an unknown fork.

Apparatus : A sonometer, some slotted weights, a set of tuning forks, at least, six (having frequencies, say 256, 320, 341, 384, 426, 512), a fork of unknown frequency, a metre scale, a padded hammer etc.

Description of the sonometer : See experiment no. 6.1

Theory : In the case of transverse vibrations of a stretched string, the frequency (n) is inversely proportional to its length (l) when tension (T) and mass per unit length of the wire (m) remain unchanged. *i.e.* $n \propto \frac{1}{l}$ when T and m are constants. Hence nl=constant. If a graph is drawn between n and l, the curve will be a rectangular hyperbola or if a curve is drawn between n and $1/l$, the curve will be a straight line. It is called $(n-l)$ curve. Finding the resonant length of wire corresponding to an unknown fork, the frequency of the unknown fork may be found out from the curve.

Experimental procedure : (1) Take a few forks of known frequencies (say six) and one of unknown frequency. Shift the movable bridge M as far away from the bridge A as possible (about 100 cm). Take a piece of thin paper and fold it in the form of the inverted $\wedge$ and place it at the centre of the portion AM.

(2) Put suitable weight (say 2 kg) on the hook. First, take the fork whose frequency is the lowest among the set of forks given and vibrate it by striking against the padded hammer. The fork will continue to vibrate. Press the handle of the vibrating fork on the sonometer board in front of the paper-rider. The vibration of the fork will be transmitted to the sonometer wire which will be thrown into transverse vibrations. Now slowly move the bridge M towards the bridge A until, due to the vibrations of the wire, the paper-rider is violently thrown off. Care should be taken that at each stage of adjustment, the paper rider is placed at the middle of the portion AM. Measure this length of the wire from A to M by a metre scale.

[For an alternative easy method for finding the resonant length, see expt no. 6.1]

(3) Repeat the above operation, at least, three times and find the mean length l.

(4) *Keeping the load in the hook unchanged,* take the known

forks one by one and following the operation no 2, find the resonant length of the wire for each fork. Last of all, take the fork of unknown frequency and find its resonant length in the same way as before.

(5) Now plotting the resonant lengths (l) along Y-axis and the known frequencies (n) along X-axis, a smooth curve is to be drawn. The curve will be a rectangular hyperbola as shown in fig. 55. From the graph, the frequency of the unknown fork corresponding to its resonant length is found out.

Measurements : (*a*) *Measurement of resonant lengths.*

Constant load put on the hook of the sonometer = .. kg

Serial No. of forks	Known frequencies of the forks (n)	Resonant length (cm)	Mean length (l cm)
1.	256	.. .. } ..	..
2.	320	.. .. } ..	..
3.	341	.. .. } ..	..
4.	384	.. .. } ..	..
5.	426	.. .. } ..	..
6.	512	.. .. } ..	..
7.	Unknown	.. .. } ..	..

(*b*) **Table for drawing graph :** (Data for illustration)

Frequency (n) →	256	320	341	384	426	512	Unknown
Resonant length (l cm) →	34·4	25·1	24·1	20·4	18·8	15·7	23·5

Notes on drawing graph : The maximum difference of frequencies is 512 − 256 = 256. Taking 250 as the origin, if we take 1 small division along the *X*-axis equal to 5Hz, then 60 divisions will cover up all the frequencies. Hence, along the *X*-axis, the scale may be chosen as 5Hz = 1 small division [Fig. 55].

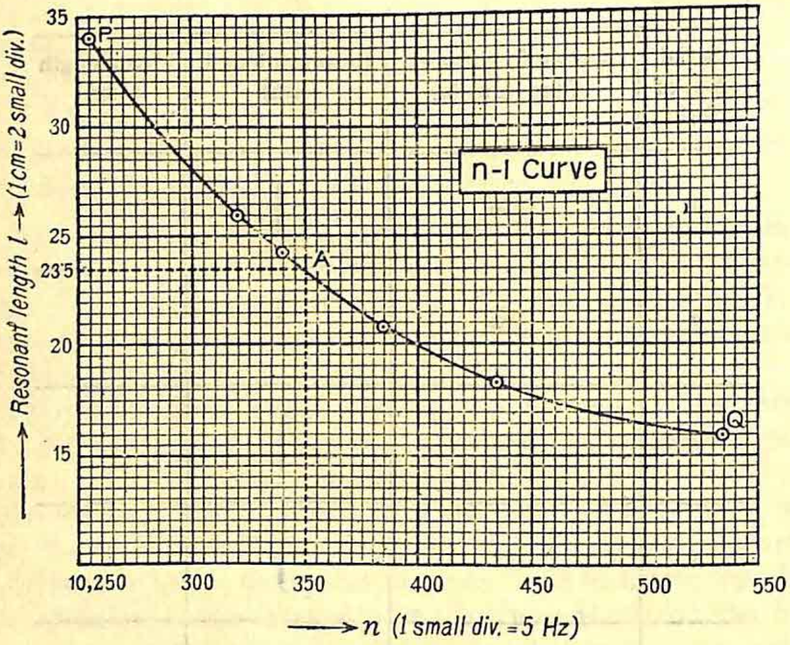

Fig. 55

The maximum difference of resonant lengths is 34·4 − 15·7 = 18·7 cm. Taking 10 cm. as the origin, if we take 1 cm equal to 2 small divisions, then 50 divisions will cover up all the resonant lengths. Hence, along the *Y*-axis, the scale may be chosen as 2 small divisions = 1 cm.

Now plotting the points we get the rectangular hyperbola PQ. Now, for the unknown frequency, the resonant length obtained is 23·5 cm. If a line parallel to X-axis be drawn through the point representing 23·5 cm on the Y-axis it will cut the curve at some point say A. A perpendicular drawn on the X-axis through the point A gives the value of the unknown frequency. From the fig. 55, the unknown frequency is 350.

Alternative method of drawing graph :

We have seen that $nl=K$ where K is constant or $n=K.\ \frac{1}{l}$.

So if n and $\frac{1}{l}$ are plotted, the curve will be a straight line. The values of $\frac{1}{l}$ are calculated from the table (b) and are tabulated as follows :

Frequency (n) →	256	320	341	384	426	512	Unknown
$1/l$ cm^{-1} →	·029	·0375	·0395	·046	·052	·064	·0415

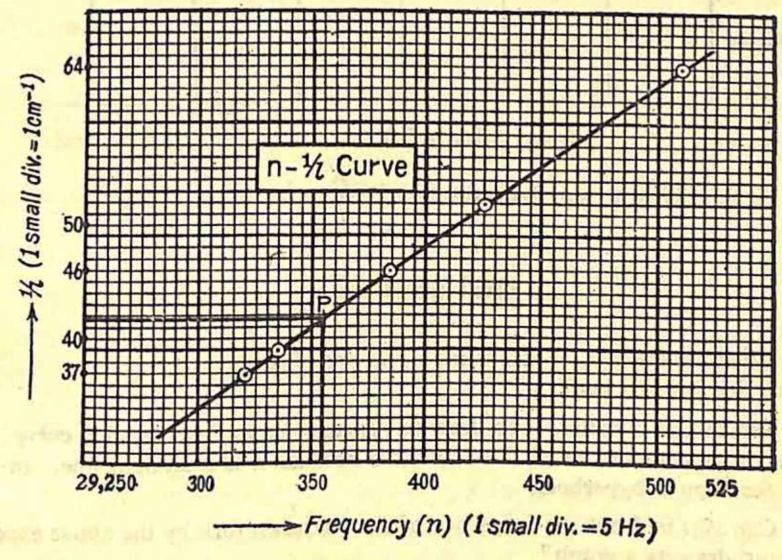

Fig. 55(a)

The maximum difference of frequencies being 256, the scale along X-axis is chosen as before, viz 5Hz=1 small division [Fig. 55(a)].

The values of $1/l$ are 29×10^{-3} to 64×10^{-3}. If we take 40 divisions along Y-axis, then 1 division may be taken to be equal to 1 cm^{-1} Hence, convenient scale along Y axis is 1 small division$=1\ \text{cm}^{-1}$. Plotting in this way, we get the straight line as shown in Fig. 55(a). The value of $1/l$ for unknown frequency being $0{\cdot}0415=41{\cdot}5\times10^{-3}$. a line is drawn through the point 41·5 on the Y-axis parallel to X-axis, cutting the straight line at P. A perpendicular drawn from P on the axis shows that the unknown frequency is 355.

Remarks : (1) Instead of $n-l$ curve, it is better to draw $n-1/l$ curve because it gives a straight line. (2) The unknown frequency can also be found out without drawing a graph. For this, the product of $n\times l$ is to be found out for each fork. The product will be found to be constant. Say, the constant is K. Then n', the unknown frequency is given by $n'=\frac{K}{l'}$ where l' is the resonant length for the unknown fork. The table may be drawn in the following way :

Frequency (n)⟶	256	320	341	384	426	512	Unknown
Resonant length (l)⟶	..	..	..	..	..	..	..(l')
Product $k=n\times l$⟶	..	..	..	..	..	..	

Mean valve of $K=..$

$\therefore$ Unknown frequency $n'=\frac{K}{l'}=..$

Oral questions

1. What are the laws of transverse vibrations of string?

Ans. See any text book.

2. Which one is better – to draw $(n-l)$ curve or to draw $(n-1/l)$ curve.

Ans. It is better to draw $(n-1/l)$ curve because it is a straight line. $(n-l)$ curve is a rectangular hyperbola.

3. Can you find out the frequency of the unknown fork by the above experiment without drawing a graph?

Ans. Yes ; See remark no. 2.

[For other questions, see experiment no. 6.1].

6.4. Determination of the frequency of a tuning fork by Melde's experiment :

Apparatus : Melde's apparatus, a common balance with weight box, a metre scale, a padded hammer, two pins whose heights can be adjusted, etc.

Description of Melde's apparatus : Fig. 56 is a form of Melde's apparatus. It consists of a tuning fork *A*, rigidly fixed to a wooden board *C* by screws. One end of a light silk thread *AB* is connected to one of the prongs of the fork. The other end of the thread passes over a smooth pulley *B* and keeps a scale-pan *S* hanging. Standard weights, placed on the scale-pan, keep the thread under tension. The height of the pulley may be so adjusted along the stand that the thread *AB* is just horizontal. In the *transverse mode*, (Fig. 56), the direction of vibration of the fork is perpendicular to the length of the thread. When the fork vibrates, transverse waves are produced in the thread.

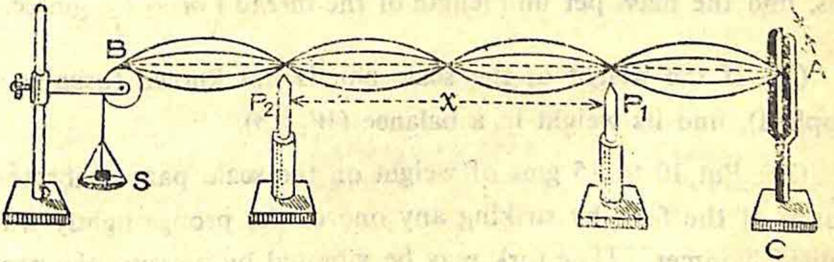

Fig. 56

Theory : When the tuning fork vibrates, a transverse wave passes along the thread, with a velocity $V=\sqrt{\frac{T}{m}}$ where T=tension of the thread and m=mass per unit length of the thread.

If L_1 be the total length of the thread (*i.e.* from the point where it is connected to the fork to the centre of the pulley) and p be the number of segments in which the thread vibrates under the given tension, then $\lambda/2=\frac{L_1}{p}=l$ (say). Then $\lambda=2l$.

In *transverse mode*, *i.e.* when the fork vibrates perpendicular to the length of the thread, the frequency of the fork becomes equal to the frequency of vibration of the thread.

$$\therefore \quad N.\lambda=\sqrt{\frac{T}{m}} \text{ where } N=\text{frequency of the fork.}$$

or, $N=\frac{1}{\lambda}\sqrt{\frac{T}{m}}=\frac{1}{2l}\sqrt{\frac{T}{m}}$

If M be the total mass of the scale pan and the standard weights placed on the scale pan, then $T=Mg$. Hence $N=\frac{1}{2l}\sqrt{\frac{Mg}{m}}$.

Experimental procedure : (1) Take a piece of light silk thread of length about 1 metre and find its weight (Wgm) accurately in a balance with the help of a rider. Measure its length L by a metre scale. From this, find the mass per unit length of the thread $\left(m=\frac{W}{L}\text{ gm/cc.}\right)$

(2) If the weight of the scale pan is not known (usually it is supplied), find its weight in a balance (W_1 gm).

(3) Put 10 to 15 gms of weight on the scale pan. Vibrate the prongs of the fork by striking any one of the prongs lightly with a padded hammer. [The fork may be vibrated by pressing the prongs with two fingers and then suddenly releasing them]. Indistinct nodes and antinodes will be found to have been produced in the thread. Slightly altering the weights on the scale-pan, make the nodes and antinodes distinct and clear. Under this circumstances, the fork will vibrate in unison with the thread. The vibrations of the thread will divide it into several sharp segments.

(4) Place two pointed pins P_1 and P_2 just below the two extreme nodes as shown in fig. 56. [Two nodes will be formed at the two ends of the thread. But do not place the pins there]. Count the number (n) of loops between the pins and measure the distance (x) between them. Then $\lambda=\frac{2x}{n}$.

(5) Repeat the operation thrice and find the mean value of λ. Hence $l=\frac{\lambda}{2}=\frac{x}{n}$.

(6) Changing the weights in the scale pan, repeat the operations (3), (4) and (5) twice. Find the value of N for each observation using the formula mentioned in the theory and get the mean value of N.

Measurements : (*a*) *Measurement of mass per unit length* (m) *of the thread.*

Length of the thread (L cm)	Mean length (L cm)	Mass of the thread (W gm)	$m=\frac{W}{L}$ gm/cm
..		..gm+..gm	
..	..	+..mg+..	..
..		=..gm	

(*b*) *Measurement of* l :

Mass of the scale pan=.. gm (W_1)

No. of Obs	Weights placed on the scale pan (W_2)	Distance between P_1 and P_2	Mean distance (x)	No. of loops between the pins (n)	$l=\frac{x}{n}$
1.	..gm	.. .. ..	..cm		..
2.	..gm	.. .. ..	..cm	..	..
3.	..gm	.. .. ..	..cm	..	..

(*c*) *Measurement of the frequency* (N) *of the fork* :

No. of Obs	Total weight ($M=W_1+W_2$)	Mass/unit length of the thread (m) [From table (*a*)]	l [From table (*b*)]	$N=\frac{1}{2l}\sqrt{\frac{Mg}{m}}$
1.	..gm	..gm/cm	..	..
2.	..gm	..gm/cm	..	..
3.	..gm	..gm/cm	..	..

Mean value of $N=..$ *Hz*

Calculations : 1. $N=\frac{1}{2l}\sqrt{\frac{M.g}{m}}=\ldots Hz$

2. $N=\ldots$

3. $N=\ldots$

Remarks : (1) The portion of the thread which hangs from the pulley should be small in length. If it is long, its weight should be added to the total weight *M*. (2) The frequency of the fork can also be found out by vibrating the thread longitudinally. In that case, the frequency of vibration of the thread will be half the frequency of the fork. (3) At the beginning of the experiment, *M* should preferably be low, because lowei value of *M* gives greater number of loops. As the value of *M* increases, the number of loops decreases. (4) The thread should have uniform mass per unit length. (5) The nodes should be made as distinct as possible.

Oral questions

1. Why are loops formed in Melde's apparatus?

Ans. When waves proceed along the thread towards any fixed end, they are reflected at the fixed end. In the mean time, another wave proceeds towards that end. These two waves get superposed and form stationery waves. For this reason, the thread vibrates in several loops.

2. Can the frequency of the fork be found out by producing longitudinal waves in Melde's experiment?

Ans. See remark no. 2.

3. Will the number of loops increase or decrease if the tension of the thread is increased?

Ans. On increasing the tension, the number of loops decreases.

4. In counting the number of loops, why do you not take measurement between the nodes at the two extreme ends of the thread?

Ans. The nodes at the extreme ends of the thread are not very distinct. If measurement is taken between them, error might come in.

5. Should the portion of the thread hanging from the hook be sufficiently long?

Ans. See remark no. 1.

6. What is the unit now attributed to frequency?

Ans. Present unit is *Hz* (Hertz) ; previously it was c.p.s. (cycles per sec).

7. ELECTRONICS

7.1. Diode valve :

It consists of a filament F and a plate P enclosed in a glass bulb which is highly evacuated [Fig 57]. When the filament is heated to incandescence by means of a battery B_1, it emits electrons by thermionic process. Now, if the plate P is given a *positive potential* with respect to the filament F by another battery B_2, the electrons are attracted towards the plate and a current passes from the filament to the plate as indicated by a milliammeter A included in the plate circuit. But if the terminals of the battery B_2 be reversed, giving the plate P, a *negative potential* with respect to the filament, no current flows. So, the current flowing between the filament and the plate is always unidirectional. For this reason, if is called a *valve*.

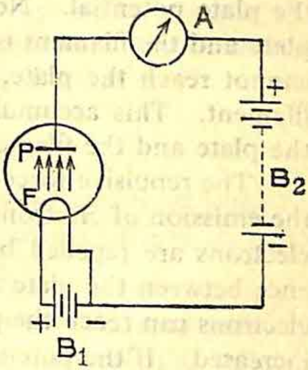

Fig. 57

Ordinarily, the voltage of the battery B_1 is not very high. On the other hand, the voltage of the battery B_2 is high. For this reason, the former is called a *low tension* (L.T.) battery and the latter a *high tension* (H.T.) battery.

Fig. 58 shows the appearance of a diode valve. Generally, the plate P of the diode is called *anode* or simply *plate* and the electron-emitting filament is called *cathode*. The cathode of a diode valve may be directly heated or indirectly heated. In a directly heated type of diode, the filament is directly connected to the battery. It is made of a ribbon of very fine wire and has a resistance such that when proper voltage is applied to it, the filament soon acquires the requisite temperature for emitting copious electrons. An indirectly heated type of diode has the advantage that it can be used with a d.c. or a.c. supply.

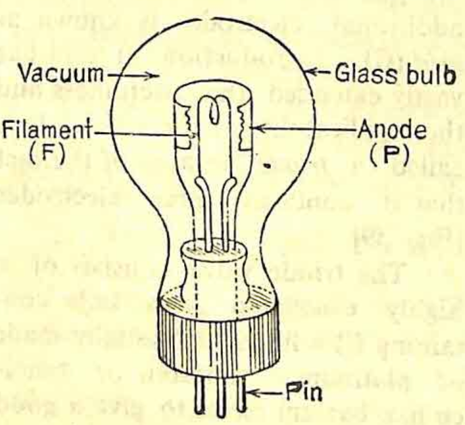

Fig. 58

7.2. Space charge :

The number of electrons emitted by a heated filament depends on the effective temperature of the filament. The positive potential of the plate has no effect on the number of electrons emitted ; it simply exerts an attractive force on the emitted electrons. So, the number of electrons reaching the plate and hence the plate current depends on the plate potential. Now, when the potential difference between the plate and the filament is low, all the electrons emitted by the filament cannot reach the plate. Some of the electrons accumulate near the filament. This accumulation of free electrons in the space between the plate and the filament is called the *space charge*.

The repulsive force that the accumulated electrons exert hampers the emission of electrons from the cathode ; even some of the emitted electrons are repelled back to the filament. As the potential difference between the plate and the filament is increased, more and more electrons can reach the plate and hence plate current is also gradually increased. If the potential difference be increased step by step in this way, a value will be soon reached when all the electrons emitted by the filament, subjected to a great attractive force, reach the plate. Then the plate current becomes the highest and it is called the saturation current.

7.3. Triode valve :

In 1907, the American experimenter Lee De Forest made an important addition to the diode by inserting a third electrode between the plate and the filament. This additional electrode is known as *grid* (*G*). Introduction of grid has vastly extended the usefulness and the applicability of the valve. It is called a *triode* because of the fact that it contains three electrodes [Fig. 59].

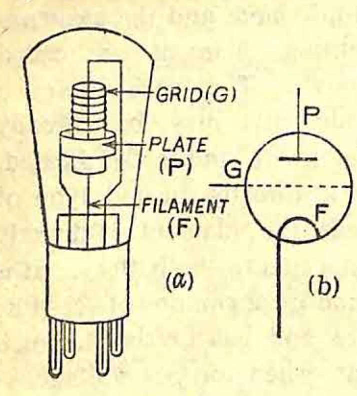

Fig. 59

The triode valve consists of a highly evacuated glass bulb containing (i) a filament *F* usually made of platinum, tungsten or tantalum and is coated with a substance like barium oxide to give a good emission of electrons at low temperature (ii) a grid *G* which may be a flat wire gauge placed above the filament or a spiral of wire mounted with the filament as axis. The grid is generally placed closer to the filament than to the plate and (iii) a plate *P* which is in the form of a flat plate if the grid is flat or in the form of a cylinder surrounding the grid if the latter is spiral. A triode is diagrammatically represented as shown in fig. 59(*b*).

The filament is provided with two external leads for connecting it to a low tension battery which supplies the heating current. The grid and the plate are each provided with a single external lead so that they may be given any desired potential—positive or negative—with respect to the filament by the use of batteries. The three electrodes of a triode are well insulted from one another.

7.4. Function of a grid :

Question may be raised as to what is the advantage of using the additional electrode grid ? The greatest advantage of grid is that it is a very efficient controller of space charge.

Ordinarily, when the plate is given a positive potential with respect to the filament, a plate current flows but it is feeble due to space charge effect. As the positive potential of the plate is slowly increased, the plate current also increases slowly and finally the current becomes saturated. If the grid which is nearer to the filament, be now given a negative potential with respect to the filament, it will repel the electrons emitted by the filament. The plate current consequently diminishes. If the grid, on the other hand, be given a positive potential, then it will help the electrons to reach the plate by exerting an additional attraction. Consequently space charge effect will decrease and the plate current will increase. In this way, we can conveniently increase or decrease the plate current by changing the grid potential. Further, it has been seen that large plate current is obtained by slightly altering the grid potential about a given value. To bring about such a change of plate current by simply altering the plate potential, the plate needs about 10 times potential than before.

7.5. To draw the static characteristic ($I_P - V_P$) of a diode

Apparatus : A diode valve (EZ 80 of B.E.L., for example) with socket and binding screw; a milliammeter (0-30 m.a. range) ; an ammeter (0-1 amp range with 0·1 amp divisions), two plug keys ; battery for sending current to the filament (as per instruction of the manufacturer ; 6·3 volt for EZ 80 for example), a high tension battery (0-100 volt), a voltmeter (0-100 volt), a wandering plug, two rheostats of suitable specification, connecting wires etc.

Circuit connections : The whole circuit arrangement can be divided into two parts : (*i*) Filament circuit and (*ii*) Plate circuit [Fig. 60].

(*i*) *Filament circuit* : Having fixed the valve socket on a wooden board, the valve is put in the socket and the terminals leading to the filament *F* are then joined to the binding screws *M* and *N*. To set up the filament circuit, an ammeter *Am* (of 0-1 amp range), a low tension battery (B_2), a suitable rheostat (R_2) and a plug key are all connected in series between the screws *M* and *N*. Closing the plug key and controlling the rheostat, if specified current [upto 0·6 amp in the case of EZ80] be sent through the filament, profuse electrons will emitted by the filament.

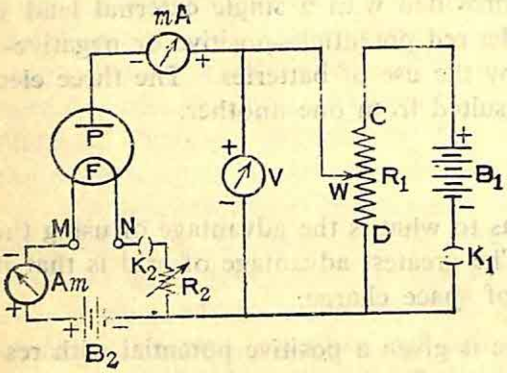

Fig. 60

(*ii*) *Plate circuit* : There is a binding screw on the wooden board, which is connected by a wire to the plate terminal of the valve socket. A wandering plug *W* is connected to this binding screw through a milliammeter *mA* (0-30 m.a. range). A potentiometer circuit is formed by joining a high tension battery B_1 (0-100V range), a suitable resistance R_1 and plug key K_1 in series. The negative end of the above potentiometric arrangement is joined to the negative end of the filament while the wandering plug *W* can move along the various points of the resistance R_1 between *C* and *D*. As a result, the plate is given a positive potential with respect to the filament which can conveniently be changed by changing the position of the wandering plug *W*. When the wandering plug *W* is in contact with the point *D*, the potential of the plate is zero. As the plug *W* moves towards *C*, the potential of the plate increases and finally becomes maximum when the plug *W* arrives at *C*. The voltmeter *V* (0-100V range) connected between the plate and the filament measures this potential.

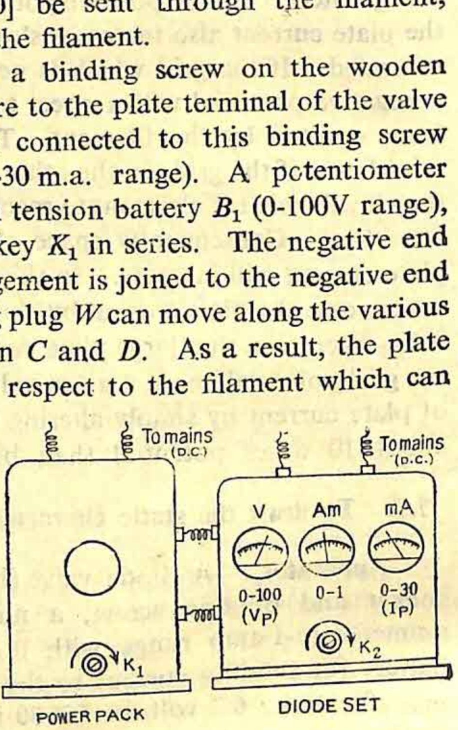

Fig. 61

[Now-a-days, a built-in circuit is generally supplied to the students. Here, all internal connections are previously made and the meters are put up in a board as shown in fig. 61. The filament heating current is derived from the mains. If the plug of the set is inserted into the mains' socket (D.C.), the valve will glow. The current is indicated the ammeter *Am*. The knob K_2 controls the heating current. The power-pack which is also, a built-in unit, supplies the plate potential. When its plug is inserted into the D.C. mains socket and the knob K_1 turned, the plate potential changes. The plate potential is recorded by the voltmeter *V* and the plate current by the milli-ammeter *mA*. The experimental procedure has, however, been described according to the circuit connections shown in Fig. 60.]

Theory : Keeping the effective temperature of the filament constant, if the plate-voltage (V_P) be changed then a graph showing the change of plate current with the change of plate-voltage is called the *characteristic curve* of a diode. A set of such curves may be drawn by keeping the filament at different effective temperatures. (Care should be taken so that the temperature does not exceed the maximum current specified by the manufacturer). From these curves, we get valuable information regarding the behaviour of the valve.

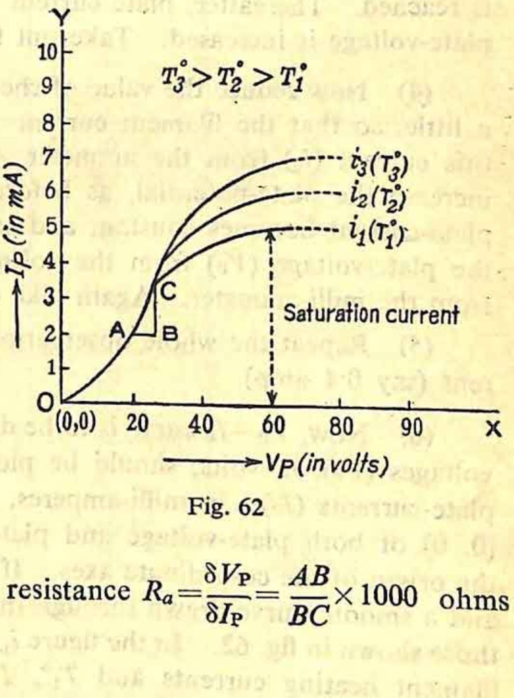

Fig. 62

Generally, the characteristics of a diode are not straight lines. So, the ratio of plate-voltage to plate-current is not constant all along the curve. For this reason, if the ratio of plate-voltage to plate-current be determined over the straight portion of the curve, the ratio is called the *plate resistance* or the *internal resistance* of the diode. From fig. 62, it is seen that the plate resistance $R_a = \frac{\delta V_P}{\delta I_P} = \frac{AB}{BC} \times 1000$ ohms

[BC milli-amp $= \frac{BC}{1000}$ amp].

Experimental procedure : (1) Make the circuit connections as shown in fig. 60. (In case a built-in circuit is used, put the plugs of the diode set and the power pack in the mains' (d.c.) socket and connect the power-pack with the valve set). Carefully connect the positive and negative terminals of the meters and other accessories as shown in the figure. Closing the plug key K_2, adjust the rheostat R_2 so that a small current (say, 0.2 amp *i.e.* much less than the specified maximum current) passes through the filament. The ammeter *Am* included in the filament circuit records this current. Note the current (i_1).

(2) Now, keeping the wandering plug *W* very near the point *D*, close the plug key K_1. This puts a small voltage (say, 10 volt) to the plate. Note the voltage (V_P) as recorded by the voltmeter *V*. The milliameter *mA* in the plate circuit records the plate current in milli-amperes. Note the current (I_P).

(3) Next, shift the wandering plug *W* slowly towards the point *C* so that the p.d. between the plate and the filament may increase by steps of 10 volts. At each step, read the plate-voltage from the voltmeter and plate-current from the milli-ammeter and tabulate them in your table. The process is to be continued till saturation current is reached. Thereafter, plate current will not increase even if the plate-voltage is increased. Take out the plug from the key K_1.

(4) Now reduce the value of rheostat R_2 in the filament circuit a little, so that the filament current increases (say 0·3 amp). Note this current (i_2) from the ammeter *Am*. Closing the plug key K_1, increase the plate-potential, as before, by steps of 10 volts till the plate-current becomes constant and saturated. At each step, record the plate-voltage (V_P) from the voltmeter *V* and plate-current (I_P) from the milli-ammeter. Again take out the plug from the key K_2.

(5) Repeat the whole observation with a different filament current (say 0·4 amp).

(6) Now, $V_P - I_P$ curve is to be drawn. For this purpose, plate-voltages (V_P), in volts, should be plotted along the *X*-axis and the plate-currents (I_P), in milli-amperes, along the *Y*-axis. Zero-values (0, 0) of both plate-voltage and plate-current should coincide with the origin of the co-ordinate axes. If the points are properly plotted and a smooth curve drawn through the points, the curves will be like those shown in fig. 62. In the figure i_1, i_2, and i_3 denote three different filament heating currents and $T_1°$, $T_2°$ and $T_3°$ the corresponding filament temperatures. From the curves it is clear that (*i*) the saturation current increases with the increase of filament temperature and (*ii*) the portion *AC* of the curve is fairly straight and linear.

(7) Taking two points A and C on the linear portion of the characteristic, draw lines AB and CB parallel to X-axis and Y-axis respectively. Find from the graph the change of plate-voltage (δV_P) denoted by AB and the change of plate-current (δI_P) denoted by CB. Then,

$$R_a = \frac{\delta V_P}{\delta I_P} = \frac{AB}{CB} \times 1000 \text{ ohms.}$$

Measurements : Valve no. = EZ 80 or UY 85 etc.

[Data given are for illustration]

No. of Obs	Filament current (amp)	Plate-voltage (Vp) in volts	Plate current (Ip) in milli-amp
1.	i_1 = 0·2 amp	10	..
2.		20	..
3.		30	..
4.		40	..
5.		..	..
etc		etc	..
			(saturated)
1.	i_2 = 0·3 amp	10	..
2.		20	..
3.		30	..
4.		40	..
5.		..	..
etc		etc	..
			(saturated)
1.	i_3 = 0·4 amp	10	..
2.		20	..
3.		30	..
4.		40	..
5.		..	..
etc		etc	..
			(saturated)

[**Note** : In some instruments, there may not be any arrangement for changing the filament heating current and hence no ammeter It has a fixed filament current. In such cases, only one set of readings will be obtained and one curve may be drawn. In drawing table in such cases, 'Filament current' column is not necessary.]

Calculations : From the graph $AB = \ldots$volt

$$BC = \ldots\text{m.a.} = \ldots\text{amp}$$

$$\therefore\ R_a = \frac{\ldots(\text{volt})}{\ldots(\text{amp})} = \ldots\text{ohms.}$$

Remarks : (1) Strict attention should be paid regarding proper connection of positive and negative ends of different accessories. (2) Highest current passed through the filament should always be less than the maximum permissible current ; otherwise the filament may burn out. (3) As the curve is not linear everywhere, in determining the plate resistance, only the linear part should be used. (4) In the case of indirectly heated diode, do not close the keys K_1 and K_2 simultaneously. First close the key K_2 and wait for some time. The emission of electrons from indirectly heated filament requires a few minutes time. After that, close the key K_1. Similarly, when the observation is over, do not take out the plugs from the keys K_1 and K_2 simultaneously. First cut off the H.T. by taking out the plug from the key K_1 and then cut off the L.T.

Oral questions

1. What is a diode? What is its utility?

Ans. A thermionic valve which contains a plate and a filament is called a diode. It is usually used to rectify an alternating potential.

2. What is plate resistance in a diode? Is it same everywhere along the characteristic curve?

Ans. The ratio of the plate-potential to the plate current is called the plate resistance in a diode. It is not same everywhere along the characteristic because the characteristic is not linear everywhere.

3. Will the plate-current be zero if the plate-potential be made zero?

Ans. No ; some plate current is obtained even if the plate-potential be zero. The reason is that when electrons are emitted from the filament, they are emitted with different velocities. Some of the electrons, due to their initial velocity, may reach the plate in absence of any attraction by the plate. As a result, some plate-current is produced.

4. What is the direction of current in the valve? In which direction flow of electrons takes place?

Ans. Electrons flow from filament to anode. Conventional direction of current is taken from plate (anode) to filament.

5. Does the plate-current obey Ohm's law?

Ans. No ; it does not obey Ohm's law. In the space-charge limited region, current is proportional to $V^{\frac{3}{2}}$ where V is the p.d. between the plate and the filament.

6. Which part of the characteristic curve denotes space-charge limited current?

Ans. The portion OAC denotes space-charge limited current.

7. What is the air-pressure inside the valve? Why is it so?

Ans. The pressure is very low—to the order of 10^{-6} mm. of mercury. If the pressure is high, the air-molecules will be ionised and positive ions will impinge upon the filament with great speed. The filament, in that case, may be damaged.

8. What are the factors on which plate current depends?

Ans. Plate current depends on (i) the temperature of the filament and (ii) the p.d. between the plate and the filament.

7.6. To draw the static characteristics of a triode and hence to determine the valve constants :

Apparatus : A triode valve (6J5 or *ECC* 82); a milli-ammeter (0-30 m.a. range), a low resistance (about 10 ohm) rheostat, an ammeter (0-5 amp. range), two plug keys, suitable battery to send current to the filament (6 volt or 4 volt according to the valve manual) a high tension battery (0 – 200 volt range), a grid-bias battery (0 – 10 volt. range ; 6 torch cells, each of 1·5 volts put in series will serve the purpose), voltmeters, wandering plug, connecting wires etc.

Circuit connections : The whole circuit arrangement can be divided into three parts : (i) Filament circuit; (ii) Grid circuit and (iii) Plate circuit. [Fig 63].

(i) *Filament circuit* : A rheostat R_h, an ammeter (0 – 5 amp range) A_m, a plug key K_1 and a low voltage (6 or 4 volts) battery (*L.T.*) are connected in series with the filament *F*. Current according to the specification is sent through the filament by suitable adjustment of the rheostat. The ammeter *Am* records this current. When specified

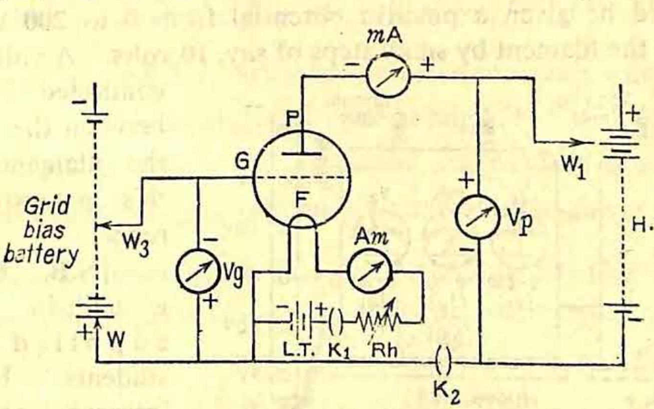

Fig. 63

current flows through the filament, profuse supply of electrons are available from the filament.

(ii) *Grid circuit* : A battery of intermediate range (0 – 10 volts) is inserted between the grid G and the filament. It is called *grid-bias* battery. With the help of grid-bias battery, the p.d. between the grid and the filament may be changed within a range of $+10$ to -10 volts by steps of 1·5 volts. To give grid a negative potential, the wandering plug W connected to the negative of the filament is joined to the positive (+) terminal of the grid-bias battery and the wandering plug W_3 connected to the grid is joined to the negative terminal of any of the cells of the grid-bias battery. If the positions of the wandering plugs W and W_3 are reversed, the grid will get a positive potential with respect to the filament. Whatever may be the grid potential—positive or negative—it is measured by a voltmeter (0 – 10 volts) V_g connected between the filament and the grid. If the grid has negative potential, the voltmeter V_g is to be connected as shown in the fig 63. If, however, the grid has positive potential, the voltmeter connections are to be reversed. (No such alteration is necessary if the voltmeter is a zero-centered instrument).

(iii) *Plate circuit* : A high voltage battery (0 – 200 V range) (H.T.), a milli-ammeter mA, a plug key K_2 are connected in series between the plate P and the filament. The wire connected to the plate P through the milliammeter ends in a wandering plug W_1 which can be brought in contact with the positive terminal of any of the cells of H.T. battery. The wire connected to the filament through the plug key K_2 is joined to the negative of H.T. battery. By this arrangement, plate could be given a positive potential from 0 to 200 volts with respect to the filament by small steps of say, 10 volts. A voltmeter V_p connected in parallel between the plate and the filament records this potential difference.

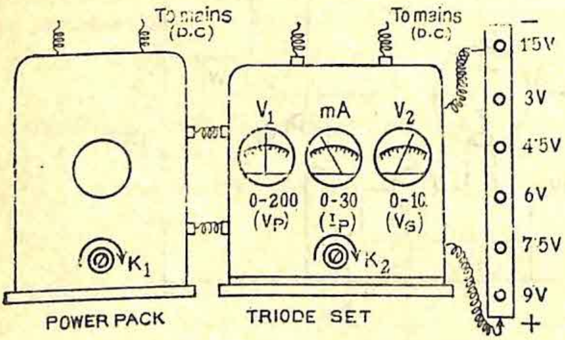

Fig. 64

[N.B. Ordinarily a built-in circuit is supplied to the students. Here all internal connections are previously made and the meters are put up in a board as shown in Fig 64. The filament heating current is derived from the mains. If the plug of the set is inserted into the mains socket (D.C.), the valve will glow. The grid-bias battery (G.B.) is usually prepared by joining six torch cells, each of e.m.f. 1·5 volts. The grid voltage (V_p) is recorded by the voltmeter V_2. With the knob K_2, a slow change of grid voltage can be achieved. The power pack is also a built-in unit. When the plug of the power pack is pushed into D.C. mains socket, the plate gets the positive potential which is recorded by the voltmeter V_1. The knob K_1 controls the plate-potential. The plate-current (I_p) is denoted by the milli-ammeter *mA*. The experimental procedure has, however been described according to the circuit connections as shown in fig 63.]

Theory : (A) Giving the plate a fixed positive potential with respect to the filament, if grid potential be changed (both positive and negative) by suitable steps, the plate-current also changes. If the grid potentials (V_g) and the corresponding plate-currents (I_p) be plotted in a graph paper, the curve obtained is called the *mutual characteristic* curve of the triode. A family of such characteristic curves may be drawn ($V_g - I_p$ curves) by keeping the plate at different fixed positive potentials. Such curves are, however, known as *static* characteristic curves when there is no resistance (*i.e.* load) in the plate circuit. Valuable information regarding the usefulness and applicability of the valve are obtained from the characteristic curves (Fig 65).

Constants of a valve : A triode has three constants which express the effect of grid-voltage (V_g) and plate-voltage (V_P) on plate-current (I_P). The three constants are (i) *mutual conductance* (g_m) (ii) *A.C. resistance* or *plate resistance* (R_p) and (iii) *amplification factor* (μ).

(i) *Mutual conductance* : Keeping plate-potential (V_p) at a fixed value, if grid-potential (V_g) be changed, plate-current (I_p) also changes. The ratio of the change of plate-current to the corresponding change of grid-potential is called the mutual conductance.

So, $g_m = \left(\frac{\delta I_p}{\delta V_g}\right) V_p =$ const.

From fig 65, it is seen that if plate-potential be kept constant at

V_3, a change of CB in the grid-potential causes a change of AB in the plate-current. Hence, $g_m = \frac{AB}{BC}$

If the plate-current is expressed in milli-ampere and grid-potential in volts, the unit of g_m will be ma/volt. If the plate-current is expressed in ampere, the unit of g_m is 'mho'.

(ii) *A.C. resistance or plate resistance* : Keeping grid-potential (V_g) at a fixed value, if plate-potential (V_p) be changed, then plate-current (I_p) also changes. The ratio of the change of plate-potential to the corresponding change of plate-current is called the a.c. resistance or plate resistance. So, $R_p = \left(\frac{\delta V_p}{\delta I_p}\right) V_g = \text{const.}$

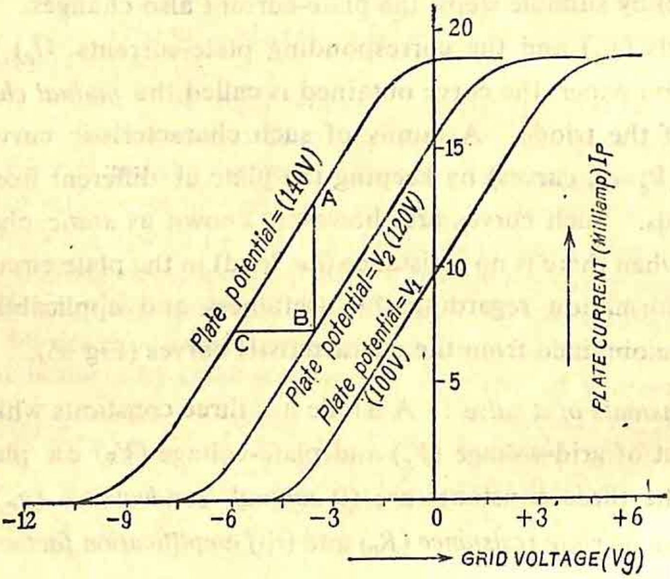

Fig. 65

From fig 65, it is clear that $R_p = \frac{V_3 - V_2}{AB}$. If the plate-current is expressed in ampere, the unit of R_p is ohm.

(iii) *Amplification factor* : The ratio of the change in the plate-potential (δV_p) required to produce a small change in plate-current to the change in grid-potential (δV_g) required to produce the same charge in plate-current is called the amplification factor.

$$\text{So} \quad \mu=\left(\frac{\delta V_p}{\delta V_g}\right) I_p=\text{const.}$$

From fig 65, it is seen that for a plate-current increase from the point A to the point B, the plate-potential needs an increment from V_2 to V_3 when grid-potential is kept constant and the grid-potential needs an increment CB when plate-potential is kept constant.

$$\text{So,} \quad \mu=\frac{V_3-V_2}{BC}$$

[**Note** : The values of g_m, Rp and μ are constants only in the linear portion of the curve. Hence, the points A, B and C should be taken on the linear portion].

From the above relations, it follows that $g_m=\frac{\mu}{R_p}$ or $\mu=g_m\times R_p$

(B) *Anode characteristics* (V_p-I_p curve): Keeping the grid-voltage (Vg) fixed, if the plate-potential be changed, the plate-current also changes. If different plate-potentials and the corresponding plate currents be plotted in a graph paper, the curve obtained is called the anode or plate characteristic curve of the triode. Keeping the grid-voltage at different fixed values, a family of (V_p-I_p) curves may be drawn. [Fig 66]. The valve constants may also be found out from anode characteristics. Further, when there is no load in the plate-circuit, the characteristic curves are known as static characteristic curves.

Experimental procedure : (*a*) *Drawing of* (V_g-I_p) *curves* :

(1) Make circuit connections as shown in fig 63. Closing the plug key K_1, adjust the rheostat R_h so that the specified current (as given by the valve manufacturer) may pass through the filament. Note this current from the ammeter (*Am*) provided in the filament circuit. [In the built-in circuit, this operation is not necessary.]

(2) Place the wandering plug W_1 at such a place that the plate gets a potential of +100 volts. Now keep the wandering plug W_3 floating

and the other wandering plug W to the extreme positive end of the grid-bias battery. Note that the voltage (negative) put to the grid is now zero.

(3) Now put the plug in the key K_2. Note the plate-voltage ($+ve$) from the voltmeter V_p and plate-current (I_p) from the milli-ammeter (mA). Take out the plug from the key K_2.

(4) Put the wandering plug W_3 now in the first hole nearest to the positive terminal of the grid-bias battery [fig. 64]. This gives grid $-1{\cdot}5$ volt potential difference. Note the voltage as recorded by the voltmeter V_g. [If the torch cell is old, its voltage may fall. For this reason, the grid-voltage as recorded by the voltmeter V_g is to be noted]. Close the key K_2 and note the plate-voltage (which remains constant) from the voltmeter V_p and plate-current (I_p) from the milli-ammeter (mA). Plate-current will slightly diminish.

(5) Gradually increase the value of the negative grid-potential by moving the wandering plug W_3 to the extreme negative end of the grid-bias till the plate-current becomes zero. At each step, note the grid-voltage, plate-voltage and the plate-current from the respective metres. Care should be taken so that at each step, the plug from the key K_2 is taken out before the grid-potential is altered. Plate-voltage should remain constant at 100 volt during the operation.

(6) Now positive potential is to be applied to the grid. For this purpose, put the wandering plug W in the extreme negative point of the grid-bias battery and the wandering plug W_3 in the first hole nearest to negative terminal. This gives grid $+1.5$ volt potential difference. Before this is done, reverse the connections of the voltmeter V_g. (If the instrument is a zero-centred one, no such reversal is necessary). Put the plug in the key K_2 and note the grid-voltage, plate-voltage and plate-current from the respective meters.

(7) Gradually increase the value of the positive grid-potential by steps of 1.5 volt by shifting the wandering plug W_3 to the extreme positive end of the grid-bias battery till the plate-current is saturated.

At each step, note the grid-voltage (V_g), the plate-voltage (V_p) and the plate-current (I_p).

(8) Repeat the whole process twice by keeping the plate-potential (V_p) fixed at two other values (say 120 volt and 140 volt).

(9) A graph is to be drawn between the different grid-voltages and the consequent plate-currents for each fixed value of the plate-potential. For this purpose, grid-voltages are plotted along X-axis and plate-currents (in milli-amperes) along Y-axis. Three curves will be obtained for three fixed plate-potentials. Write the value of the fixed plate-potential by the side of the curve to which it belongs [Fig 65]. These are the mutual characteristic curves or ($V_g - I_p$) curves.

(10) Take a point A on the curve no. 3 (corresponding to $V_p =$ 140 volts) and draw a line AB parallel to Y-axis to meet the curve no. 2 (corresponding to V_p=120 volts) at B. Then draw BC parallel to X-axis such that it meets the curve no. 3 at C. Find the valve constants from the values of AB, BC, V_3 (*i.e.* 140 volt) and V_2 (*i.e.* 120 volt). Also find the valve constants separately using the curves no. 3 and 1 (corresponding to V_p=100 volt) and curves no. 2 and 1. Then calculate the mean value of each constant from the three values obtained from three curves.

(*b*) *Drawing* ($V_p - I_p$) *curves* :

Here grid potential is to be kept fixed (say, at 0, −1·5, −3 volt etc.) and in each case, the wandering plug W_1 connected to the plate, is to be moved from one cell of the H.T. battery to another, giving the plate a series of increasing positive potentials. The consequent plate-currents (I_p) are to be noted from the milli-ammeter and plate-potentials from the voltmeter V_p. Plotting the values of plate-voltage (V_p) along X-axis and plate-currents (I_p) along Y-axis, we will get ($V_p - I_p$) curve. In this way, ($V_p - I_p$) curves are to be drawn for each fixed value of grid-potential [fig. 66]. Find the constants of the valve from the curves in a way described earlier.

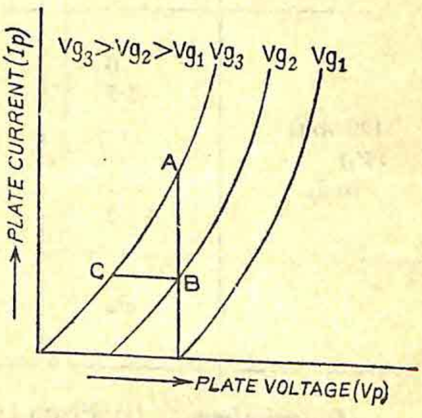

Fig. 66

Measurements : (*a*) *Table for* $(V_g - I_p)$ *graph* :

No. of valve = 6*J*5 or *ECC* 82
Voltage applied to the filament = 6 volts.
Current in the filament = . . . amp.

Plate voltage (Vp)	Grid voltage (Vg)	Plate current (Ip) (milli-amp)	gm (mho)	Rp	μ
100 volts (V_1) (Fixed)	0	..			
	−1·5	..	..	.. (From curves no. 2 & 1)	..
	..	..			
	..	..			
	..	..			
	+1·5	..			
	+..	..			
	..	..			
	etc	etc			
120 volts (V_2) (Fixed)	0	..			
	−1·5	..			
	..	..	..	.. (From curves no. 2 & 3)	..
	..	..			
	..	..			
	+1·5	..			
	+..	..			
	..	..			
	etc	etc			
140 volts (V_3) (Fixed)	0	..			
	−1·5	..			
	..	..			
	..	..	..	.. (From curves no. 3 and 1)	..
	..	..			
	+1·5	..			
	..	..			
	..	..			
	etc	etc			

Calculations : (i) From curves no. 3 and 2:

$AB = \ldots mA = \ldots$ amp
$BC = \ldots$ volts
$V_3 - V_2 = \ldots$ volts.

$\therefore \quad g_m = \dfrac{AB}{BC} = \ldots$ mho.

$R_p = \dfrac{V_3 - V_2}{AB} = \ldots$ ohms.

$\mu = \dfrac{V_3 - V_2}{BC} = \ldots$

(ii) From curves no. 2 and 1 : as before.

(iii) From curves no. 3 and 1 : as before.

$\therefore$ Mean $g_m = (\ldots + \ldots + \ldots)/3 = \ldots$ mho

,, $R_p = (\ldots + \ldots + \ldots)/3 = \ldots$ onm

,, $\mu = (\ldots + \ldots + \ldots)/3 = \ldots$

(*b*) *Table for* ($V_p - I_p$) *graph* :

No. of the valve=6*J*5 or *ECC* 82

Voltage applied to the filament=6 volts

Current in the filament=...amp.

Grid voltage (Vg)	Plate potential (Vp)	Plate current (Ip)	g_m (mho)	Rp (ohm)	μ
0	.. volts	.. ma			
(Vg_1)	.. ,,	.. ,,			
	.. ,,	.. ,,			
	.. ,,	.. ,,	..	..	..
	.. ,,	.. ,,			
	.. ,,	.. ,,			
	.. ,,	.. ,,			
	.. ,,	.. ,,			
−1·5 volts	.. ,,	.. ,,			
(Vg_2)	.. ,,	.. ,,	..	..	..
	.. ,,	.. ,,			
	.. ,,	.. ,,			
	etc	etc			
	.. ,,	.. ,,			
	.. ,,	.. ,,			
−3 volts	.. ,,	.. ,,	..	..	..
(Vg_3)	.. ,,	.. ,,			
	.. ,,	.. ,,			
	.. ,,	etc			

Calculations : (i) From curves no. 3 and 2 :

$AB = \ldots ma = \ldots$ amp ; $BC = \ldots$ volts ; $V_{g2} - V_{g3} = \ldots$ volts.

$$g_m = \frac{AB}{V_{g2} - V_{g3}} = \ldots \text{mho} \; ; \; R_p = \frac{BC}{AB} = \ldots \text{ohm and } \mu = \frac{BC}{V_{g2} - V}$$

(ii) From curves no. 2 and 1 : as before.

(iii) From curves no. 3 and 1 : as before.

$\therefore$ Mean $g_m = (\ldots + \ldots + \ldots)/3 = \ldots$ mho

,, $R_p = (\ldots + \ldots + \ldots)/3 = \ldots$ ohm

,, $\mu = (\ldots + \ldots + \ldots)/3 = \ldots$

Oral questions

1. What are the constants of a triode valve?

Ans. See the theory of the experiment.

2. What is the utility of grid in a triode?

Ans. Grid is a very efficient controller of the flow of electrons from the filament to the plate. If the grid is kept at a negative potential with respect to the filament, the flow of electrons will be retarded and the plate current will diminish. On the other hand, if the grid is given a positive potential with respect to the filament, the flow of electrons will be fecilitated and thereby, the plate-current will increase.

3. For what purposes is a triode valve used?

Ans. A triode valve is used to (i) rectify an alternating potential or alternating current (ii) to amplify a voltage signal (iii) to generate undamped high frequency oscillations.

4. What are the characteristic curves of a triode?

Ans. Keeping *Vg* constant, a graph between *Vp* and *Ip* and keeping *Vp* constant, a graph between *Vg*—*Ip* are known as the characteristic curves of a triode.

5. When are the characteristic curves called 'static' and 'dynamic'?

Ans. When the plate circuit of a triode does not include any load (i.e. resistance), the characteristic curves obtained are known as static curves. When the plate circuit includes a load, the characteristic curves are known as dynamic curves.

7.7. Semi-conductors :

Substances through which electricity can easily pass are called *conductors.* Metals, like copper, silver, gold, aluminium etc. are all good conductors of electricity. Substances through which electricity can not pass easily are called *insulators.* Quartz, mica, sulphur, ebonite, wood etc are examples of insulators. Conductors have a large number of free electrons available as charge carriers but insulators have practically no free electrons available to conduct current.

There are, however, a large number of solids that are neither good conductors of electricity nor good insulators. Their conductivity lies between those of good conductors and good insulators. These substances are called *semi-conductors*. In them, electrons are capable of being moved by the application of heat, strong light or strong electric field. Researches are now being carried out with various semi-conductors—specially germanium and silicon—because with these semi-conductors transistors have been devised.

Movement of charge carriers in semi-conductors :

Silicon and germanium atom, each have four valence electrons in their outermost shell. The atomic pattern of atoms in silicon and germanium crystals is a tetrahedral structure such that each atom shares one of its electrons with each neighbour and the neighbour in turn shares one of its four with it. This is known as 'covalent bond' which maintains the crystalline solid structure. Since there is no free electrons in absolutely pure germanium (called *intrinsic* crystal), there can be no conduction of electricity at low temperatures. At room temperature, however, the thermal energy of a valence electron may become greater than the energy binding it to its nucleus. The covalent bond is then broken. The electron leaves the atom and becomes a free electron. This leaves the atom with a valency or *hole* (upper right and lower left of fig 67). Since that part of the crystal was neutral beforehand, it now lacks an electron and the vacant 'hole' is equivalent to a net positive charge.

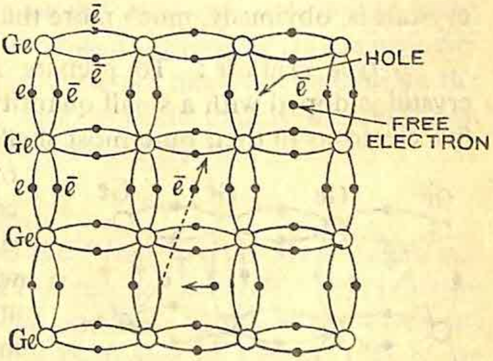

Fig. 67

Due also to thermal agitation, a *bound* electron next to a hole can move across to fill the gap, the net motion of the negative charge from one bounded position to another being, in effect, equivalent to the motion of a hole in the opposite direction. The motion of a hole is, therefore, equivalent to the motion of a positive charge, equal in amount to the negative charge of an electron. This has been shown at the lower part of the fig 67.

In the case of a semi-conductor, however, the increase in thermal energy of the valence electrons due to temperature rise enables more of them to break the covalent bonds and become free electrons. Thus more electron-hole pairs are produced which can act as charge carriers and facilitate conduction of electricity.

7.8. N-type and P-type crystals :

A pure or intrinsic semi-conductor has charge carriers which are thermally generated. These are relatively few in number. If a small amount of impurity is introduced into a semi-conductor (a procedure known as *doping*), such as one part in a million, a so-called *extrinsic* semi-conductor is formed where a large number of charge carriers are available. Both *N*-type and *P*-type crystals may be prepared by doping pure germanium with suitable quantities of impurities. In *N*-type crystals, conduction of electricity is done by negative electrons only, *N* representing the negative charge on an electron, while in *P*-type crystals the conduction is done by positive holes, *P* representing the positive charge on a hole. The conductivity of these extrinsic crystals is, obviously, much more than that of pure or intrinsic crystals.

N-type crystals : To prepare *N*-type crystals, pure germanium crystal is doped with a small quantity of *arsenic*. Arsenic atoms have five electrons in their outermost shell or valence shell. When an atom of arsenic is added to a germanium crystal, the atom settles in a lattice site with four of its electrons shared with neighbouring germanium atoms [Fig 68]. The fifth electron may thus become free to wander through the crystal. As each arsenic atom donates one free electron to the system, the arsenic here is known as a *donor*. With arsenic present in quantities of one to a million, there are about 10^{17} donor atoms, contributing 10^{17} free electrons per cubic centimeter of the crystal. In a good conductor like copper, there are approximately 10^{23} free electrons per c.c. of the metal.

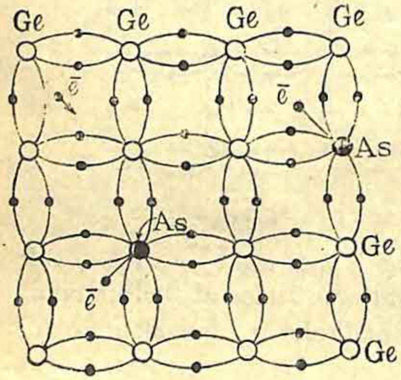

Fig. 68

Now due to thermal agitation, when a few bound electrons are broken free of their covalent bonds, an equal number of holes are thereby created and the free electrons donated by the impurity rush to fill up the gaps. Since in this case, the number of donor electrons far exceeds the number of unbound electrons or holes, the conduction is done mainly by electrons. For this reason, the crystal is known as *N-type crystals.*

P-type crystal : *P*-type crystals are made by doping germanium crystal with foreign atoms like aluminium, boron, indium etc. which are trivalent. When an atom of, say aluminium, is added to a germanium crystal, the atom settles in a lattice site and attracts an electron from a neighbouring atom, thereby completing the four valence bonds and forming a hole in the neighbouring atom (Fig 69). So each aluminium atom provides one hole to the system which is free to accept an electron. For this reason, the aluminium here is known as *acceptor*. By thermal agitation, some of the bound electrons are shaken loose and an equal number of holes are produced. Since by far the majority of the charge carriers are holes and these act like positive charges the conduction of electricity, in this case, is done mainly by the holes. For this reason, the crystal is called a *P-type crystal.*

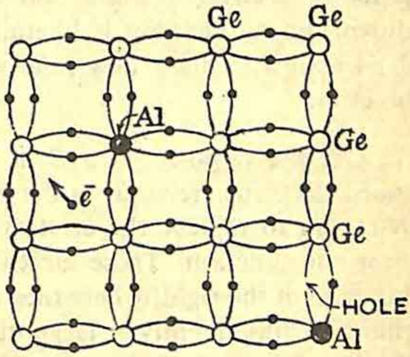

Fig. 69

7.9. Semi-conductor diode :

When two semi-conductors of the *P* and *N*-types are brought into contact they form what is called a *P-N junction* or *diode*. In a junction thus formed, electrons and holes are available in the *N* and *P*-regions respectively of the semi-conductors as carrier of charges. They are spoken of as *majority carriers* because in *N*-region, the donor electrons far outnumber the holes and in *P*-region, the donor holes far outnumber the electrons. Each region is electrically neutral when total charges of all the atoms are considered.

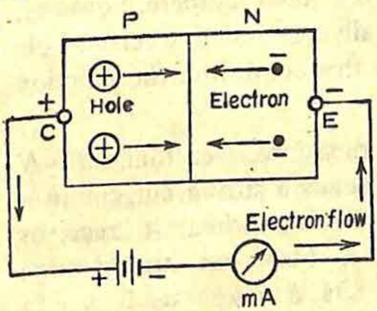

Fig. 70

If now, a potential is applied with the help of a battery to make the *P*-region positive [Fig 70], the (positive) holes are repelled by the battery voltage towards the junction. Simultaneously, the electrons in the *N*-region are repelled by the negative battery voltage towards the junction. Although there is normally a potential barrier at the *P-N* junction that prevents electrons and holes from moving across and combining, yet under the influence of the electric field of the battery, the holes and the electrons cross over the junction where they meet each other and combine. Thus, they cease to exist as mobile charge carriers. For each electron-hole combination taking place near the junction, a covalent bond near the positive battery terminal *C*, breaks down and an electron is liberated which enters the positive terminal. This action creats a new hole which moves to the right towards the junction.

At the opposite end *E* in the *N*-region near negative terminal, more electrons arrive from the negative battery terminal and enter the *N*-region to replace the electrons lost by combination with the holes near the junction. These electrons, in their turn, move towards the junction at the right, where they again combine with new holes arriving there. Consequently a large current flows in the circuit as long as the e.m.f. is applied, in a direction shown in fig 70. A milli-ammeter (*mA*) connected with the external circuit indicates this current by the deflection of its pointer.

The battery connection that permits current to flow across the junction in a manner described above, is known as *forward bias*. In this condition the junction offers low resistance to the passage of current.

On the other hand, if the *P*-region is made negative and the *N*-region positive by reversing the polarities of the battery, both holes and electrons are attracted towards the respective terminals and away from the junction. As a result current flow stops almost completely and the junction appears to offer a very high resistance to the passage of current. A small reverse current of a few micro-ampere, however, flows across the junction due to thermally generated electron-hole pairs within both the *P* and *N* region. In this condition, the junction is said to have a *backward bias*.

It is, therefore, seen that a $P-N$ junction sends a strong current in a given direction when it receives a forward bias but almost none when it gets a backward bias. In this respect, a $P-N$ juntion is very similar to a diode valve.

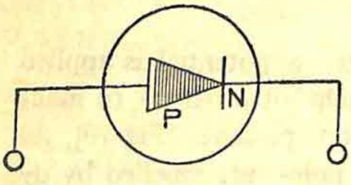

Fig. 71

Fig. 71 shows a symbolic representation of a semi-conductor diode.

7.10. To draw the characteristic curves of a semi-conductor diode i.e. $P-N$ junction.

Apparatus : A semi-conductor diode (for example *BY*127 of Bel) fixed on a wooden board with suitable binding screws, a voltmeter (0-5 volt range with 0·1 volt division), a plug commutator, a milli-ammeter (0-20 m.a. range), a micro-ammeter (0-20μ a range), a battery (9 volt), a plug key, a resistor of suitable value, a wandering plug etc.

Circuit connections : Let the *P*-type crystal of the diode is connected to the binding screw *C* and the *N*-type one to the screw *D* by wire below the wooden board. [Fig. 72].

P-type crystal circuit : A potentiometric arrangement is made by connecting a battery *E* of low voltage (9 volt), a suitable resistor *R* and a plug key *K* in series. The end *A* of the resistor *R* is joined to the positive terminal and the end *B* to the negative terminal of the battery. A wandering plug *W* which is connected to the binding screw no. 2 of a plug commutator can slide over the resistor *R*. The binding screw no. 1 of the commutator is joined to the screw *C* fixed on the wooden board. If the wandering plug *W* moves along the different points of the resistor *R*, the $P-N$ junction will get different voltages which will be recorded by the voltmeter *V* (0-5 volt). One terminal of the voltmeter is joined to the wandering plug *W* and the other terminal to the point *B* of the resistor, which is again joined to the binding screw no. 4 of the commutator.

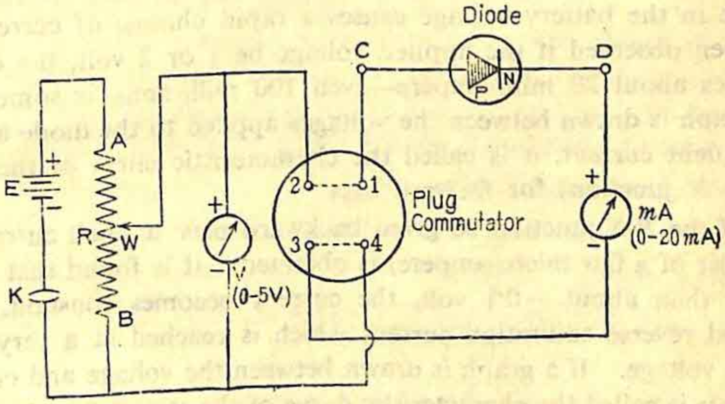

Fig. 72

N-type crystal circuit : The binding screw no. 3 of the commutator is connected to the screw *D* fixed on the board, through a milli-ammeter *mA* (0-20 m.a. range). If now a plug be inserted into the gap between the screws 1 and 2 and another plug into the gap between the screws 3 and 4 of the commutator, the *P*-region of the diode becomes connected to the positive end of the battery and the *N*-region to the negative end of the battery. Under this condition, the semi-conductor diode gets forward bias. But if the positions of the plugs in the commutator be reversed *i.e.* if the gaps between the screws no. 2 and 3 and the screws no. 1 and 4 be closed, the *N*-region gets connected with the positive end and the *P*-region with negative end of the battery. In this condition, the semi-conductor diode gets reverse or backward bias. The connection of the voltmeter *V* are to be reversed now (if it is not a zero-centred instrument) and the milli-ammeter is to be replaced by a micro-ammeter (0-20μ a range) because the current obtained in reverse bias is of the order of few micro-amperes.

Theory : In a $P-N$ junction diode under forward bias, a slight change in the battery voltage causes a rapid change of current. It has been observed if the applied voltage be 1 or 2 volt, the current becomes about 20 milli-ampere—even 100 milli-amp. in some cases. If a graph is drawn between the voltages applied to the diode and the consequent current, it is called the characteristic curve of the diode (or $P-N$ junction) for forward bias.

If the *P-N* junction be given backward bias a small current (of the order of a few micro-ampere) is obtained. It is found that if *V* is greater than about $-0{\cdot}1$ volt, the current becomes constant. This is called reverse saturation current which is reached at a very small reverse voltage. If a graph is drawn between the voltage and current, the curve is called the characteristic curve of the semi-conductor diode for backward bias :

Experimental procedure : (1) Make connections as shown in fig 72. Connect the $+ve$ and $-ve$ terminals of the voltmeter and the milli-ammeter to appropriate points.

(2) Closing the plug key *K* and putting the wandering plug *W* very near to the point *B*, apply a small potential difference (say 0·1 volt) to the junction. Put two plugs—one in the gap between the screw no. 1 and 2 and the other in the gap between the screw no. 3 and 4 of the plug commutator. This puts the junction in the forward bias. Note the voltage from the voltmeter *V* and the current, if any, from the milli-ammeter (*mA*). [A silicon diode begins to conduct

at a minimum voltage of about 0·5 volt and a germanium diode at about 0·3 volt).

(3) Shifting the wandering plug *W* a little towards the point *A*, increase the forward bias voltage from 0·1 volt to 0·2 volt. In this condition, take the readings of the voltmeter and the milli-ammeter, if any. Gradually increase the bias voltage by steps of 0·1 volt till the current increases very high (say about 15 m.a.). At every step, note the voltmeter and milli-ammeter readings.

(4) Draw a graph with forward bias voltages (in volt units) along *OX* axis and the current (in milli-amp. units) along *OY* axis [Fig. 72(*a*)]. This gives the characteristic curve for forward bias.

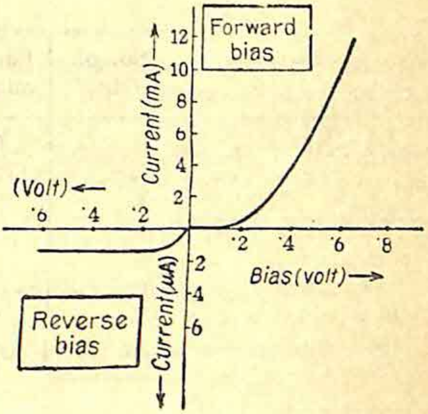

Fig. 72(*a*)

(5) Now reverse bias is to be applied to the diode. Reverse the connection of the voltmeter *V* *i.e.* connect its +*ve* terminal with the point *B* of the resistor *R* and the −*ve* terminal with the wandering plug *W*. Replace the milli-ammeter by a micro-ammeter (0-20μ.a.). Join its −*ve* terminal with the binding screw *D* and the +*ve* terminal with the binding screw no. 3 of the plug commutator. Now put two plugs—one in the gap between the screws no. (2—3) and the other in the gap between the other screws no. (1—4). Read the voltameter *V* and the micrometer which may not give any reading.

Put the wandering plug *W* at a point very near the point *B* of the resistor *R*, so that reverse bias applied to the diode is small (say 0·1 volt).

(6) Increase the reverse bias voltage by steps of 0·5 volt and in each step read the voltmeter and the micro-ammeter. It will be found that the current remains constant at a very small reverse potential. This is known as reverse saturation current.

(7) Draw a graph plotting bias voltages (in volts) along *OX′* axis and the current (in micro-ampere) along *OY′* axis. This gives the characteristic curve for reverse bias [Fig. 73].

Measurements :

No. of semi-conductor diode = .. (say, *BY*127)

(*a*) *Table for forward bias* :

No. of Obs.	Forward bias voltage (volts)	Current (*mA*)
1.	0·1	..
2.	0·2	..
3.	..	..
4.	..	..
5.	..	
6.	..	..
7.	0·7	etc
8.	0·8	

(*b*) *Table for backward bias* :

No. of Obs	Backward bias voltage (volts)	Current (μ*A*)
1.	0·1	..
2.	0·5	..
3.	1·0	.. (Saturation)
etc	..	..
	..	..
	..	
	..	.. (")
	4·5	..
	5	etc

[*N.B.* Both the curves—forward bias and backward bias—may be drawn on a single graph paper.]

Remarks : (1) While taking readings for reverse bias, the connections of the voltmeter and micro-ammeter are to be reversed. (2) Silicon diode starts conducting current at a minimum voltage of about 0·5 volt and germanium diode at a minimum voltage of about 0·3 volt.

Oral questions

1. What is a semi-conductor ? Name a few semi-conductors.

Ans. Substances whose electric conductivity is intermediate between those of good conductors and bad conductors are called semi-conductors. Germanium and silicon are two very important semi-conductors.

2. What are *P*-type and *N*-type semi-conductor crystals ?

Ans. If a silicon and a germanium crystal be doped with suitable amount of pentavalent impurity, large number of free electrons are available as charge carriers. In this condition, the crystal is called a *N*-type crystal. on the other hand, if the crystal be doped with trivalent impurity, a large number of holes are created which act as positive charge carriers. The crystal, then, is called a *P*-type crystal.

3. What is a semi-conductor diode ? What is its similarity with a valve diode ?

Ans. If a *P*-type crystal and a *N*-type crystal be joined together, we get a semi-conductor diode. Like valve diode, semi-conductor diode also produces unidirectional current. For this reason, voth valve diodes and semi-conductor diodes are used for rectification purpose.

4. Does a diode offer greater or smaller resistance in the path of current flow during forward bias ?

Ans. It offers smaller resistance because in forward bias, current is much greater than in the case of reverse bias.

5. What effect does temperature produce on the resistance of a conductor and a semi-conductor ?

Ans. For a conductor, resistance increases with the increase of temperature ; for a semi-conductor, resistance decreases with the increase of temperature.

6. What is the difference between forward bias and reverse bias ?

Ans. When the *P*-region of a $P-N$ junction is given a positive potential and the *N*-region a negative potential, it is called a forward bias. The junction becomes conducting in forward bias. When the potentials are reversed, it is called a reverse bias. The junction becomes almost non-conducting during reverse bias.

7. What is Zener voltage ?

Ans. When the junction diode is given a reverse bias, the current is very small. But if the reverse bias voltage is increased gradually, at a given high voltage, the current increases sharply. This particular voltage is called Zener voltage.

7.11. Semi-conductor triode :

A transistor or a semi-conductor triode is composed of three semi-conductor elements, two of the *P*-type crystal and one of *N*-type [Fig. 74(i)]. This combination is referred to as $P-N-P$ type transistor. We can also have $N-P-N$ type transistor [Fig. 74(ii)] composed of two *N*-type and one *P*-type crystals.

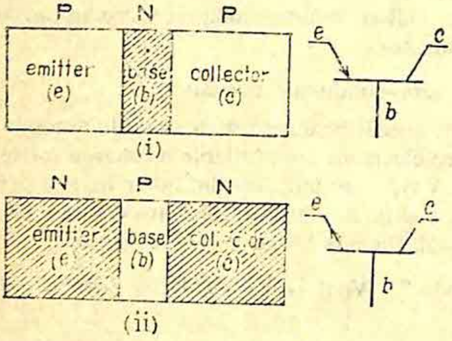

Fig. 74

In the first case, the *P*-region on the left is called the *emitter* (*e*), the *P*-region on the right is called *collector* (*c*) and the *N*-region in between is known as the *base* (*b*). The diagram by the side of fig. (i) shows the diagramatic representation of $P-N-P$ transistor. Similarly, in the second case, the left-hand *N*-region is the emitter (*e*), the right-hand *N*-region is the collector (*c*) and the *P*-region in between is the base (*b*). Its diagramatic symbol has been shown by the side of fig(ii). It is to be noted that as a triode valve has three electrodes —the plate, the filament and the grid, the transistor has three electrodes—the collector, the emitter and the base.

Action: Consider the $P-N-P$ transistor first. A low voltage battery E_1 is so connected between the base and the emitter that the emitter gets the positive potential while the base gets the negative potential [fig. 75]. On the other hand, a high voltage battery E_2 is connected between the collector and the base with its positive terminal joined to the base and the negative terminal to the collector. Note that the left-hand $P-N$ junction gets the forward bias while the right-hand $P-N$ junction gets the reverse bias. Now, due to the influence of the battery E_1, the positively charged holes from the *P*-region of the left hand side $P-N$ junction will cross the potential barrier at the junction and enter into the base (*i.e.* *N*-region). As the base is very thin (about 0·001″ thick) and very lightly doped, almost 95% of the holes will cross over the base and enter into the *P*-region of the right

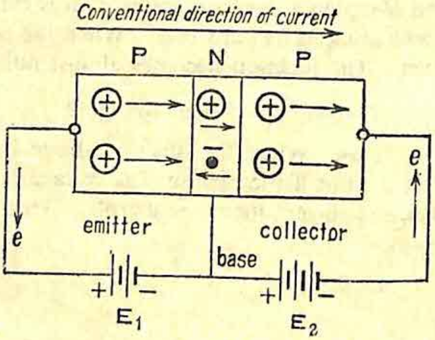

Fig. 75

side *i.e.* into the collector. About 5% of the holes will combine with the electrons in the base and will lose their conducting character. The holes which enter the collector are attracted by the negative terminal of the battery E_2 where they unite with electrons. As soon as a hole disappears in this way either in the base region or at the negative terminal of the battery E_2, the covalent bond of an atom near the terminal of the emitter breaks down setting free an electron which enters into the positive terminal of the battery E_1. The hole created by the breaking of the co-valent bond travels towards the base, and the whole process is repeated. It is seen that by the above process, electricity is conducted from the emitter to the collector by the holes inside the crystal while in the external circuit, the conduction is carried out by the electrons. The conventional direction of current, in this case, is from the emitter to the collector.

The current in the collector circuit is somewhat less than the current in the emitter circuit, the difference being proportional to the number of holes that unite with electrons in the base region. The ratio of the collector current (I_c) to the emitter current (I_e) is called the current amplification factor and it is denoted by a_o. From the above, it is clear that a_o can never be more than 1.

Now, consider $N-P-N$ transistor. The battery connection is shown in fig 76. The negative terminal of the low tension battery E_1 is joined to the N-type crystal (here the emitter) of the left hand side and the positive terminal to the base. This gives forward bias to the left hand side $N-P$ junction. The positive terminal of the high-tension battery E_2 is joined to the N-type crystal (here, the collector) of the right hand side and the negative terminal to the base. This gives the right-hand side $N-P$ junction a backward or reverse bias. Note that the same type of bias was also given to $P-N-P$ transistor.

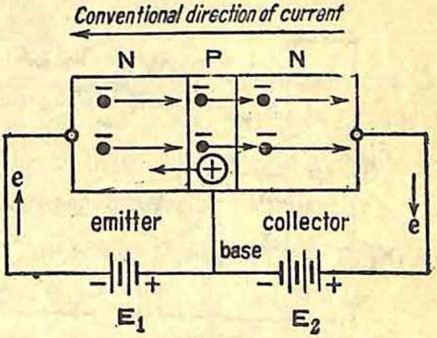

Fig. 76

Here, the process of conduction of charges is exactly the same and in the case of $P-N-P$ transistor. Due to the influence of the battery E_1, electrons are repelled from the emitter towards the base where the electrons cross the potential barrier and enter the collector region. As the base is very thin and lightly doped, most of the electrons cross over into the collector region and being attracted by the positive terminal of the high tension battery E_2, enter into the positive electrode. So inside the crystal, conduction of electric charge is carried out by electrons from the emitter to the collector. As soon as an electron enters into the positive electrode of the battery E_2, an electron from the negative terminal of the battery E_1 enters into the emmitter region and the whole process is repeated. Here, the conventional direction of current inside the crystal is from the collector to the emitter.

7.12. To draw the static characteristic of a common emitter type transistor :

Apparatus : A wooden board with suitable socket for holding a transistor (AC126 of B.e.l. which is a $P-N-P$ crystal) and binding screws (one each for the emitter, collector and the base), a low voltage battery E_1 (about 4 volt), a high voltage battery E_2 (about 20 volts), two plug keys, two resistors of suitable value (R_1 and R_2), a milli-ammeter mA (0-15mA range with 0·2mA divisions), a micro-ammeter μA(0-300μA range with 2·5μA divisions), a voltmeter V_2 (0-10 volt range with 0·2 volt division), a milli-voltmeter V_1 (0-500 mV range), two wandering plugs W_1 and W_2 etc.

Circuit connections : Fig 77 shows the circuit connections. The arrangement is known as common emitter type arrangement because both the base (b) and the collector (c) are connected to the emitter

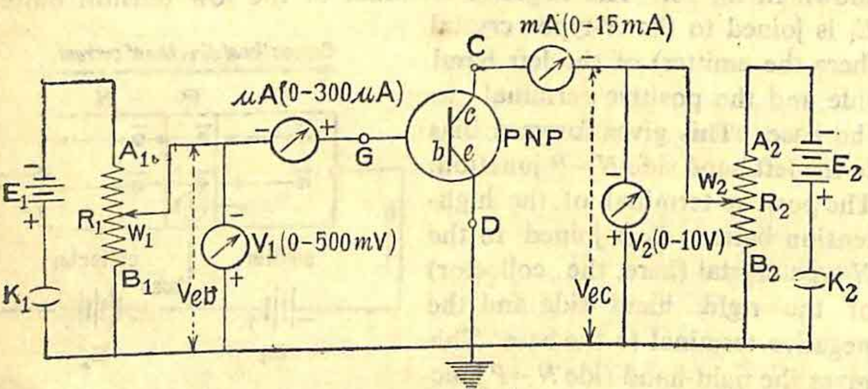

Fig. 77

which is earth—connected. The whole circuit may be divided into two parts :—(i) emitter-base circuit which is known as *'input circuit'* and (ii) emitter-collector circuit which is known as the *'output circuit'*

Emitter-base circuit : Suppose the three elements of the transistor—the emitter (e), the base (b) and the collector (c) are connected to the binding screws D, G and C respectively fixed on the wooden board. These connections are made by drawing wire below the board. If any voltage be applied between the emitter and the base, it will be called emitter-base voltage (V_{eb}). The voltage is usually small and is recorded by a millivoltmeter V_1 connected across the screws G and D. The voltage is applied by a potentiometric arrangement consisting of the low tension battery E_1, a resistor R_1 and a plug key K_1- all connected in series. The negative terminal of the battery E_1 is joined to the end A_1 and the positive terminal to the end B_1 of the resistor. The binding screw D to which the emitter (e) of the transistor is connected, is joined directly to the point B_1 of the resistor R_1 and finally to the earth. A wandering plug W_1 is connected to the binding screw G through a micro-ammeter (μA). The wandering plug can slide over the resistor R_1. When the wandering plug W_1 moves from B_1 towards A_1, the value of V_{eb} gradually increases. The current that flows in the emitter-base circuit (I_b) is recorded by the micro-ammeter (μA). Note that by this arrangement, the base is given a negative potential while the emitter a positive potential (ref. fig 75).

Emitter-collector circuit : If any voltage be applied between the terminals C and D, it will be called emitter-collector voltage (V_{ec}). The value of this voltage is usually high and it is recorded by the voltmeter V_2 connected across the terminals C and D. This voltage is applied by another potentiometric arrangement consisting of a high tension battery E_2, a resistor R_2 and a plug key K_2—all connected in series. The negative terminal of the battery is joined to the end A_2 and the positive terminal to the end B_2 of the resistor. The binding screw D is joined directly to the end B_2 of the resistor and finally to the earth. A wandering plug W_2 is connected to the binding screw C through a milli-ammeter (mA). The wandering plug W_2 can slide over the resistor R_2. When the wandering plug W_2 moves from B_2 towards A_2, the value of V_{ec} gradually increases. The current that flows in the emitter-collector circuit (I_c) is recorded by the milli-ammeter.

Note that by this arrangement, the emitter is given a positive potential while the collector a negative potential (ref. fig 75).

The $+ve$ and $-ve$ terminals of the voltmeters and ammeters are to be carefully connected to the proper places according to the diagram. [Fig 77].

Theory : (i) Keeping the emitter-collector voltage (V_{ec}) constant, if a graph is drawn between the various values of V_{eb} and the corresponding base currents (I_b), the curve obtained is called the *input characteristic curve.* A family of such curves may be drawn by giving V_{ec} different fixed values.

(ii) Keeping the base current (I_b) constant, if a graph is drawn between the various values of V_{ec} and the corresponding collector current (I_c), the curve obtained is called the *output characteristic curve.* A family of such curves may be drawn by giving I_b different fixed values.

Experimental procedure :

(*i*) *Input characteristic curve* ($V_{eb}-I_b$)

(1) Make circuit connections as shown in fig 77. Connect the $+ve$ and $-ve$ terminals of the meters properly as shown in the figure. Make the point D earth-connected.

(2) Close the plug key K_2. Move the wandering plug W_2 slowly from the point B_2 of the resistor R_2 towards the point A_2. This gives increasing values to V_{ec}. Find the point on the resistor R_2 where the wandering plug W_2 is to be put inorder that the voltmeter V_2 may read 1 volt. This makes $V_{ec}=1$ volt. During the subsequent operations, this value of V_{ec} will be kept constant.

(3) Now close the plug key K_1 and place the wandering plug W_1 very near the point B_1 of the resistor R_1. Place the wandering plug W_1 at such a position that the milli-voltmeter V_1 may read 50 mV. *i.e.* V_{eb} becomes equal to 50 mV. When this is done, read the micro-ammeter (μA). The reading will give base current (I_b) at that moment.

(4) Keeping the wandering plug W_2 fixed in its position, change the values of V_{eb} by steps of 50 mV by changing the position of the wandering plug W_1. At each step, take the micro-ammeter reading. Take, at least, five such readings.

(5) Now take the plug out of the key K_1. Push the wandering plug W_2 a little towards the end A_2 so that the voltmeter V_2 now reads 3 volt. Keeping this value of V_{ec} constant, close the plug key K_1 and repeat the operations no. 3 and 4.

(6) In this way, the operations are to be repeated again for a constant value of V_{ec} equal to 5 volts. Take out the plugs from the keys K_1 and K_2.

(7) Draw one curve, between V_{eb} and I_b for each fixed value of V_{ec} (i.e. 1 volt. 3 volt and 5 volt). This gives the family of input characteristic curves [Fig 78].

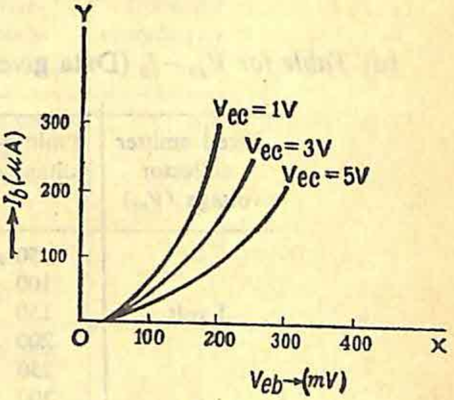

Fig. 78

(*ii*) *Output characteristic curve* ($V_{ec}-I_c$):

(8) Close the plug key K_1. Shift the wandering plug W_1 slowly from the end B_1 towards the end A_1 of the resistor R_1. This increases the value of V_{eb} and consequently the base current (I_b). Keep the wandering plug W_1 fixed at a position where the micro-ammeter (μA) reads, say 40 μ-ampere.

(9) Now close the plug key K_2 and put the wandering plug W_2 at such a position near B_2 of the resistor R_2 that the voltmeter reads 0·5 volt. In this case, the emitter-collector potential *i.e.* V_{ec} becomes 0·5 volt. Read the milli-ammeter (*mA*). This gives the corresponding value of I_c.

(10) Keeping the wandering plug W_1 fixed in its position, change the values of V_{ec} by steps of 0·5 volt by changing the position of the wandering plug W_2. At each step take the milli-ammeter reading. Take, at least, five such readings till the voltmeter V_2 reads 6 or 7 volts.

(11) Repeat the above operations keeping I_b at different fixed values (say, $80\mu A$, $100\mu A$ etc). For every fixed value of I_b, draw a curve between V_{ec} and I_c. This gives the family of output characteristic curves [Fig 79].

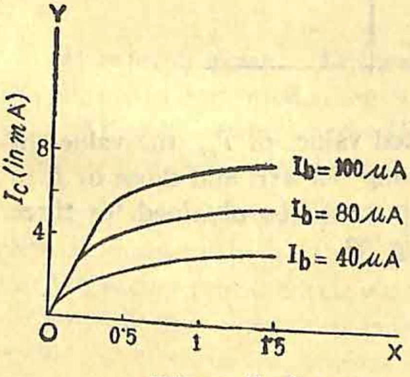

Fig. 79

Measurements :

No. of the transistor=... (For example *AC* 126, $P-N-P$ type)

(*a*) *Table for* $V_{eb}-I_b$ (Data given as illustration)

Fixed emitter -collector voltage (V_{ec})	Emitter-base voltage (V_{eb})	Base current (I_b)
1 volt	50 mV	.. μA
	100 ,,	.. ,,
	150 ,,	.. ,,
	200 ,,	.. ,,
	250 ,,	.. ,,
	300 ,,	.. ,,
3 volts	50 mV	..
	..	..
	..	..
	..	..
	etc	..
	300 ,,	etc
5 volts	50 mV	..
	..	..
	..	..
	..	
	..	etc
	..	
	etc	
	300 ,,	

Drawing of graph : For every fixed value of V_{ec}, the values of V_{eb} in milli-volts are to be plotted along OX axis and those of I_b in micro-ampere in OY axis. Three curves will be obtained for three fixed values ($1V$, $3V$ and $5V$) of V_{ec} [Fig 78].

(*b*) *Table for* $V_{ec} - I_c$ (Data given as illustrations)

Fixed base current (I_b)	Emitter-collector voltage (V_{ec})	Collector current (I_c)
	0·5 volt	.. mA
	1·0 ,,	.. ,,
40µA	1·5 ,,	.. ,,
	..	.. ,,
	..	.. ,,
	..	
	6·0 ,,	.. ,,
	0·5 volt	..
	..	
	..	..
80µA	..	..
	..	..
	etc	..
	6·0 ,,	..
	0·5 volt	..
	..	..
	..	..
100 µA	..	..
	..	..
	..	..
	etc	..
	6·0 ,,	..

Drawing of graph : For every fixed value of I_b, the values of V_{ec} in volts are to be plotted along OX axis and those of I_c in milli-ampere, long OY axis. Three curves [Fig. 79] will be obtained for three fixed values (40µA, 80µA and 100µA) of base current (I_b).

Remarks : (1) From the input characteristic curves, it is clear that the output voltage (V_{ec}) has very little effect on the input characteristics except that the base current (I_b) increases with the increase of output voltage. The reason is that with the increase of output voltage, greater number of holes cross the base and enter into the collector region. (2) In common emitter mode, the current amplification is high because a slight change of base current appreciably affects emitter cuirent. So, a transistor in common emitter mode is frequently used as an amplifier. (3) The input current is usually very small— of the order of few micro-ampere ; but the output current is farely high— of the order of few milli-amperes. (4) The specification of meters, resistance etc as stated in the experiment may vary.

Oral questions

1. What is a transistor ? How many types of transistors are there ?

Ans. A transistor consists of a semi-conductor crystal sandwitched between two opposite kinds of semi-conductor crystals. There are two types of transistor : (*i*) $P-N-P$ type and (*ii*) $N-P-N$ type.

2. Which type of transistor are you using ? What difference would there be if it were an opposite type of transistor ?

Ans. I am using $P-N-P$ type transistor. Had it been $N-P-N$ transistor, the polarities of the batteries E_1 and E_2 should have been reversed. The connections of the meters need also be reversed. Experimental procedure is the same.

3. What are the similarities between a transistor and a triode valve ?

Ans. There are many similarities, both functional and constructional, between a transistor and a triode valve. As a triode valve has three electrodes, the transistor has also three electrodes. As a triode valve may be used as an amplifier so also a transistor is used as an amplifier.

4. What are the advantages of a transistor over a triode valve ?

Ans. A transistor has many advantages over a triode valve. Its size is very small, it requires very little voltage to work, it does not require any easily breakable vacuum chamber like a valve, it does not contain a fragile filament. A transistor lasts much longer than a triode valve.

5. What are the different configurations of a transistor ?

Ans. There are three configurations : (*i*) Common emitter mode (*ii*) Common base mode (*iii*) Common collector mode.

6. Why a transistor, in common emitter mode, can work as an amplifier ?

Ans. See remark no. 2.

[Questions given in the expt. 7.10 are also applicable here]

Table of some important physical constants and conversion tables

1. Conversion table :

Length	Mass	Volume
1 inch=2·54 cm	1 lb=453·6 gm	1 litre=1000 c.c.
1 foot=30·48 cm	1 lb=0·454 kg	1 gallon=4·54 litres
1 metre=39·37 inch	1 gm=0·0022 lb.	1 cu.ft=28·31 litres
1 metre=3·28 ft	1 kg=2·2 lb.	1 litre=61·03 cu inches
1 mile=1·6 km	1 ton=2240 lb.	1 cu inch=16·39 c.c.
1 km=0·62 mile	1 ton=1016 kg	1 cu inch=0·01639 litre

2. Specific gravities of common substances.

Solid		Liquid		Gas	
Substance	Sp. gr.	Substance	Sp. gr.	Substance	Sp. gr.
Copper	8·9	Kerosene	0·8	Hydrogen	0·000089
Brass	8·4−8·7	Olive oil	0·02	Air	0·001293
Iron	7·86	Glycerine	1·28	Carbon dioxide	0·001977
Aluminium	2·7	Mercury	13·6	Nitrogen	0·001251
Gold	11·3	Alcohol	0·81	Oxygen	0·001429
Zinc	7·1	Paraffin oil	0·9	Steam	0·000581
Steel	7·8	Milk	1·03		
Wax	0·87−0·93	Turpentine	0·87		
Wood	0·7 −0·93	Petrol	0·68−0·92		
Marble	2·5−2·3				

3. Density of water at different temperatures.

Temperature (°C)	Density (gm/c.c.)	Temperature (°C)	Density (gm/c.c.)
0°	0·999871	30	0·99560
4	1·00000	31	0·99537
8	0·99988	33	0·99473
10	0·99970	35	0·99371
16	0·99897	40	0·99220
20	0·99820	50	0·99180
26	0·99681		
27	0·99654		
28	0·99626		
29	0·99576		

4. Elastic constants of some materials.

Materials	Young's mod (Y) (dynes/cm^2)	Rigidity mod(n) (dynes/cm^2)	Bulk mod (k) (dynes/cm^2)	Poisson's ratio (σ)
Brass	$9\cdot7-10\cdot2\times10^{11}$	$3\cdot5\times10^{11}$	6×10^{11}	0.34−0.4
Copper	$12\cdot4-12\cdot9\times10^{11}$	$3\cdot6-4\cdot6\times10^{11}$	$13-14\cdot3\times10^{11}$	0.25−0.35
Iron	$19-20\times10^{11}$	$7\cdot4-7\cdot6\times10^{11}$	$16\cdot5-17\cdot5\times10^{11}$	0.28
Steel	$19\cdot5-20\cdot6\times10^{11}$	$8-8\cdot9\times10^{11}$	$16-19\times10^{11}$	0.25−0.31
Aluminium	7×10^{11}	$2\cdot5\times10^{11}$	$7\cdot5\times10^{11}$	0.34

5. Breaking weight for some common materials.

Materials	Breaking wt. (kilo/cm^2)	Materials	Breaking wt. (kilo/cm^2)
Brass	3160 to 3980	Steel (ordinary)	11230
Copper	3000	,, (tempered)	15810
Iron	6000	,, (pianoforte)	21380

6. Surface tensions of some common liquids in contact with air

Liquids	Surface tension (dynes/cm)	Liquids	Surface tension (dynes/cm)
Water	72	Paraffin oil	26·4
Copper-sulphate solution	70	Olive oil	32
Turpentine	27·3	Alcohol	22

7. Viscosity of liquids and gases.

Liquids	Viscosity (Poise)	Gases	Viscosity (Poise)
Water (at 25°C)	0·00893	Air (0°C)	0·000183
Olive oil	0·89	Hydrogen	0·000086
Alcohol	0·0099	Oxygen	0·000192
Paraffin oil	0·02	Steam (100°C)	0·000120

8. Coefficients of linear expansion of solids.

Substances	Coefficients (per °C)	Substances	Coefficients (per °C)
Cast iron	$10{\cdot}2\times10^{-6}$	Aluminium	$25{\cdot}5\times10^{-6}$
Steel	$11{\cdot}9\times10^{-6}$	Copper	$16{\cdot}7\times10^{-6}$
Brass	$18{\cdot}9\times10^{-6}$	Glass	9×10^{-6}

9. Specific heats of solids and liquids.

Solids	Sp. heat	Liquids	Sp. heat
Silver	0·056	Alcohol	0·55
Lead	0·03	Glycerine	0·58
Zinc	0·09	Paraffin oil	0·53
Copper	0·09	Turpentine	0·42
Aluminium	0·219	Mercury	0·034
Iron	0·119	Castor oil	0·508
Marble	0·22	Mastard oil	0·501
Ice	0·5	Aniline	0·514
Glass	0·16	Water (at 70°C)	1·000
Tin	0·055		
Nickel	0·106		

10. Coefficients of cubical expansion of liquids.

Liquids	Coefficient (per °C)	Liquids	Coefficient (per °C)
Mercury	$18{\cdot}18\times10^{-5}$	Water (10° − 20°)	15×10^{-5}
Alcohol	108×10^{-5}	Water (20° − 40°)	$30{\cdot}2\times10^{-5}$
Turpentine	94×10^{-5}	Water (40° − 60°)	$45{\cdot}8\times10^{-5}$
Glycerine	47×10^{-5}	Paraffin	90×10^{-5}

11. Coefficient of expansion of air at constant pressure or at constant volume = 0·0036.

12. Thermal conductivities of some materials. (good and bad conductors)

Solids	Thermal conductivity (c.g.s.)	Solids	Thermal conductivity (c.g.s.)	Liquids	Thermal conductivity (c.g.s.)
Brass	0·26	Steel	0·115	Water	0.00147
Copper	0·918	Constantan	0·054	Mercury	0.0201
Aluminium	0·48	Rubber	$0{\cdot}45\times10^{-3}$	Paraffin oil	0.0003
German Silver	0·07	Asbestos	3×10^{-4}	Castor oil	0.00043
Silver	1·006	Glass	25×10^{-4}		
		Ebonite	4.2×10^{-4}		

13. Mechanical equivalent of heat :

$J=4{\cdot}2\times10^{7}$ *ergs/cal* (C.G.S.)

$J=778$ *ft-lb/B.t.u.* (F.P.S.)

14. Saturation vapour pressure of water at different temperatures

Temp (°C)	Pressure (mm of Hg)	Temp (°C)	Pressure (mm of Hg)	Temp (°C)	Pressure (mm of Hg)	Temp (°C)	Pressure (mm of Hg)
0	4·58	14	11·972	27	26·71	55	118·1
1	4·924	15	12·771	28	28·32	60	149·4
2	5·290	16	13·617	29	30·01	65	187·7
3	5·679	17	14·511	30	31·79	70	233·7
4	6·094	18	15·457	31	33·66	75	289·3
5	6·536	19	16·456	32	35·63	80	355·2
7	7·505	20	17·51	33	37·69	85	433·8
8	8·036	21	18·63	34	39·86	90	525·9
9	8·600	22	19·80	35	42·14	95	634·4
10	9·198	23	21·04	40	55·29	100	760·00
11	9·831	24	22·35	45	71·92		
12	10·504	25	23·73	50	92·49		
13	11·217	26	25·18				

15. Melting and Boiling points

Substance	Melting point (°C)	Substance	Boiling point (°C)
Bees Wax	61° – 64°	Water	100°
Napthalene	80°	Carbontetra chloride	76·8°
Ice	0°	Ether	34·6°
Paraffin wax	52° – 56°	Aniline	189°
Aluminium	660·1°	Carbon di-sulphide	46·3°
		Acetone	56·5°

16. Refractive indices and dispersive powers of common substances

Substances	Refr. index (μ)	Dispersive power (ω)	Substances	Refr. index (μ)	Dispersive power (ω)
Crown glass	1·48 – 1·61	0·018	Water	1·33	0·018
Flint glass	1·58 – 1·96	0·027	Paraffin oil	1·44	..
Diamond	2·42	..	Turpentine	1·47	..
Mica	1·60	..	Glycerine	1·47	..
Canada Balsam	1·530	..	Ether	1·352	0·017

17. Specific rotation of some common solutions

Active substance	Solvent	Sp. rotation	Active substance	Solvent	Sp. rotation
Cane sugar	Water	+66·7°	Dextrose	Water	+52·5°
Glucose	,,	+52·6°	Camphor	Alcohol	+41·0°
Fructose	,,	−91·1°	Turpentine	Pure	−37·0°

18. Wave lengths of some of the important spectral lines (in Angstrom units ; $1 A.U. = 10^{-8}$ cm)

Substance	Wave-length (λ)	Substance	Wave-length (λ)	Substance	Wave-length (λ)
Sodium chloride	5896 *y* (D_1) 5890 *y* (D_2)	Argon	4159 *v* 4192 *v* 4198 *v* (3 lines very close) 4259 *b* 4703 *b* 6031 *o*	Helium	3889 *v* 4026 *v* 4471 *b* 4713 *b* 4922 *bg* 5016 *g* 5876 *y* 6678 *r* 7065 *r*
Potassium chloride	4047 *v* 7668 *r* 7702 *r*				
Neon	5853 *y* 5945 *o* 6402 *o* 6507 *r*	Mercury (Vapour lamp)	5461 *g* 5770 *y* 5791 *y*	Hydrogen	3970 *v* 4102 *v* 4340 *b* 4861 *gb* 6563 *r*

19. Velocity of sound in different media

Solid medium	Velocity (at 20°C) metre/s	Liquid medium	Velocity (at 20°C) metres/s	Gas medium	Velocity (at 0°C) metre/s
Aluminium	5100	Alcohol	1210	Air	331·7
Brass	3400	Mercury	1407	Hydrogen	1262·0
Copper	3560	Water	1457	Oxygen	316
Glass	5000			Nitrogen	338
Iron	5130			Carbon dioxide	259·0
Steel	4990				

20. Specific resistance and temperature coefficient of resistance

Material	Sp. resistance (ohm-cm)	Temp. coeffn ($^\circ C^{-1}$)	Material	Sp. resistance (ohm-cm)	Temp. coeffn ($^\circ C^{-1}$)
Iron	12×10^{-6}	62×10^{-4}	German silver	40×10^{-6}	$2{\cdot}3$ to 6×10^{-4}
Copper	$1{\cdot}2$ to 2×10^{-6}	$42{\cdot}8 \times 10^{-4}$	Manganin	$44{\cdot}5 \times 10^{-6}$	$0{\cdot}02$ to 0.5×10^{-4}
Eureka	49×10^{-6}	$-0{\cdot}4$ to 0.1×10^{-4}	Nickel	$11{\cdot}8 \times 10^{-6}$	27×10^{-4}
Aluminium	$3{\cdot}21 \times 10^{-6}$	38×10^{-4}	Nichrome	110×10^{-6}	$1{\cdot}7 \times 10^{-4}$
Silver	$1{\cdot}63 \times 10^{-6}$	40×10^{-4}	Antimony	$40{\cdot}5 \times 10^{-6}$	$-39{\cdot}8 \times 10^{-4}$

21. E.M.F. of some common cells

Cells	E.M.F. (in volts)
Leclanche	1·4
Daniel	1·08
Bunsen	1·85
Storage (Acid)	2·0
Storage (Alkali)	1·4 5o 1·1

22. E.M.F. of common thermo-couples

[Equation : $E = at + bt^2$; values of a and b when one junction is at 0°C and the other at t°C]

Couple	a	b
Copper-eureka	37.54×10^{-6}	0.0445×10^{-6}
Copper iron	13.40×10^{-6}	-0.0137×10^{-6}

$\pi = 3{\cdot}142$; $\pi^2 = 9{\cdot}8696$; $\log \pi = 0{\cdot}4972$; $1/\pi = 0{\cdot}1318$

$\sqrt{2} = 1{\cdot}414$; $\sqrt{3} = 1{\cdot}7321$; $g = 980 \text{ cm/s}^2$ (c.g.s.)

$= 32 \text{ ft/s}^2$ (f.p.s.)

LOG TABLES

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170						5 9 13	17 21 26	30 34 38
						0212	0253	0294	0334	0374	4 8 12	16 20 24	28 32 36
11	0414	0453	0492	0531	0569						4 8 12	16 20 23	27 31 35
						0607	0645	0682	0719	0755	4 7 11	15 18 22	26 29 33
12	0792	0828	0864	0899	0934						3 7 11	14 18 21	25 28 32
						0969	1004	1038	1072	1106	3 7 10	14 17 20	24 27 31
13	1139	1173	1206	1239	1271						3 6 10	13 16 19	23 26 29
						1303	1335	1367	1399	1430	3 7 10	13 16 19	22 25 29
14	1461	1492	1523	1553	1584						3 6 9	12 15 19	22 25 28
						1614	1644	1673	1703	1732	3 6 9	12 14 17	20 23 26
15	1761	1790	1818	1847	1875						3 6 9	11 14 17	20 23 26
						1903	1931	1959	1987	2014	3 6 8	11 14 17	19 22 25
16	2041	2068	2095	2122	2148						3 6 8	11 14 16	19 22 24
						2175	2201	2227	2253	2279	3 5 8	10 13 16	18 21 23
17	2304	2330	2355	2380	2405						3 5 8	10 13 15	18 20 23
						2430	2455	2480	2504	2529	3 5 8	10 12 15	17 20 22
18	2553	2577	2601	2625	2648						2 5 7	9 12 14	17 19 21
						2672	2695	2718	2742	2765	2 4 7	9 11 14	16 18 21
19	2788	2810	2833	2856	2878						2 4 7	9 11 13	16 18 20
						2900	2923	2945	2967	2989	2 4 6	8 11 13	15 17 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8

LOGARITHMS

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52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
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97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

ANTILOGARITHMS

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·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
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·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
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·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
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·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
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·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6794	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	24	16	18	20

NATURAL SINES

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	 2	 3	 4	 5
45	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	·7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	3
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	·9998	9999	9999	9999	9999	1·000	1·000	1·000	1·000	1·000	0	0	0	0	0
90	1·000														

NATURAL COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
0	1·000	1·900	1·000	1·000	1·000	1·000	9999	9999	9999	9999	0	0	0	0	0
1	·9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	·9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	·9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	·9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	·9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	·9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	·9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	·9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	·9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	·9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	·9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	·9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	·9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	·9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	·9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	·9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	·9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	·9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	·9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	·9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	·9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	·9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	5
23	·9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	4	6
24	·9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	3	5	6
25	·9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	2	4	5	6
26	·8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	·8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	6
28	·8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	5	7
29	·8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	·8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	·8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	·8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	·8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	·8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	·8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	·8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	·7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	·7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	·7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	·7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	·7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	·7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	·7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	·7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

NATURAL COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1	2	3	4	5
45	·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	·6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	·6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	·6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	·6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	·6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	·6018	6004	5990	5970	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	·5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	·5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	·4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	·2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	·2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	·2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	·1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	15
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	·0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15
90.	·0000														

NATURAL TANGENTS

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences				
											1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	15
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	12	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	9	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	6	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	16
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	13	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	3	7	10	14	17
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	14	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	7	11	15	18
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	11	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	15	19
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	12	16	20
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	8	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	17	21
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	4	9	13	18	22
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	18	23
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	9	14	19	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	24
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	15	20	25
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	10	16	21	26
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	5	11	16	21	27
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	22	28

NATURAL TANGENTS

Degrees	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Differences				
	0°·0	0°·1	0°·2	0°·3	0°·4	0°·5	0°·6	0°·7	0°·8	0°·9	1	2	3	4	5
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	2·0057	2·0145	2·0233	2·0323	2·0413	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	3·0061	3·0237	3·0415	3·0595	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	5·0045	5·0504	5·0970	Mean differences cease to be sufficiently accurate.				
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	6·0405	6·1066	6·1742	6·2432					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	7·0264					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	8·0285					
83	8·1443	2636	3863	5126	6427	7769	9152	9·0579	9·2052	9·3572					
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					
90	∞														